Implementation of numerical modelling in anisotropic media by pseudo-spectral method

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ABSTRACT

Numerical simulation or forward modelling of wave propagation is an efficient method to produce test data and examine the mechanism of wave propagation. In this paper, implementation of the numerical modelling in anisotropic media by the pseudo-spectral method is examined, examples of forward modelling in azimuthally anisotropic media are presented. It proves that the pseudo-spectral method is feasible and successful in modelling wave propagation in anisotropic media.

INTRODUCTION

Anisotropy analysis is of great interest to exploration geophysicists. Due to the complexity of anisotropy, analytical solutions are usually not available. Physical and numerical modelling are necessary means for the analysis of wave propagation in anisotropic media. Numerical method is usually preferred when physical modelling is relatively expensive and restrictive.

Numerical modelling in anisotropic media has been a topic for many researchers and several algorithms have been proposed and used in the last decades, including ray tracing method (Guest, 1993), finite-difference method (Carcione, 1990; Dong and McMechan, 1995) and pseudo-spectral method (Lou and Rial, 1995). Pseudo-spectral method uses Fourier transform to compute the spatial derivatives and finite-difference method to compute the time derivative. Compared to other methods, pseudo-spectral method is faster and accurate and easy to implement. It has been successfully applied to the isotropic media case (e.g., Kosloff et al., 1984). Lou and Rial (1995) successfully used the pseudo-spectral method to compute wavefields in 2-D inhomogeneous anisotropic media. In this paper, the pseudo-spectral method is used to compute the wavefields in azimuthally anisotropic media. Examples of forward modelling show that this method is feasible.

PRINCIPLES

The wave equation governing wave propagation in elastic media is:

$$\rho \ddot{u}_i = C_{ijkl} u_{k,lj} + \rho g_i \tag{1}$$

where ρ is the density, u_i is the infinitesimal displacements vector, "$,lj$" denotes the partial derivatives with x_l and x_j, C_{ijkl} is the stiffness tensor, g_i is the body force per unit mass (Cheadle et al., 1991). And C_{ijkl} relate the stress tensor σ_{ij} and strain tensor ε_{kl} as Hooke's law:

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl} \tag{2}$$

To compute the spatial derivative by Fourier transformation, we forward-transform the displacement to the wavenumber domain, perform complex multiplication in the wavenumber domain and then reverse-transform it back to the space domain. For example, the derivative

$$\frac{\partial u_k}{\partial x_j \partial x_l} = -\frac{1}{2\pi}\int_{-\infty}^{+\infty}(k_j)(k_l)U_k(k_i)\exp(-ik_i x_i)dx_i \tag{3}$$

where $U_k(k_i)$ is the Fourier transform of $u_k(x_i)$, which can be calculated by a 3-D FFT, and k_i is the circular wavenumber in the x_i direction.

Based on the second-order finite-differencing approximation of $\ddot{u}_i$ in equation (1), the displacement $u_i(t+\Delta t)$ can be expressed as:

$$u_i(t+\Delta t) = (\Delta t)^2 \ddot{u}_i(t) + 2u_i(t) - u_i(t-\Delta t) \tag{4}$$

where Δt is the sampling interval in time.

The source function for the modelling computation is simulated by introducing a force to the source point and its vicinity. The source force gradually decrease to zero away from the source point to prevent aliasing (Lou and Rial, 1995).

By multiplying a weighting function to make the wave-fields to be attenuated when approaching the boundary, good non-reflecting boundary condition has been achieved. Exponential function proves to be a good weighting function (Cerjan et al., 1985).

Though the pesudo-spectral method is relatively fast, it is still not feasible to run 3-D anisotropic modelling in a Sun workstation, because of the huge computation of 3D modelling.

2-D ANISOTROPIC MODELLING RESULT AND ANALYSIS

2D modelling assumes the model is infinite and identical along x_2 direction and the sources are line sources. Here we present two modeling examples.

Model 1

Model 1 is a homogeneous azimuthally anisotropic model created by introducing a set of fractures that are at an angle of 45° from the x_1 direction. The crack density is 0.07. Other parameters are shown in Figure 1. A source polarized in the x_2 direction is introduced at grid point (128,128). The model is 256 grid point by 256 grid point in size, with the grid size being 25 m by 25 m. The time step is 2 ms. The stiffness tensor is computed by Hudson's theory (Hudson, 1981, 1982; Crampin 1984).

Figure 3 shows the snapshots of the wave propagation in this model. Due to the shear-wave splitting in anisotropic media, waves are also recorded in x_1 direction from a source polarized in x_2 direction. Also boundary reflections is greatly reduced.

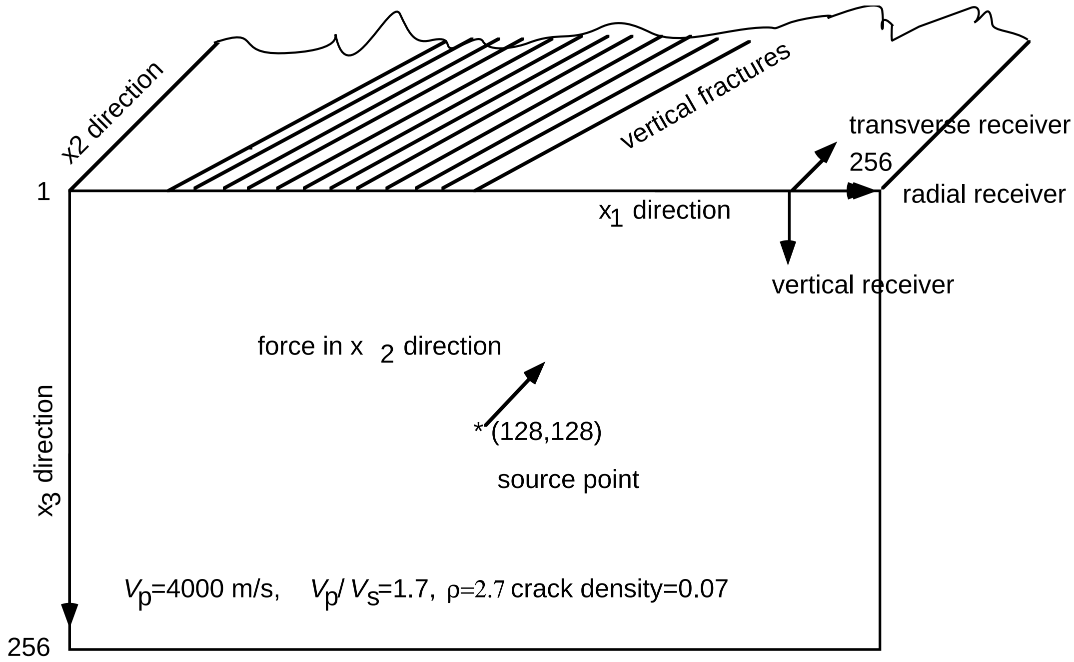

Figure 1. Illustration of model 1.

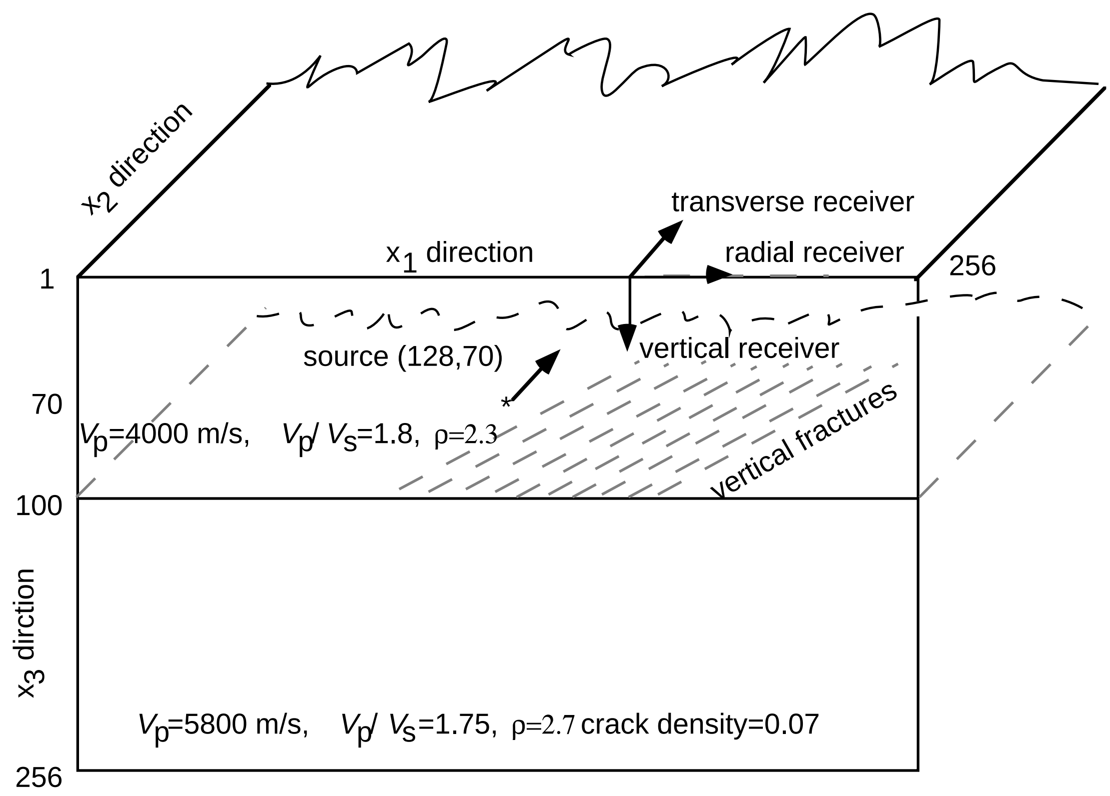

Figure 2. Illustration of model 2.

Model 2

Shown in Figure 2 is a two-layer model with the first layer being isotropic and second layer being anisotropic. The anisotropic layer has the same setting as model 1 and the parameters of the isotropic layer is shown in the figure. The source point is in the isotropic layer and also polarized in x_2 direction. At first, there is only wavefields in x_2 direction, due to the isotropy. Upon the propagation of waves into the second layer, the anisotropic layer, shear-wave splitting occurs and wavefields is recorded in x_1 direction, as shown in Figure 4.

CONCLUSIONS

Pseudo-spectral method proves to be successful in modelling wave propagation in anisotropic media. It is accurate and fast. 3D modelling of wave propagation in anisotropic modelling could be feasible by using super-computer.

ACKNOWLEDGMENTS

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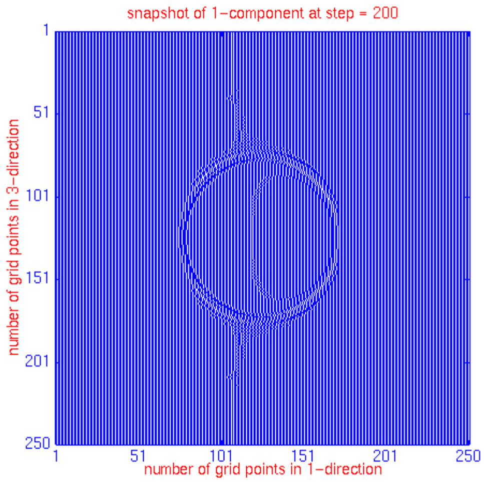

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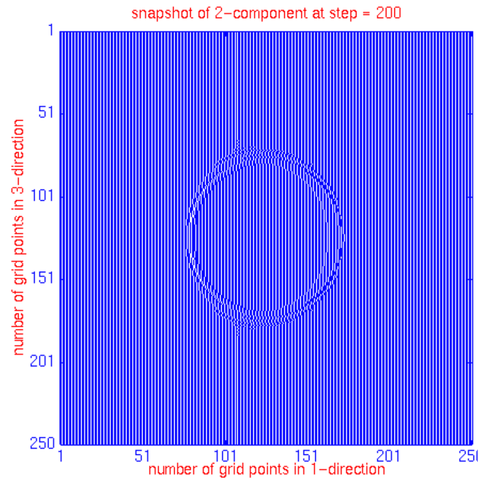

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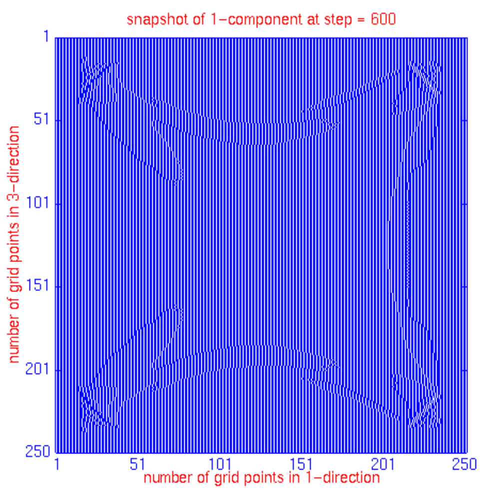

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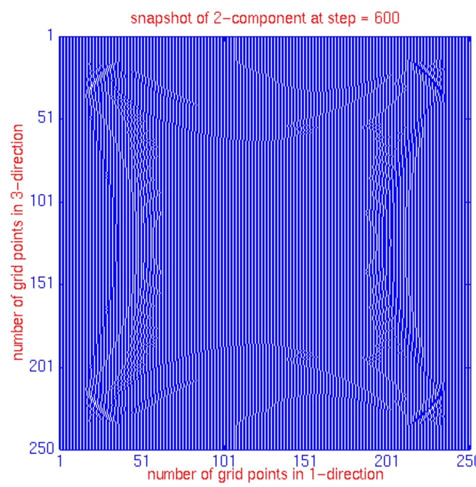

(3.d)

Figure 3. Snapshots of wave propagation in model shown in Figure 1.

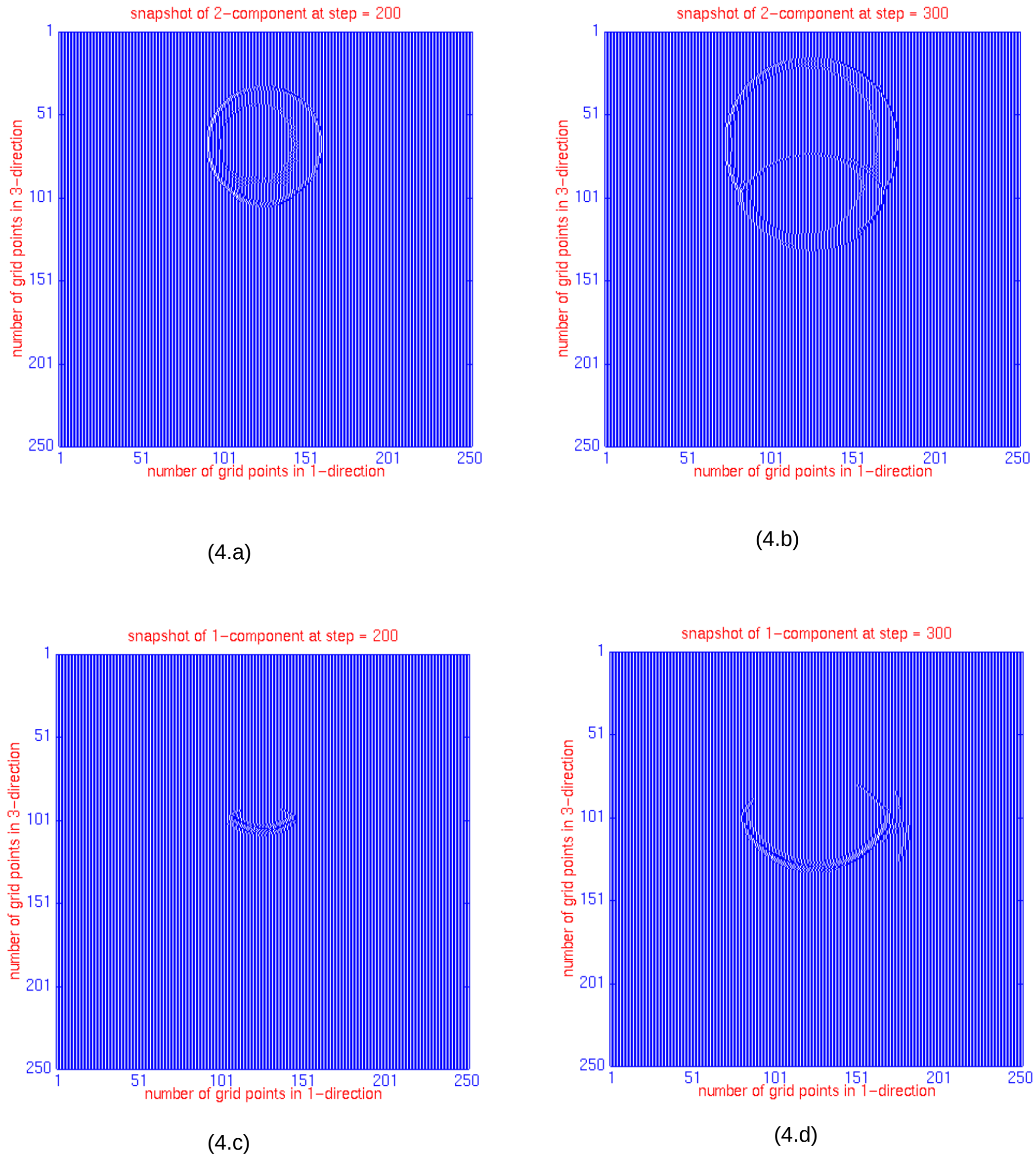

Figure 4. Snapshots of wave propagation in the model shown in Figure 2.