P/S wavefield separation in the presence of statics

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ABSTRACT

Existing wavefield separation techniques for 4C ocean-bottom cable data are formulated with the assumption that the earth is essentially one-dimensional at the sea-bottom; i.e. the equations assume that the sea floor is flat and that the medium properties of the ocean bottom are homogeneous. These assumptions are often violated. For example, shear-wave receiver statics often exist, which are an indication of rapid variation in shear velocity within a wavelength along the sea floor. It would appear, therefore, that a method that accounts for a heterogeneous sea floor is needed. However, this is only true if the analysis is performed on common-shot gathers. Performing the decomposition on common-receiver gathers is a simple solution to this problem. All traces in the receiver gather are subject to the same S-wave and P-wave receiver static, so the sea-floor appears to be homogeneous in this domain. It is necessary to perform the analysis on a common-shot gather only if the spatial sampling within each common-receiver gather is coarse enough to cause data aliasing. This is often the case in land 2D acquisition, where the source interval is usually several times larger than the receiver interval. A technique for performing the wavefield separation on shot gathers in this situation is presented here. The approach assumes that a homogeneous layer exists below the heterogeneous near-surface layer that causes the statics. Localized ray-tracing is used to model the upgoing wave propagation through the heterogeneous layer to each receiver. The forward modeling equations are inverted in order to resolve the plane-wave components of the P and S wavefields at a fixed depth below the heterogeneous layer. The near-surface medium properties, as well as the different geophone component response functions, are required input parameters to this decomposition.

INTRODUCTION

Many methods for achieving multiple attenuation on dual-sensor ocean-bottom cable (OBC) data by combination of the hydrophone and geophone recordings have been proposed over the last several years (e.g. Barr and Sanders, 1989; Paffenholz and Barr, 1995; Soubaras, 1996; Ball and Corrigan, 1996; Bale, 1998; Jiao et al., 1998). There is similar interest in P and S-wave separation and multiple attenuation on 4-C OBC data, since the hydrophone and three components of the geophone all record receiver-side reverberations in the water column (e.g. Wapenaar et al., 1990; Amundsen and Reitan, 1995; Osen et al., 1996; Schalkwijk et al., 1998; Soubaras, 1998).

Generally, the equations that are used to describe the data for each of these algorithms have gradually been getting closer to modeling physical reality. For example, several methods now exist for taking into account the differences between hydrophone and geophone response functions and variable coupling to the sea floor, whereas they were not considered in the earliest algorithms. In addition, algorithms now exist for taking into account non-vertical propagation angles, whereas vertical rays were assumed initially. The formulas that are used in existing methods generally assume, either implicitly or explicitly, that the ocean bottom is one-dimensional in character. In other words, it is assumed that the sea floor is flat, and that the medium properties (P-wave velocity, S-wave velocity and density) in the layer immediately below the ocean bottom are essentially constant within a record length. Recently, attention has been paid to formalisms for overcoming this assumption (Amundsen et al., 1998), and to variations of existing techniques in order to handle local variations in medium properties at least to some extent (Osen et al., 1998).

In this paper a new approach for overcoming the 1-D assumption is proposed, but first I analyse when and why this alternative approach would be needed. It turns out that a heterogeneous ocean bottom is not as much of a problem as it would at first appear to be.

WHY HOMOGENEITY?

In the literature on P/S separation on 4C data, results of algorithms on synthetic data are usually demonstrated on common-source gathers, but real data examples are demonstrated on common-receiver gathers, with little or no explanation of why the switch in domains has been made. The paper by Schalkwijk et al. (1997) is an exception. In explanation of their switch to a common-receiver gather in their real data analysis they state that they have assumed that the sub-sea-bottom is more or less one-dimensional, so they could consider their common receiver gather to be a common source gather. They seem to have invoked an argument based on reciprocity of shot and receiver gathers when near-surface layers are one-dimensional. However, it is not obvious that this reciprocity argument is correct, or even that the argument is necessary in order to justify performing P/S separation on receiver gathers. If the reciprocity argument is made, it is likely to be incorrect. The vector reciprocity theorem essentially does not apply to P-S converted waves due to the fact that the polarisation of the S-wavefield at the receiver is almost perpendicular to the polarisation of the P-wavefield at the source (Thomsen, 1998; Knopoff and Grangi, 1959).

In actual fact, the issue of the reciprocity of shot gathers and receiver gathers is irrelevant to the P/S separation issue. The fundamental reason that homogeneity of the sub-sea-bottom layer is important for P/S separation is because the equations that are used for decomposition into P and S upgoing and downgoing waves are formulated in terms of plane waves (in the Fourier or Radon domain), and the plane-wave decomposition can only be done correctly on a data gather that "senses" that the sea floor is homogeneous.

Although the mathematical procedures for transforming data into plane waves with Fourier or Radon transforms can be performed on any type of data gather, the physical interpretation of the transformed data as plane wave components arriving at the receivers is only correct when the receivers are located in a layer in which velocity, v, is laterally constant (Treitel et al., 1982). Only then do the well-known relations ($p = \sin\theta / v = k / 2\pi f$) between ray parameter, p, horizontal waveslowness,

$\sin\theta/v$, frequency, f, and wavenumber, k, strictly apply. Consequently, if the static model that assumes vertical raypaths through the near-surface layer is reasonably correct, all traces in a common-receiver gather are subject to the same static delay on passing through the sub-sea floor layer. For all intents and purposes, each common-receiver gather "senses" a homogeneous sea floor.

Consequently, common-receiver gathers yield reliable plane-wave decompositions of the wavefield recorded at each receiver when the near-surface is heterogenous, as long as there are enough sources recorded by each receiver to prevent data aliasing. Receiver gathers are usually well sampled in 2D, 4C OBC surveys, so there is no problem for marine 2-D data. However, data aliasing of receiver gathers often occurs with 2-D land data because the source interval is typically 4 or 5 times the receiver interval. So in this situation a method is required that can work on shot gathers, which are usually well sampled, but are subject to the heterogeneous near-surface problem.

The situation for both marine and land 3-D multicomponent data acquired with orthogonal shot and receiver lines is different. In this case, both shot and receiver gathers will usually be well-sampled in either the inline or crossline direction, but aliased in the orthogonal direction. Cross-spreads, which are well-sampled in both directions, are the natural sub-set of the data to analyse in this situation. Since cross-spreads involve traces from multiple receivers, a method that performs P/S separation in the presence of P-wave and S-wave statics will be required for 3-D land and marine multicomponent data, as well as for 2-D land data.

In the next section, a method that applies to the 2-D land situation is outlined. The extensions to the 3-D land and marine situations have not yet been done, but will use many of the same concepts presented here.

P/S SEPARATION WITH STATICS

For the problem of wavefield separation at a solid/air interface, Dankbaar (1985) and Donati (1996) apply separation filters to the vertical and radial geophone components that have been transformed to either the $f-k$ or the $\tau-p$ domain. We assume that the plane-wave decomposition has to be performed on common-shot gathers because common-receiver gathers are aliased. The P-P and P-S reflections recorded by both the vertical and radial components on a single shot gather can have very different receiver statics, so in order to obtain a correct plane-wave decomposition, P-statics and S-statics have to be applied beforehand. But how can two different sets of receiver statics be applied simultaneously to the same traces?

This problem can be resolved by computing the $\tau-p$ transforms of the P-P and P-S wavefields below the near-surface layer causing the statics, instead of at the surface. After separation, the P-P and P-S wavefields in the x,t domain can be reconstructed below the near-surface layer by inverse $\tau-p$ transforms. The problem can also be solved using Fourier transforms instead of Radon transforms. A reasonably homogeneous layer is assumed to exist below the heterogeneous layer that causes the statics. If this is not the case, wavefield separation would probably have to be incorporated into an elastic prestack depth migration.

Global transform techniques like Fourier and Radon transforms are less useful for modeling wavefield propagation through media with rapid changes in physical properties than techniques like ray tracing. We therefore formulate the forward problem of modeling the observed data at the surface from the plane-wave data below the near-surface layer using ray-tracing. Statics are used to indicate the local P and S velocities below each receiver. Standard least-squares inverse theory is then used to solve the linear set of forward modeling equations.

The forward problem, assuming only upcoming waves and a traction-free surface, can be stated as (Dankbaar, 1985; Holvik et al., 1998):

$$D_v(\tau, p) = R_V^{PP}(p)U^{PP}(\tau, p) + R_V^{PS}(p)U^{PS}(\tau, p)$$

$$D_R(\tau, p) = R_R^{PP}(p)U^{PP}(\tau, p) + R_R^{PS}(p)U^{PS}(\tau, p)$$

where $D_V(\tau, p)$ and $D_R(\tau, p)$ are the τ, p transforms of the recorded vertical and radial particle velocities, U^{PP} and U^{PS} are the τ, p transforms of the upgoing and downgoing P- and S-wave potentials, and $R_{V,R}^{PP}, R_{V,R}^{PS}$ are the so-called receiver characteristics of the geophone, which describe the interaction of the wavefields with the free surface. The full expressions for $R_{V,R}^{PP}, R_{V,R}^{PS}$ are given in Dankbaar (1985) and Holvik et al. (1998).

Now suppose that the P-P and P-S receiver statics, $\Delta\tau_{x_i}^{PP}$ and $\Delta\tau_{x_i}^{PS}$, for each receiver x_i, have been resolved elsewhere in the processing flow. We interpret these statics as the time for a $p = 0$ plane-wave to propagate up through the near-surface layer at position x_i (see Figure 1). To be physically consistent, we should allow the statics to be non-surface consistent, $\Delta\tau_{x_i} = \Delta\tau_{x_i}(p)$, and let the near-surface velocity be variable in the x direction (we assume that statics are due to velocity, not depth, variations). We assume straight rays through the near-surface layer, and also require

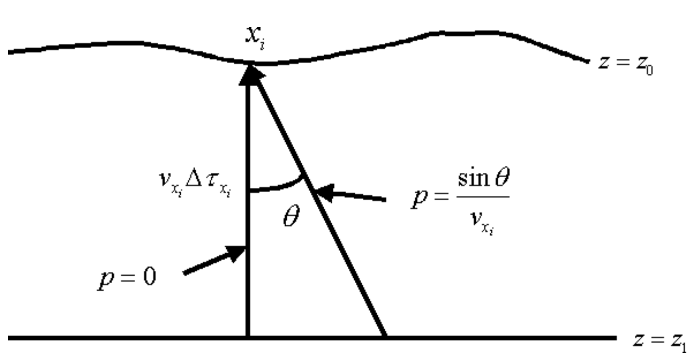

Figure 1: Ray paths through a heterogeneous near-surface layer

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$\Delta\tau_{x_i} > 0$ for all x_i so that the application of the static corrections represents a downward continuation of the receivers to a level below the near-surface layer causing the statics. From Figure 1,

$$\Delta\tau(p) = \frac{\Delta\tau(p=0)}{\cos\theta},$$

where $\theta = \sin^{-1}(pv_{x_i})$. We now choose a fixed depth, z_1, that is below the near-surface heterogeneous layer. Knowing the surface elevation, z_0, and vertical traveltime, $\Delta\tau_{x_i}(p=0)$, we can define a local velocity through the near-surface layer:

$$v(x_i) = \frac{z_1 - z_0(x_i)}{\Delta\tau(x_i)}.$$

These expressions for v_{x_i} and $\Delta\tau_{x_i}(p)$ are used in the decomposition equations below.

The fact that v_{x_i} depends on the arbitrarily chosen depth z_1 would seem to indicate that the velocities in these equations are somewhat arbitrary. In actual fact it is the average value of the statics, $\Delta\bar{\tau}(x_i)$, that is essentially arbitrary in the statics analysis because the average static depends on a poorly-constrained replacement velocity. In order to make this dependence on the replacement velocity more explicit, the problem can be stated in terms of relatively well-known differences in velocity in the near-surface layer, as revealed by the relative static shifts from station to station, and a relatively poorly-known average velocity of the entire near-surface layer:

$$\frac{1}{v_{x_i}} = \frac{1}{\bar{v}} + \frac{1}{\Delta v_{x_i}} = \frac{\Delta\bar{\tau}(x_i)}{z_1 - z_0(x_i)} + \frac{\Delta\tau(x_i) - \Delta\bar{\tau}(x_i)}{z_1 - z_0(x_i)}.$$

By stating the velocity dependence in this way, the number of input parameters to the inversion equations can be reduced. The P/S decomposition will be much more strongly dependent on the replacement velocity, $\bar{v}$, than on small variations in Δv_{x_i}, so it should be sufficient to keep Δv_{x_i} fixed, and vary only $\bar{v}$ during the search for optimum parameters.

We state the composition equations in the frequency domain. The static shift that each plane wave undergoes on passing through the near-surface layer is represented by a phase rotation, $e^{-i\omega\Delta\tau_{x_i}(p)}$. At the surface, the solid/vacuum boundary conditions modify the amplitude of each plane wave by the factor $R(p, v^S_{x_i}, v^P_{x_i})$. So a plane wave that starts out as $U(\tau = t - p_j x_i, p_j)$ below the near-surface layer becomes $R(p_j, v^P, v^S)U(\tau = t - p_j x_i - \Delta\tau_{x_i}, p_j)$ upon propagating to the surface. Each seismogram at position x_i on the surface is modelled as a synthesis of the modified plane waves (an inverse τ, p transform):

$$D(t, x_i) = \sum_j R(p_j, v^P_{x_i}, v^S_{x_i})U(t - p_j x_i - \Delta\tau_{x_i}, p_j).$$

This approximate modeling of the propagation through the near-surface layer is similar to the method of WKBJ/Maslov synthetic seismogram modeling (Chapman, 1978; Chapman and Drummond, 1982) . There will be additional modification of the recorded seismograms by filters, $\alpha^V(\omega)$ and $\alpha^R(\omega)$, which describe the different response of each geophone component and the coupling of each component to the ground. Before the P/S decomposition can properly be done, these different response functions need to be equalized on each component.

Assuming that this equalization has been done, the amount of P-S wavefield present on the vertical component is

$$\underline{D}_V^{PS}(\omega,\underline{x}) = \underline{\underline{B}}_V^{PS}(\omega,p,\Delta\tau_{x_i}^S,v_x^P,v_x^S)\underline{U}^{PS}(\omega,p),$$

or in expanded form:

$$\begin{pmatrix} D_V^{PS}(\omega,x_1) \\ D_V^{PS}(\omega,x_2) \\ D_V^{PS}(\omega,x_3) \\ . \\ . \\ D_V^{PS}(\omega,x_N) \end{pmatrix} = \begin{pmatrix} e^{-i\omega p_1 x_1} R_V^{PS} e^{-i\omega\Delta\tau_{x_1}^S} & e^{-i\omega p_2 x_1} R_V^{PS} e^{-i\omega\Delta\tau_{x_1}^S} & . & . & . \\ e^{-i\omega p_1 x_2} R_V^{PS} e^{-i\omega\Delta\tau_{x_2}^S} & e^{-i\omega p_2 x_2} R_V^{PS} e^{-i\omega\Delta\tau_{x_2}^S} & . & . & . \\ e^{-i\omega p_1 x_3} R_V^{PS} e^{-i\omega\Delta\tau_{x_3}^S} & e^{-i\omega p_2 x_3} R_V^{PS} e^{-i\omega\Delta\tau_{x_3}^S} & . & . & . \\ . & . & . & & \\ . & . & & . & \\ e^{-i\omega p_1 x_N} R_V^{PS} e^{-i\omega\Delta\tau_{x_N}^S} & e^{-i\omega p_2 x_N} R_V^{PS} e^{-i\omega\Delta\tau_{x_N}^S} & & & . \end{pmatrix} \begin{pmatrix} U^{PS}(\omega,p_1) \\ U^{PS}(\omega,p_2) \\ U^{PS}(\omega,p_3) \\ . \\ . \\ U^{PS}(\omega,p_M) \end{pmatrix}.$$

Similarly,

$$\begin{aligned} \underline{D}_V^{PP}(\omega,\underline{x}) &= \underline{\underline{B}}_V^{PP}\underline{U}^{PP}(\omega,p), \\ \underline{D}_R^{PP}(\omega,\underline{x}) &= \underline{\underline{B}}_R^{PP}\underline{U}^{PP}(\omega,p), \\ \underline{D}_R^{PS}(\omega,\underline{x}) &= \underline{\underline{B}}_R^{PS}\underline{U}^{PS}(\omega,p). \end{aligned}$$

Since the total wavefield recorded on each geophone component is the sum of the P-P and P-S wavefields, then

$$\begin{aligned} \underline{D}_V &= \underline{D}_V^{PS} + \underline{D}_V^{PP} = \underline{\underline{B}}_V^{PS}\underline{U}^{PS} + \underline{\underline{B}}_V^{PP}\underline{U}^{PP}, \\ \underline{D}_R &= \underline{D}_R^{PS} + \underline{D}_R^{PP} = \underline{\underline{B}}_R^{PS}\underline{U}^{PS} + \underline{\underline{B}}_R^{PP}\underline{U}^{PP}. \end{aligned}$$

Therefore,

$$\begin{pmatrix} \underline{D}_V \\ \underline{D}_R \end{pmatrix} = \begin{pmatrix} \underline{\underline{B}}_V^{PS} & \underline{\underline{B}}_R^{PP} \\ \underline{\underline{B}}_R^{PS} & \underline{\underline{B}}_R^{PP} \end{pmatrix} \begin{pmatrix} \underline{U}^{PS} \\ \underline{U}^{PP} \end{pmatrix},$$

or

$$\underline{D} = \underline{\underline{B}}\,\underline{U}.$$

A least-squares solution gives

$$\underline{U} = \left(\underline{\underline{B}}^H \underline{\underline{B}}\right)^{-1} \underline{\underline{B}}^H \underline{D}.$$

It is possible to work with these equations in the τ, p domain, or in the ω, k domain, with the substitution, $p = k/\omega$. These equations require the average velocities and the density at the surface as input parameters. In addition, the vertical and horizontal component response functions have to be calibrated and equalized. A method for determining these parameters from the data, such as the method suggested by Schalkwijk et al. (1998), needs to be investigated.

As in any least-squares fitting procedure, the inversion can be expected to work well only if the forward model is accurate, and if non-random noise does not contaminate the recording to a significant degree. If this is not true, it will be necessary to condition the data beforehand, or to use a more robust method of inversion than the standard least-squares method in order to make the inversion less sensitive to non-Gaussian noise.

CONCLUSIONS

A heterogeneous sea-floor is not an impediment to the application of P/S wavefield separation methods that assume a homogeneous sea-floor, as long as common-receiver gathers are well sampled spatially, and ray paths arriving at the receiver are close to vertical (so that the statics model applies). For the land 2-D case, where receiver gathers are often poorly sampled, a method of P/S wavefield separation that can be applied to shot gathers that suffer from P- and S-statics is required. A method that applies to this situation has been outlined here. Further development is required to extend this technique to the 3-D case in both land and marine situations.

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