User's guide to elastic wavefield modeling with the ELMO software package

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ABSTRACT

The Elmo software package, available as a Matlab program, is an elastic wavefield modeling package. The software models the earth response of 2-D horizontally layered media for a cylindrical P wave source. Within the Fourier domain, each component plane wave of the source is propagated through a computational grid by phase shift and all multiples and mode conversions can be calculated. The software also offers the flexibility of isolating specific wave types and the ability to decompose a seismogram into component wavefields. Finally, Elmo contains a number of parameters designed to attenuate the wavefield in order to reduce algorithm artifacts.

THEORY

The phase shift cascade method is a form of elastic modeling in 2-D horizontally layered media. Based on the elastic wave equation, the method propagates P and S wave potentials to model an earth response. Elmo (elastic modeling) refers to the package developed within Matlab and described in Silawongsawat and Margrave (1998) and Silawongsawat (1998).

First, the program defines the area in which the model will be constructed in f-k_x space. This region is limited by two variables: maximum frequency and the limiting ray parameter. The first is determined by the user, and has a limit of Nyquist frequency. The second is computed from the velocity included in the model (keeping in mind that k_x/f= ray parameter = $\sin\theta$/velocity).
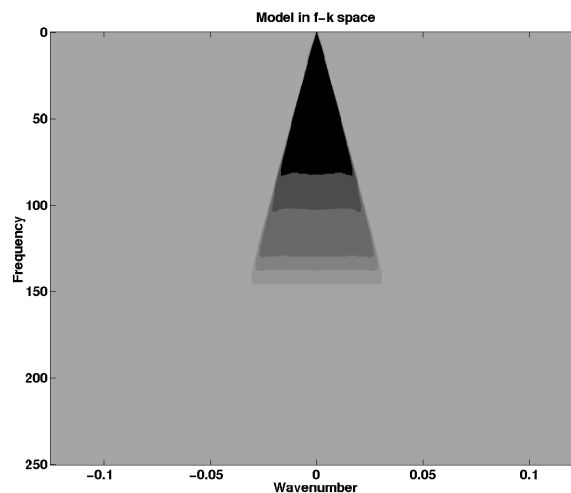

Fig. 1. The area the model will be constructed in f-k_x space

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Once this area has been defined, a complex valued velocity matrix is created. Complex velocities cause a numerical attenuation, whose effect increases with traveltime, and which reduces algorithm artifacts such as Fourier "wrap-around". Within the model area, complex velocities are negligible. At the limiting ray parameter, the percent of complex velocity is gradually increased until the maximum value desired by the user is attained. The order of a Butterworth filter (Figure 2), chosen by the user, defines how quickly the maximum value is attained. A Butterworth filter of order infinity is a step function. There is no limit to the order of the filter, but values less than 100 are suggested.
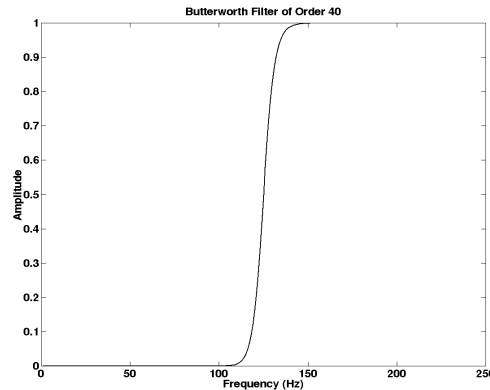

Fig. 2. A Butterworth filter of order 40

Elmo now proceeds to the modeling algorithm. A cylindrical source, always composed of P waves, is decomposed into Fourier plane waves. Plane wave potentials, P and S are advanced in depth by phase shift across each layer of the stratified system. Each point in the shaded triangle of Figure 1 represents a Fourier plane wave that will be contributed to the model. Every Fourier plane wave is propagated through a computation grid and all multiples and mode conversions can be computed in a phase shift cascade (Figure 3).
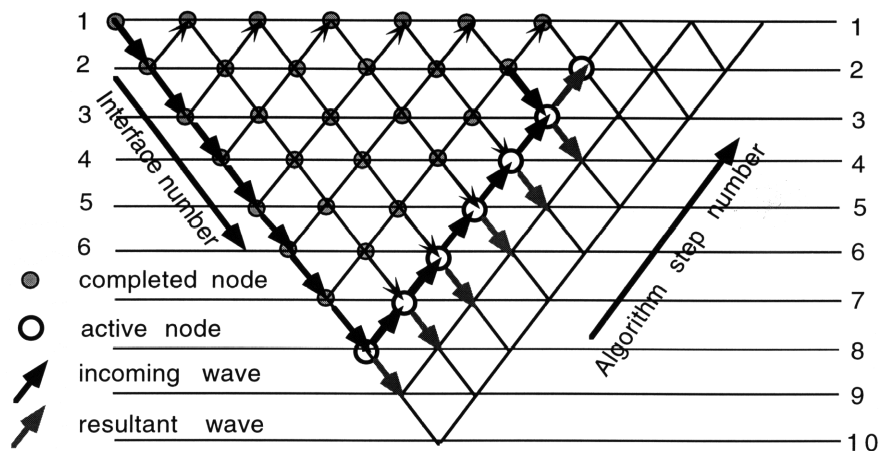

Fig. 3. Plane wave cascade

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At any interface, the four incoming potentials are related to the four scattered potentials (Figure 4a) by the 4x4 scattering matrix, in which scattering coefficients are computed using the exact Zoeppritz equations presented in Aki and Richards (1980). The explicit use of the scattering matrix allows a "partitioned modeling" which can be depth dependent. Partitioned modeling refers to the ability to turn off any wavefield scattering by simply setting elements of the scattering matrix to zero. In this way, various wavefield effects can be isolated and a complete seismic response can be partitioned into component wavefields.

A.
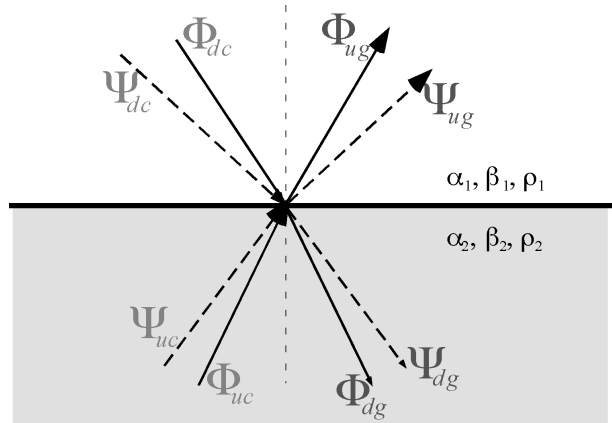
Elastic scattering at an interface. Potentials for P waves (Φ) and S-waves (Ψ) are shown for incident and scattered waves. The subscripts uc and dc refer to "upcoming" and "downcoming" while ug and dg are "upgoing" and "downgoing".

B.

The scattering matrix relates the incident and scattered wavefields. Matrix elements are given by the Zoeppritz equations as adapted for potentials.

Fig.4. (a) Elastic scattering at an interface (b) The scattering equation in matrix form

The notation of the scattering matrix is as follows: T_{sp} is the transmission coefficient for an S wave converting to a P wave, where the S wave is incident from above. T^-_{sp} is similar except the S wave is incident from below. R represents reflections.

After scattering is complete, the elastic seismograms are obtained by inverse Fourier transformation. Either the P and S potentials may be output as "separated" wavefields, or vertical and horizontal displacement components can be computed. The displacements involve derivatives of the P and S potentials and are computed in the Fourier domain as well.

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PROGRAM DESCRIPTION

Elmo has been implemented as a Matlab function. Users will need to purchase both Matlab and the signal processing toolbox separately. The function Elmo has the following format:

```
function [pm,sm,pp,sp,x,t,uz,ux,uzp,uxp]
=elmo(vp,vs,rho,z,dx,dt,params,zmsk,scmas,waveparam);
```

where variables on the left hand side of the '=' are output variables, and those on the right hand side are input variables.

Input Variables

The input values are chosen by the user to define the model and control the output. Default values are the values are effective when the user enters "nan" as a value.

1.   **vp** – vector of P wave velocities for each layer (any length units are acceptable, as long as they are consistent)

2. **vs** – vector of S wave velocities for each layer (in consistent units)

3. **rho** – vector of densities for each layer (in consistent units)

4. **z** – vector of depths for each interface (in consistent units)
    - Note that the first value must always be z=0

5. **dx** – the desired receiver interval for the output seismogram (in consistent units)

6. **dt** – the desired time sample rate for the output seismogram (seconds)

7. **params** – a vector of 11 parameters (will be described fully later in this section)

8. **zmsk** – a 2 element vector indicating the depth zones in which to alter the scattering matrix OR can choose the following scalars (default = 1):

    > (1) full wave effects at all depths

    > (2) P and S wave primaries at all depths

9. **scmas** – a 4x4x3 user defined mask to generate different effects from three depth zones defined by zmsk
    - one 4x4 matrix is composed for each depth zone
    - each 4x4 matrix is a set of ones and zeros which effectively turn "on" or "off" the wave effects described by each element of the matrix in Figure 5
    - a script to construct the 4x4x3 matrix is illustrated in the last example of the Sample Program section

    OR the scalar options (default = 1):

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(1) full wave effects in the defined zone with primaries elsewhere

(2) primaries and multiples without mode conversions inside, and primary waves outside the zone of interest

(3) P and S wave primaries inside, and transmissions without conversions outside the zone of interest

$$
\begin{bmatrix}
T_{pp} & T_{sp} & R_{pp}^{-} & R_{sp}^{-} \\
T_{ps} & T_{ss} & R_{ps}^{-} & R_{ss}^{-} \\
R_{pp} & R_{sp} & T_{pp}^{-} & T_{sp}^{-} \\
R_{ps} & R_{ss} & T_{ps}^{-} & T_{ss}^{-}
\end{bmatrix}
$$

Fig.5. The scattering matrix

waveparam – an input vector [f1 fw1 f2 fw2]defining a wavelet with gaussian tapers
- only used if params(4)=2
- if params(4)=1, waveparam will be ignored, but values must be entered to run the program ([0 0 0 0] can be used)
- the shape of the wavelet is specified by:

f1 – low frequency at which the gaussian taper begins (in Hz)

fw1 – 1/e width of gaussian taper on low end (Hz)

f2 – high frequency at which the gaussian taper begins (Hz)
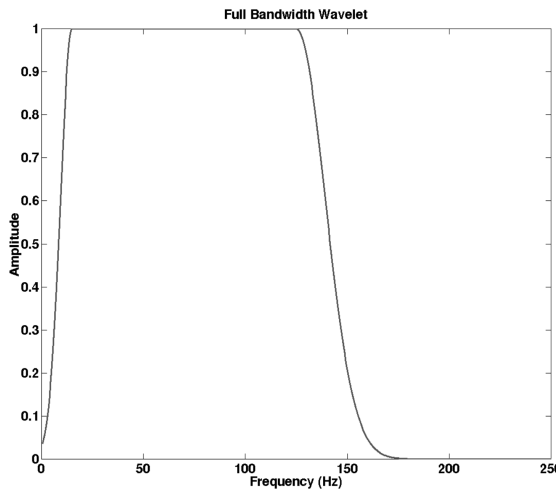
fw2 – 1/e width of gaussian taper on high end (Hz)

Fig. 6. Wavelet with waveparam=[15 8 125 20] for Nyquist frequency of 250 Hz

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The Params Vector

The input vector 'params' has 11 components that specify different parameters within the model.

params=[xmax, tmax, fmax,wavelet,nbutter,raypmax,…

rpbutter,percvim,dspflag,fsflg, tshift];

The components of "params" are:

1. **xmax** – the maximum offset of the seismogram (consistent length units)
   - default = 120*dx

2. **tmax** – the maximum output time of the seismogram (s)
   - default = 1.5*two way travel time

3. **fmax** – the maximum fraction of Nyquist frequency
   - the gaussian wavelet requires fmax =1.0 (100% or fmax = $f_{Nyquist}$)
   - default = 0.6*fNyquist

4. **wavelet** – can choose two scalars (default = 1):

   (1) bandlimited wavelet (Figure 7)– Hanning wavelet centered at fmax/2
       - values entered for waveparam will be ignored

   (2) gaussian wavelet (Figure 6) - wavelet with gaussian tapers on spectral limits
       - determined by waveparam=[f1 fw1 f2 fw2]
       - params(3) must = 1 (fmax=$f_{Nyquist}$)
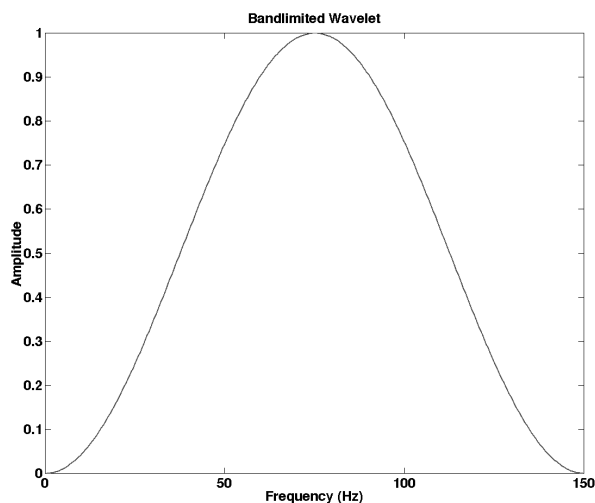   - note that the gaussian wavelet will increase run times considerably because all frequencies must by computed

Fig. 7.Bandlimited wavelet for fmax=0.6*fNyquist where Nyquist frequency is 250 Hz

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5. **nbutter** - order of the Butterworth filter (integer) which controls the onset of the numerical attenuation
   - default = 90
   - the larger the order, the sharper the transition from no dispersion to maximum dispersion

6. **raypmax** – the offset at which the maximum ray parameter is calculated at
   - expressed as a fraction of xmax
   - default = 1.1*xmax

7. **rpbutter** – the offset to compute the ray parameter at which complex velocity is turned "on" and attenuation begins
   -this offset occurs at the center of the Butterworth shape
   - expressed as fraction of xmax
   - default = 0.95*xmax

8. **percvim** – the maximum value of complex (imaginary) velocity to be used in attenuation
   - expressed as a fraction of the real velocity
   - default = 0.3*vcomplex

9 . **displacement flag** – scalar value that indicates the potentials and/or displacements to be returned
   - default = 0

   (0) P and S potentials

   (1) P and S potentials, and horizontal and vertical displacements

   (2) Horizontal and vertical displacements

10. **free surface effect flag** – this is under development, and must be set to = 0. Elmo does not yet compute a free surface effect.

11. **tshift** – a time shift of the seismogram to reduce temporal wraparound
    - input in seconds
    - default = 0

Output Variables

Again, the function Elmo has the following format:

```
function [pm,sm,pp,sp,x,t,uz,ux,uzp,uxp]
=elmo(vp, vs, rho, z, dx, dt, params, zmsk, scmas,waveparam);
```

The 10 output variables are:

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1. **pm** – the P wave potential for primaries and multiples
    - a matrix of one trace (offset) per column
    - are P waves at the receivers, but may have mode conversions anywhere else

2. **sm** – the P-S wave potential for primaries and multiples
    - are S waves at the receivers

3. **pp** – the P wave potential for primaries
    - are P waves at the receivers
    - there are no reflections when incident from below
    - may have P-S or S-P on transmission

4. **sp** – the P-S wave potential for primaries
    - are S waves at the receivers
    - there are no reflections when incident from below
    - may have P-S or S-P on transmission

5. **x** – the offset vector for the seismogram

6. **t** – the time vector for the seismogram

7. **uz** – vertical displacement

8. **ux** – horizontal displacement

9. **uzp** – vertical displacement of primaries

10. **uxp** – horizontal displacement of primaries

Any of the matrices 1 through 4 and 7 through 10 can be conveniently viewed with "plotimage". For example:

```
plotimage(pm,t,x);
```

will display the P wave potential for primaries and multiples.

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SAMPLE PROGRAMS

The two synthetic models in Figure 8 will be used to illustrate how to use Elmo.

Model #1

z=0 m

| vp=2000 m/s |
| vs=980 m/s |
| rho=1.0 g/cm3 |

z=500 m

| vp=2700 m/s |
| vs=1300 m/s |
| rho=1.0 g/cm3 |

Model #2

vp=2000 m/s	z=0 m
vs=1000 m/s	till
rho=2.2 g/cm3	

vp=6000 m/s	z=50 m
vs=3000 m/s	basalt
rho=2.9 g/cm3	

vp=2500 m/s	z=200 m
vs=1250 m/s	sediment
rho=2.5 g/cm3	

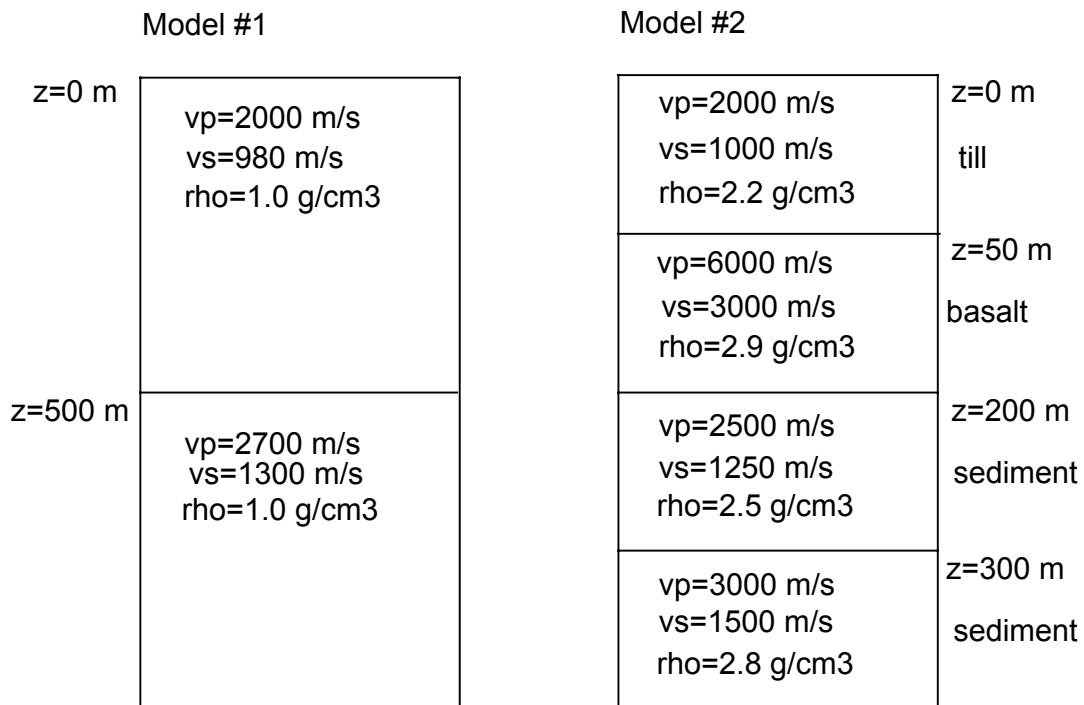| vp=3000 m/s | z=300 m |
| vs=1500 m/s | sediment |
| rho=2.8 g/cm3 | |

Fig. 8. Configuration of two models where: vp is P wave velocity, vs is S wave velocity, rho is density, and z is the depth

The first model has a receiver spacing of 10 m and a time sample rate of 2 milliseconds. The second model has a receiver spacing of 1 m and a time sample rate of 1 millisecond.

A script to generate the potentials and multiples from the first model would be:

```
z=[0; 500];                          rho=[1; 1];
vp=[2000; 2700];                     vs=[980; 1300];
params=[1200 1.2 1.0 2 40 nan 0.5 0.8 1 nan nan];
waveparam=[15  8 125 20];
zmsk=1;              scmas=1;
[pm,sm,pp,sp,x,t,uz,ux,uzp,uxp]=elmo(vp,vs,rho,z,dx,dt,params,zmsk, scmas,waveparam);
plotimage(pm,t,x);
```

Params assigns the following parameters:

```
xmax=1200m                   rpbutter=0.5*xmax
tmax=1.2 s                   percvim=0.8*vcomplex
fmax=1*f_Nyquist             dspflag=1  = (potentials and displacements returned)
wavelet=2 = (full bandwidth) fsflag=nan = (off)
nbutter=40                   tshift=nan (0 second shift)
raypmax=nan  = (1.1*xmax)
```

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The waveparam vector specifies the bandwidth and tapers on the wavelet. Finally, zmsk and scmas set the program to return all primaries and multiples. Model 1 has only one interface, and so there will only be reflections off of that layer. No response from the second layer will be seen.

The parameters set above were decided by testing a number of different values to determine which were most appropriate for this model. It was found that when the values for fraction of complex velocity (percvim) and the offset at which the filtering begins (rpbutter) are set correctly, the values of xmax, tmax, and nbutter (the order of the Butterworth filter) have little effect on the quality of the data. One should keep in mind, however, that larger values of xmax and tmax prolong the processing time. The importance of choosing rpbutter and percvim is illustrated in Figures 9 and 10.
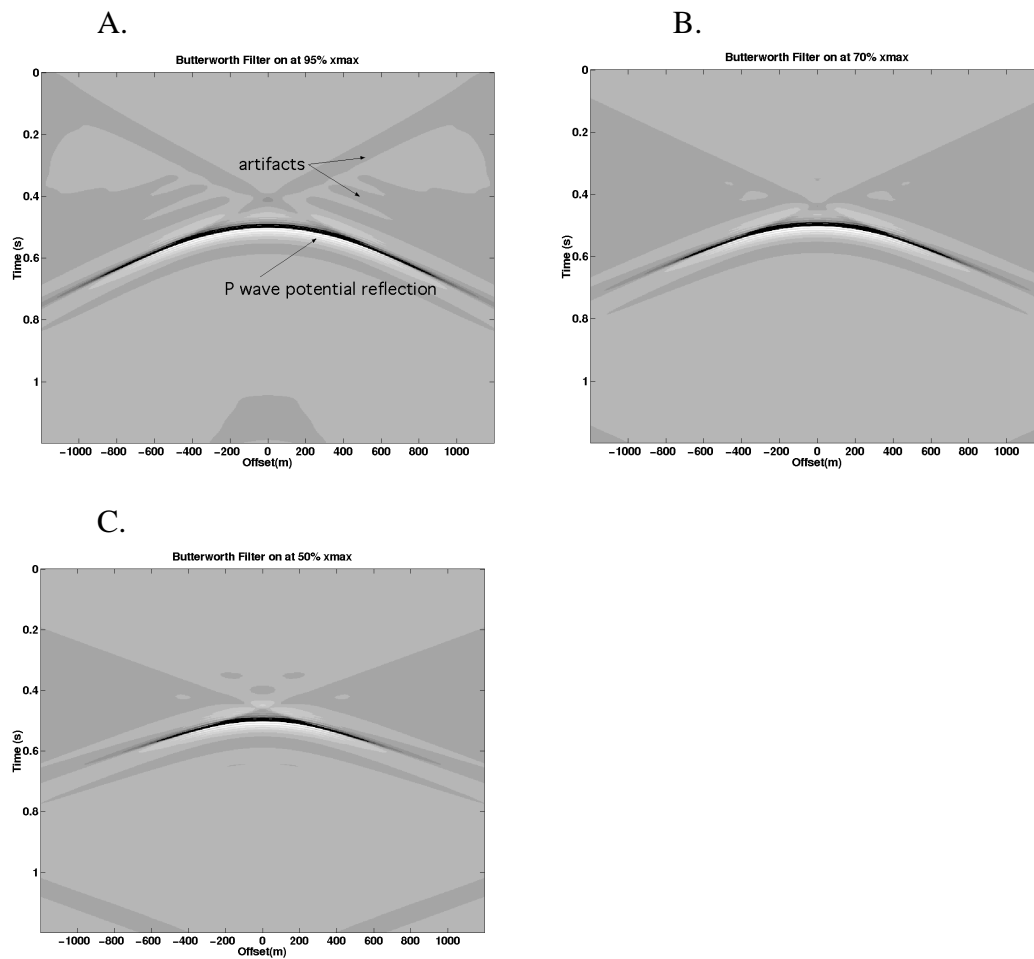
A.

B.

C.

Fig. 9.(a) P Potentials for Butterworth filters (which control complex velocity) beginning at ray parameter at 95% xmax (b) 70% xmax (c) 50% xmax

In Figure 9, the artifacts are fewer for models (b) and (c). Note that because of the attenuation, model (c) is only reliable up to an offset of 0.5*xmax.
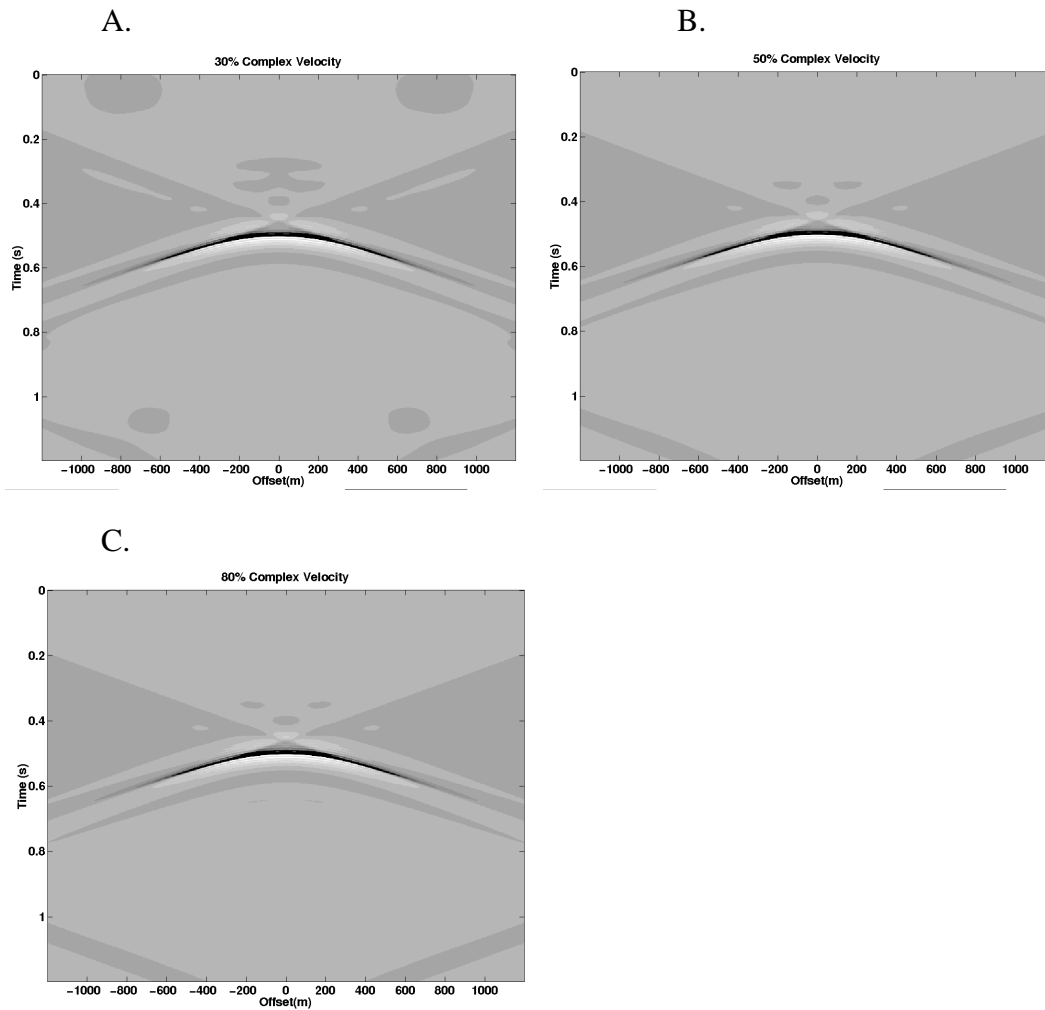
Contents

A.

B.

C.

Fig. 10. The P Potentials of models containing (a) 30% complex velocity (b) 50% complex velocity (c) 80% complex velocity

In Figure 10, note the decrease in artifacts for models (b) and (c).

It can be seen from Figures 9 and 10 that a value of rpbutter = 50% xmax and percvim = 80% give images with the least amount of artifacts. Therefore, parameters that control the attenuation are the most important. A comparison of the P potential for model 1 with and without attempts to control artifacts is seen in Figure 11. Because of attenuation, model (b) is accurate only to an offset of 0.5*xmax, while model (a) is accurate for all offsets.

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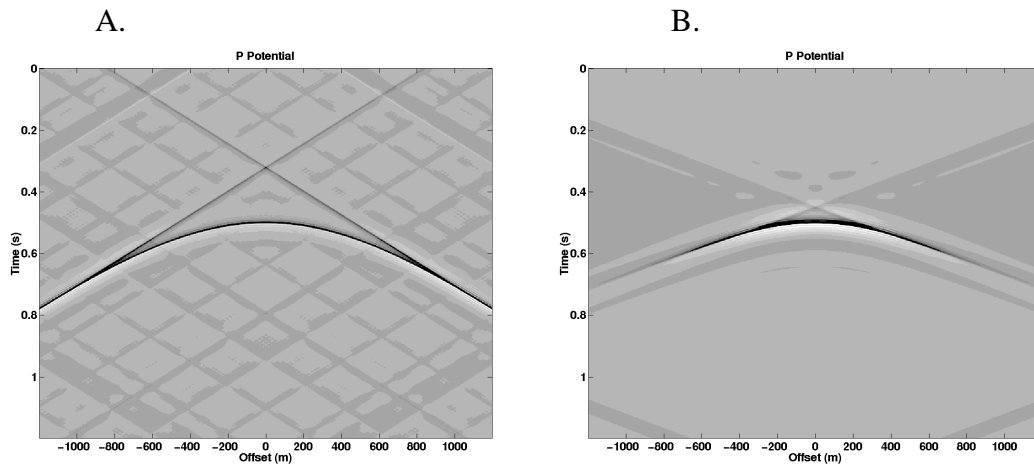A.

B.

Fig. 11. P Potential with (a) no attenuation (b) attenuation

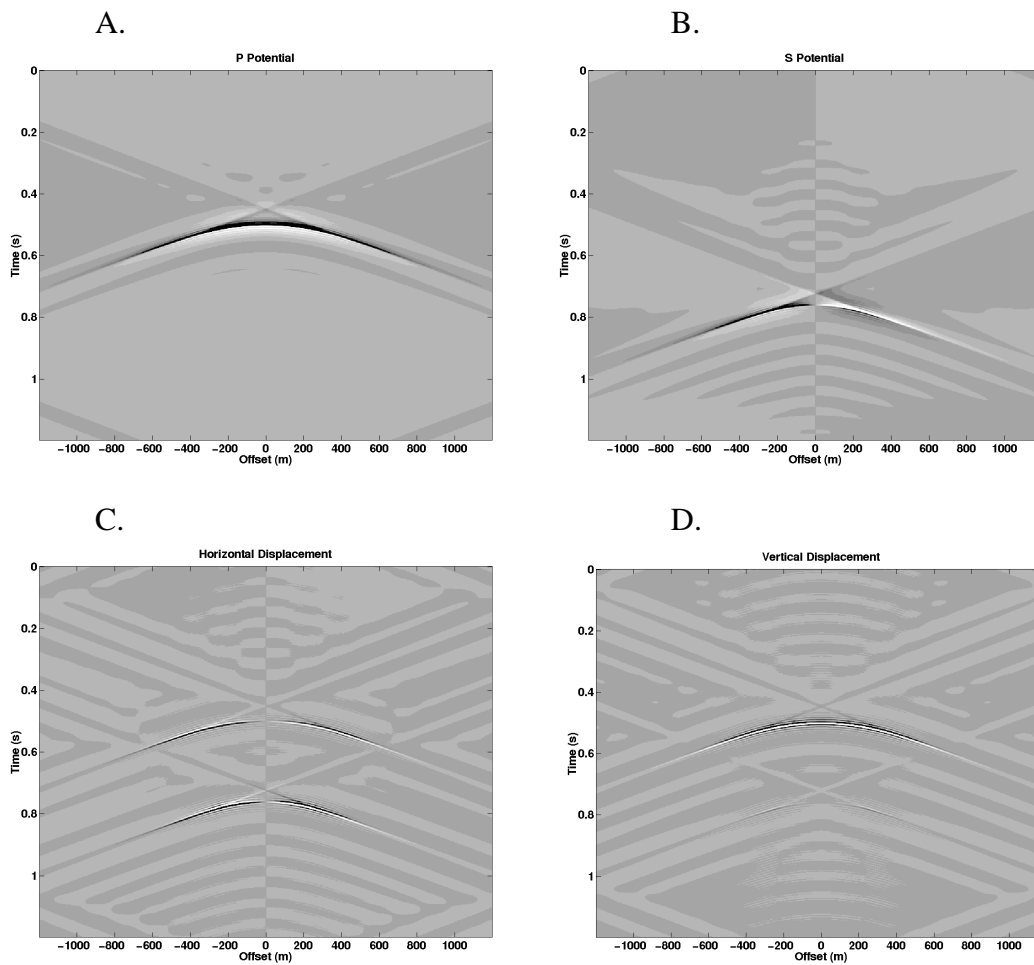The full results for model 1 are shown in Figure 12.

A.

B.

C.

D.

Fig. 12. The results for Model #1

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The second model uses the same parameters as the first model, with the exceptions of those that describe the model layers and the wavelet. Because the second model is more complicated, and has finer sampling rates, the processing time is longer. In an effort to reduce the time, a bandlimited wavelet of maximum frequency 60% of the Nyquist frequency is chosen.

The P potential (Figure 13a), has a larger offset scale than necessary. This can be used as another way to reduce artifacts. Calculating the potential or displacement over a larger area than needed, and then viewing only portion of that image will cut out many artifacts. The results for second model in Figure 14 focus in on the area of interest.

The dynamic range of Figure 13a is much larger than the range used in the more conventional display of wiggle traces. This means that a much wider range of responses will be visible. The comparison in Figure 13b illustrates that the artifacts present are negligible on the more customary display. Therefore, it is important to realize that though the artifacts exist, they are not as problematic as one might expect when viewed on a more familiar scale.
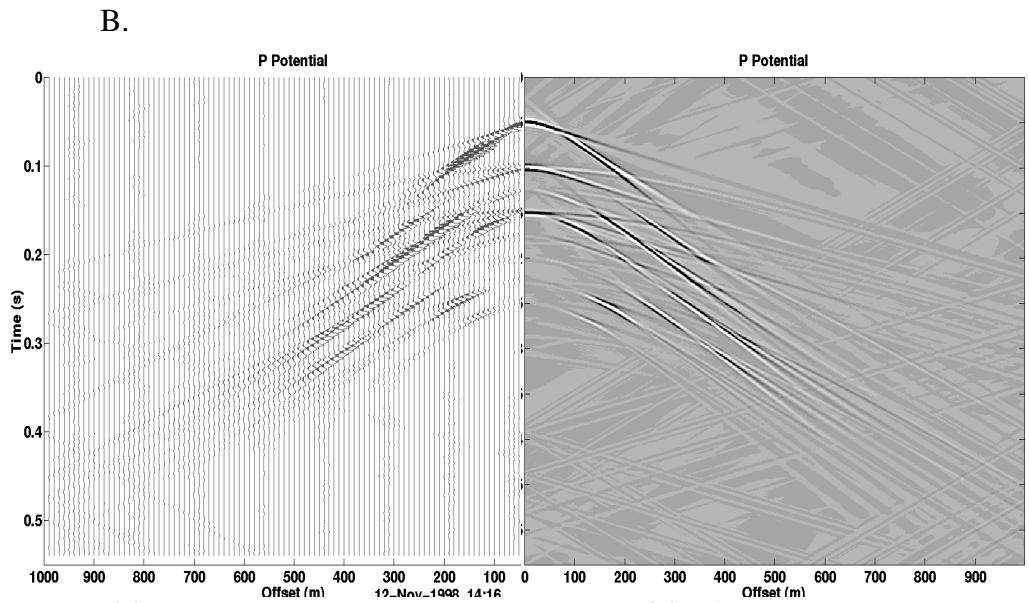
A.

Contents

B.
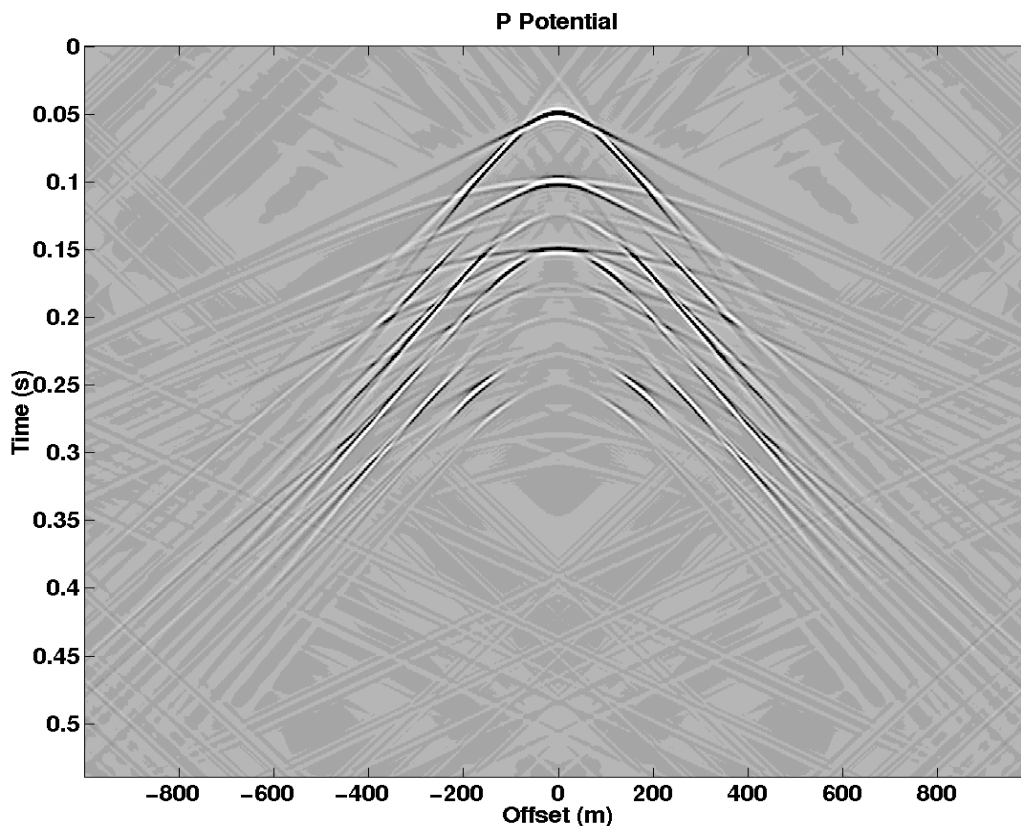
Fig. 13. (a) P Potential displayed using 'plotimage' (b) left hand side displayed in wiggle traces(using 'plotseis') and right hand side displayed using 'plotimage'
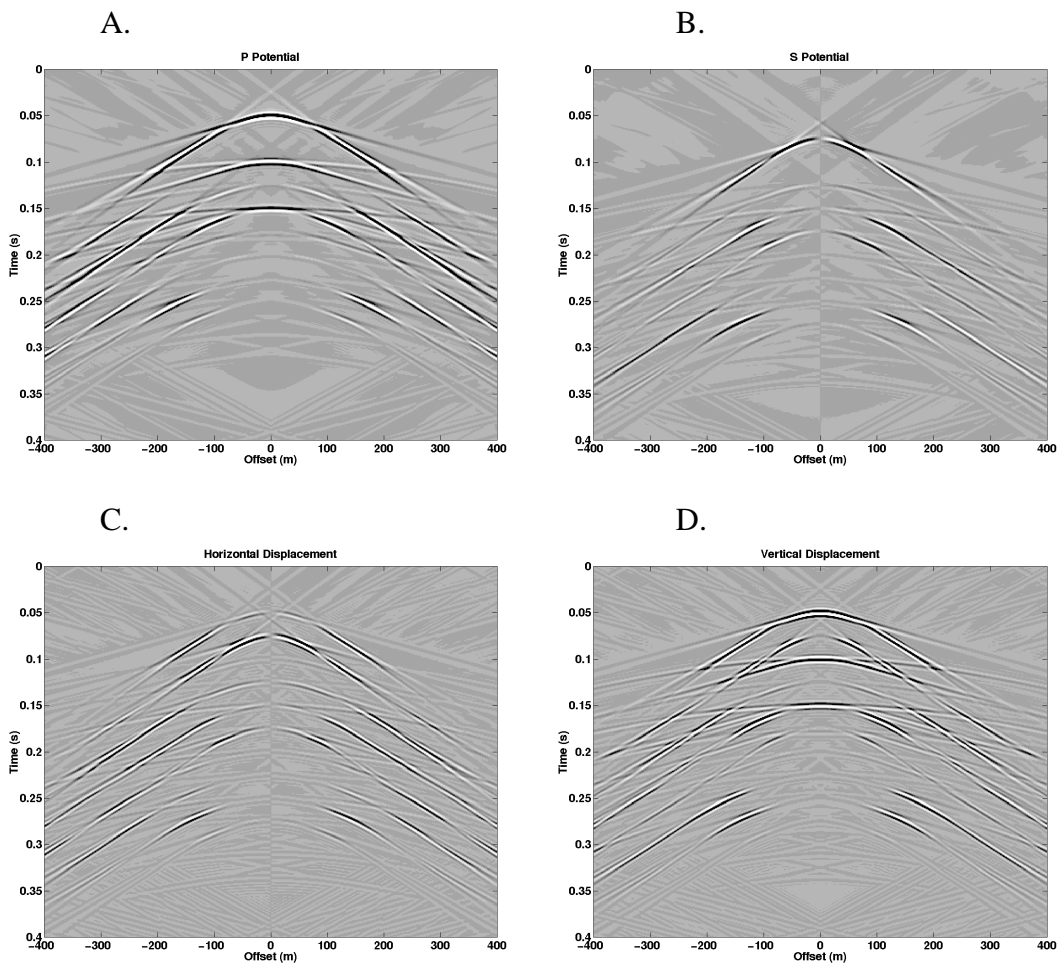
A.

B.

C.

D.

Fig. 14. The results of Model #2

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The next step was to test some of Elmo's masking features. The new parameters are:

<div align="center">

zmsk=[200 300];   scmas=1;

</div>

where scmas=1 returns full effects inside the zone of interest (defined by zmsk), and primary waves outside the zone of interest. The intent is to image the response of the third layer, and to prevent obscuring this by the multiples and mode conversions generated by the second (basalt) layer.
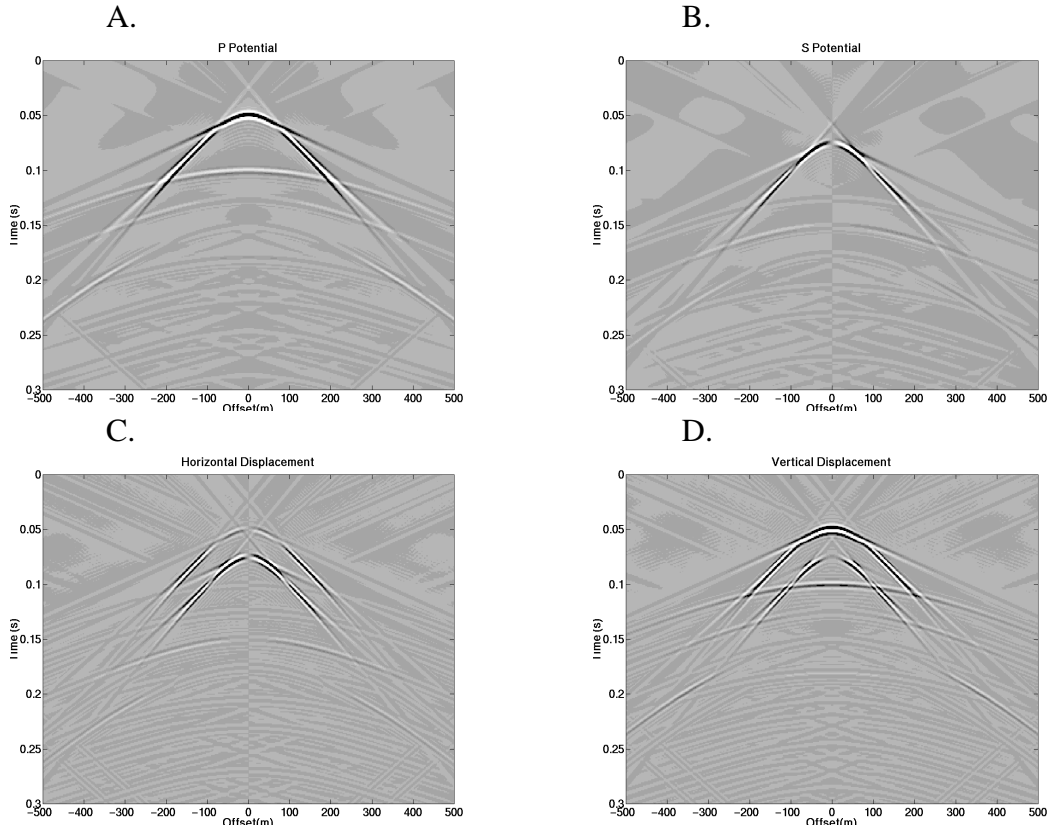
A.

B.

C.

D.

Fig. 15. The results for the full responses from layer 3, and primaries elsewhere generated by Model #2

The final variation uses the subtraction scheme (Figure 16) outlined in Silawongsawat (1998) to isolate multiples and mode conversions of the first two interfaces. Specifying zmsk=[50 200] isolates the responses from these interfaces.

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First Model

$$\begin{bmatrix} T_{pp} & 0 & 0 & 0 \\ 0 & T_{ss} & 0 & 0 \\ 0 & 0 & T_{pp}^- & 0 \\ 0 & 0 & 0 & T_{ss}^- \end{bmatrix}$$ **Zone 1 Transmissions only**

$$\begin{bmatrix} T_{pp} & T_{sp} & R_{pp}^- & R_{sp}^- \\ T_{ps} & T_{ss} & R_{ps}^- & R_{ss}^- \\ R_{pp} & R_{sp} & T_{pp}^- & T_{sp}^- \\ R_{ps} & R_{ss} & T_{ps}^- & T_{ss}^- \end{bmatrix}$$ **Zone 2 Full response**

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ **Zone 3 Turned off**

Minus

Second Model

$$\begin{bmatrix} T_{pp} & 0 & 0 & 0 \\ 0 & T_{ss} & 0 & 0 \\ 0 & 0 & T_{pp}^- & 0 \\ 0 & 0 & 0 & T_{ss}^- \end{bmatrix}$$ **Zone 1 Transmissions only**

$$\begin{bmatrix} T_{pp} & 0 & 0 & 0 \\ 0 & T_{ss} & 0 & 0 \\ R_{pp} & 0 & T_{pp}^- & 0 \\ 0 & R_{ss} & 0 & T_{ss}^- \end{bmatrix}$$ **Zone 2 Primaries only**

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ **Zone 3 Turned off**
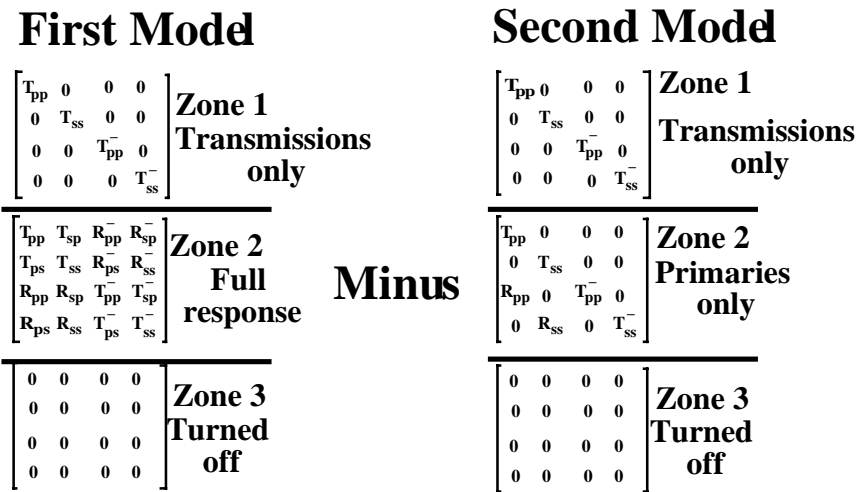
Fig. 16.Subtraction scheme to isolate multiples and mode conversions in zone 2 described by Silawongsawat (1998)

The script to image the multiples and mode conversions of zone 2 is:

```
z=[0; 50; 200; 300];                            rho=[2.2; 2.9; 2.5; 2.8];
vp=[2000; 6000; 2500; 3000];                    vs=[1000; 3000; 1250; 1500];
dx=1;                                           dt=0.001;
params=[1000 nan 0.6 1 40 nan 0.5 0.8 1 nan nan];
waveparam=[0 0 0 0];
zmsk=[50 200];                   %specifies interfaces at depth 50m and 200m be included

% create the mask for the first model illustrated in Figure 16

sc1=[1 0 0 0; 0 1 0 0; 0 0 1 0; 0 0 0 1];       %the 4x4 mask for zone 1
sc2=[1 1 1 1; 1 1 1 1; 1 1 1 1; 1 1 1 1];       %the 4x4 mask for zone 2
sc3=[0 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0];       %the 4x4 mask for zone 3

% create the mask for the second model illustrated in Figure 16

sc1b=[1 0 0 0; 0 1 0 0; 0 0 1 0; 0 0 01];       %the 4x4 mask for zone 1
sc2b=[1 0 0 0; 0 1 0 0; 1 0 1 0; 0 1 0 1];      %the 4x4 mask for zone 2
sc3b=[0 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0];      %the 4x4 mask for zone 3
scmas1=cat(3,sc1,sc2,sc3);       %"cat" concatenates matrices into a single matrix
scmas2=cat(3,sc1b,sc2b,sc3b);
%generate response for first model
[pm1,sm1,pp1,sp1,x1,t1,uz1,ux1,uzp1,uxp1]= …
elmo(vp,vs,rho,z,dx,dt,params,zmsk, scmas1,waveparam);

%generate response for second model
[pm2,sm2,pp2,sp2,x2,t2,uz2,ux2,uzp2,uxp2]= …
elmo(vp,vs,rho,z,dx,dt,params,zmsk,scmas2,waveparam);
```

The P and S primaries are found in the matrices pp2 and sp2 respectively. To view the P multiples and mode conversions, subtract pp2 from pm1. Similarly, subtract sp2 from sm1 for the S multiples and mode conversions.

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The multiples and mode conversions from the first two interfaces are seen in Figure 17. If only the response from the second interface is desired, a mask of zmsk=[51 200] will give the desired results.
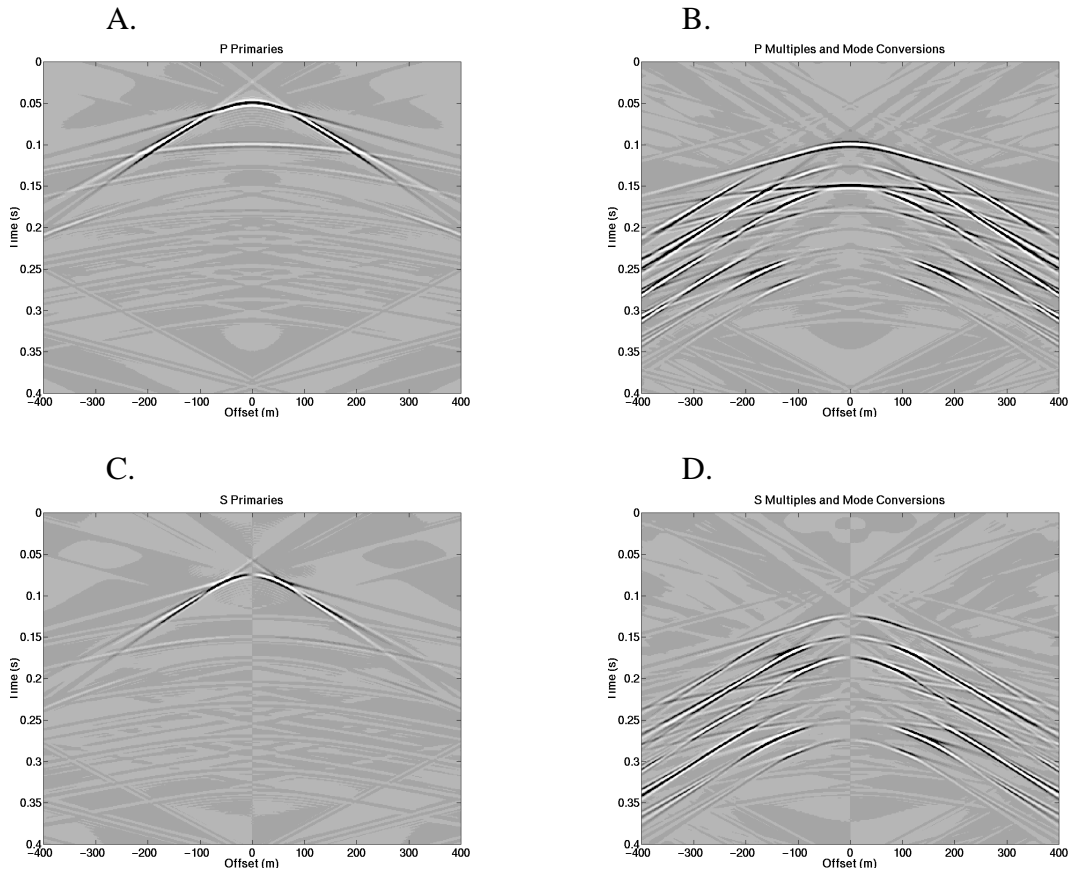
A.

B.

C.

D.

Fig. 17. The responses from the first two interfaces: (a) P primaries (b) P multiples and mode conversions (c) S primaries and (d) S multiples and mode conversions

CONCLUSIONS

Elmo, available as a Matlab program, is an elastic modeling package for 2-D horizontally layered media. Based on the elastic wave equation, the method propagates P and S wave potentials to model an earth response. Because the algorithm uses wave potentials, displacements can also be computed if desired. As well, the explicit use of the scattering matrix enables "partitioned modeling". The complete seismic response can be partitioned into component wavefields, and various wave effects can be isolated.

The software contains three parameters that reduce algorithm artifacts by inducing numerical attenuation. These parameters include the order of the Butterworth filter, the ray parameter at which filtering begins, and the maximum fraction of complex velocity. The order of the Butterworth filter is recommended to be less than 100. It was found that when the values for fraction of complex velocity and the ray

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parameter at which filtering begins are set correctly, all other parameters have little effect on the quality of the data.

In order to effectively reduce artifacts, the complex velocity may need to be introduced at a lower offset (0.5*xmax rather than 0.95*xmax). If this is the case, it is important to realize that the potentials and/or displacements will only be accurate up to the offset at which attenuation becomes effective.

The means of displaying the seismogram can change the apparent effect of artifacts. The 'plotimage' function, chosen for its fast processing time, has a larger dynamic range than users are familiar with. The image of more conventional wiggle traces shows the artifacts to be less prominent than one would expect.

Elmo allows the user control over the parameters that determined the output seismogram. This flexibility allows the reduction of artifacts, the ability to balance detail against processing time, and the opportunity to separate specific wavefields or responses. Elmo can handle a complete range of geology: from simple synthetic models to information from well logs.

ACKNOWLEDGMENTS

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Silawongsawat, C., and Margrave, G.F., 1997, Elastic modeling by phase-shift cascade with scattering matrix control, 1997 Annual Research Report of the CREWES Project.

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