Computational speed of EOM relative to standard Kirchhoff migration

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ABSTRACT

The computational speed of equivalent offset migration (EOM) is compared with a standard method of prestack time migration.

INTRODUCTION

Standard method

Scattered energy from one scatterpoint in a 2-D seismic line is visually displayed in a 3-D prestack volume (*x, h, t*) on a surface known as Cheops pyramid. In this volume *x* is the surface distance between a CMP and the scatterpoint, *h* is the half source-receiver offset, and *t* the time on an input trace. An inefficient Kirchhoff method of prestack migration would gather and sum all the energy for one scatterpoint before proceeding to the next scatterpoint, i.e. sum the energy from each input trace at the time defined by Cheops pyramid. A more efficient method moves all the relevant energy from one input trace to the migrated trace; I will refer to this method as the standard method, and will compare its computational efficiency with that of EOM.

A typical standard approach to Kirchhoff migration would;

- loop over all migrated traces,
- loop over all input traces, then
- sum the appropriate sample from an input trace into the migrated trace,
- end loops.

This arrangement of loops allows the velocity function and other related parameters to be defined once for each migrated trace. This method is efficient when all the input data can be stored within the computer memory.

Kirchhoff migration requirements

Energy in the input trace, that is gathered into the migrated trace, must have four numerical components applied:

1. compute the time *t* at the given displacement,
2. antialias filter the data in the neighbourhood of the sample,
3. interpolate the data from the quantized input samples, and

4. scale the amplitude of the data.

Often the antialiasing filter and interpolator are combined into one step.

The input data should also be filtered by a root differential filter ($rj\omega$) for 2-D data, or differential filter ($j\omega$) for 3-D data. (The designation of $rj\omega$ and $j\omega$ for these filters is derived from the Fourier transform of a derivative.)

EOM method

Equivalent offset migration (EOM) (Bancroft et al 1998) rearranged the order of the data into a more efficient form by first forming common scatter point (CSP) gathers. Each CSP gather contains energy from all traces within the prestack migration aperture similar to the loops described above, but none of the four numerical components are applied.

After the CSP gathers have been completed, the four numerical components are applied to each bin of the CSP gathers; the combination of these processes is referred to as Kirchhoff NMO. Stacking the CSP gathers complete the prestack migration.

Assumptions for both methods

- Time migration; assume an RMS type velocities at each scatter point.
- Velocity varies at each migrated sample.
- Data set are large enough to ignore reduced computations at boundaries.
- The number of bins in a CSP gather is one quarter of the number of traces in a two-sided migration aperture.
- The number of migrated traces equals the number of stacked traces.
- The number of samples migrated from an input trace is the same for both methods.
- Fold computations are ignored.

Definitions

The following definitions will use the lower case "n" to represent some form of computer cycles for a numerical step or process, and a capital "N" to represent a geometrical parameter on a data set.

n_{AAF}	Computations for antialias filter
n_{interp}	Computation for interpolation
n_{scale}	Computations for amplitude scaling the input sample

$n_{T/T0}$	Computations to compute time shift from T_0 to T at given displacement
n_{sum}	Computations to sum one sample within a "do loop"
$n_{set\text{-}up}$	Computations to set up each CSP gather (T_a, T_{0max}, T-T_0-V arrays)
N_{tr}	Number of input traces
N_{samp}	Number of time samples
N_{bins}	Number of bins in a CSP gather
N_{CSPs}	Number of CSP gathers (equals the number of CMP gathers)
$N_{mig\text{-}ap}$	Number of traces in two-sided migration aperture $N_{bins} \equiv \frac{N_{mig-ap}}{4}$
$R_{2D\text{-}gath}$	2-D data ratio of SCP gathering time to standard time
$R_{2D\text{-}KNMO}$	2-D data ratio of Kirchhoff NMO on SCP gathers to standard time
$R_{3D\text{-}gath}$	3-D data ratio of SCP gathering time to standard time
$R_{3D\text{-}KNMO}$	3-D data ratio Kirchhoff NMO on SCP gathers to standard time

DERIVING THE COMPUTING RATIOS

Ratios for 2-D data

Let the number of four component computations and summing be represented by

$$nn = n_{AAF} + n_{\text{int}\,erp} + n_{T/T0} + n_{scale} + n_{add} \quad . \tag{1}$$

The number of computations for forming the CSP gathers may be fined by

$$N_{2-D-EOM-gath} = N_{CSPs} \times \left(n_{set-up} + N_{mig-ap} \times N_{ave-fld} \times N_{samp} \times n_{add}\right) \tag{2}$$

The number of computations from performing Kirchhoff NMO is given by

$$N_{2-D-EOM-KNMO} = N_{CSPs} \times N_{bins} \times N_{samp} \times nn \tag{3}$$

The total number of computations for the <u>standard method</u> is given by

$$N_{2-D-Bru} = N_{CSPs} \times N_{mig-ap} \times N_{ave-fld} \times N_{samp} \times nn \tag{4}$$

The ratio of computations for EOM gathering to the standard method is

$$R_{2-D-gath} = \frac{N_{2-D-EOM-gath}}{N_{2-D-Bru}} = \frac{N_{CSPs} \times \left(n_{set-up} + N_{mig-ap} \times N_{ave-fld} \times N_{samp} \times n_{add}\right)}{N_{CSPs} \times N_{mig-ap} \times N_{ave-fld} \times N_{samp} \times nn} \tag{5}$$

The setup time n_{set-up} is relatively small giving

$$R_{2-D-gath} = \frac{N_{CSPs} \times N_{mig-ap} \times N_{ave-fld} \times N_{samp} \times n_{add}}{N_{CSPs} \times N_{mig-ap} \times N_{ave-fld} \times N_{samp} \times nn} \tag{6}$$

or

$$R_{2-D-gath} = \frac{n_{add}}{\left(n_{AAF} + n_{\mathrm{int}erp} + n_{T/T0} + n_{scale} + n_{add}\right)}. \tag{7}$$

The ratio for Kirchhoff NMO to the standard method is given by

$$R_{2-D-KNMO} = \frac{N_{2-D-EOM-KNMO}}{N_{2-D-Bru}} = \frac{N_{CSPs} \times N_{bins} \times N_{samp} \times nn}{N_{CSPs} \times N_{mig-ap} \times N_{ave-fld} \times N_{samp} \times nn} \tag{8}$$

or

$$R_{2-D-KNMO} = \frac{N_{bins}}{N_{mig-ap} \times N_{ave-fld}} = \frac{N_{bins}}{4 \times N_{bins} \times N_{ave-fld}} \tag{9}$$

giving

$$R_{2-D-KNMO} = \frac{1}{4 \times N_{ave-fld}} \tag{10}$$

Ratios for 3-D Data

The number of computations for forming the CSP gather is given by

$$N_{3-D-EOM-gath} = N_{CSPs} \times \left(n_{set-up} + \frac{\pi}{4} N^2_{mig-ap} \times N_{ave-fld} \times N_{samp} \times n_{DO-add} \right), \tag{11}$$

where the $\pi N^2/4$ term represents the surface coverage of the migration aperture. The Kirchhoff NMO computations are given by

$$N_{3-D-EOM-KNMO} = N_{CSPs} \times N_{bins} \times N_{samp} \times nn \quad . \tag{12}$$

The number of computations for the standard method is given by

$$N_{3-D-Bru} = N_{CSPs} \times \frac{\pi}{4} N^2_{mig-ap} \times N_{ave-fld} \times N_{samp} \times nn \tag{13}$$

The ratio of forming the CSP gathers to the standard method is given by

$$R_{3-D-gath} = \frac{N_{3-D-EOM-gath}}{N_{3-D-Bru}} = \frac{N_{CSPs} \times \left(n_{set-up} + \frac{\pi}{4} N_{mig-ap}^{2} \times N_{ave-fld} \times N_{samp} \times n_{add} \right)}{N_{CSPs} \times \frac{\pi}{4} N_{mig-ap}^{2} \times N_{ave-fld} \times N_{samp} \times nn} \tag{14}$$

Once again, n_{set-up} is relatively small giving

$$R_{3-D-gath} = \frac{n_{add}}{\left(n_{AAF} + n_{\mathrm{int}\,erp} + n_{T/T0} + n_{scale} + n_{add} \right)} \tag{15}$$

The ratio of computations for Kirchhoff NMO to the standard method is given by

$$R_{3-D-KNMO} = \frac{N_{3-D-EOM-KNMO}}{N_{3-D-Bru}} = \frac{N_{CSPs} \times N_{bins} \times N_{samp} \times nn}{N_{CSPs} \times \frac{\pi}{4} N_{mig-ap}^{2} \times N_{ave-fld} \times N_{samp} \times nn} \tag{16}$$

or

$$R_{3-D-NMO} = \frac{4N_{bins}}{\pi \times N_{mig-ap}^{2} \times N_{ave-fld}} \tag{17}$$

giving

$$R_{3-D-NMO} = \frac{1}{4\pi \times N_{bins} \times N_{ave-fld}} \tag{18}$$

DISCUSSION

The ratios for the gathering processes in 2-D and 3-D data sets are given by similar equations (7) and (15). The relative computations for EOM and the standard method are given by summing one sample verses Kirchhoff NMO and summing.

The ratios of Kirchhoff NMO and stacking verses the standard method for 2-D and 3-D data set are given in equations (10) and (18). The 2-D data shows a computation ratio proportional to $4N_{ave\text{-}fld}$, while that for 3-D data is $4\pi N_{ave\text{-}fld} N_{bins}$.

When accurate methods of antialiasing filters are used (sinx/x types), the Kirchhoff NMO computations dominate the overall computations of the EOM process. In these cases, the computational speed of EOM over the standard method may increase from 10's for 2-D data and 100's for 3-D data. Less accurate but more efficient antialiasing filters (i.e. attenuate both *signal* and noise) speed the overall processing times of both methods and reduce the relative computational speeds. Elimination of the antialiasing filter and the interpolation step further increases the speed of both process and reduces relative processing times that may tend to those given by of equations (7) or (15).

Contents

CONCLUSIONS

A comparison of arithmetic computations between EOM and a standard Kirchhoff migration show a reduction for the EOM method. The relative increase in speed of the EOM method is dependent on the choice of antialiasing filters.

REFERENCES

Bancroft, J. C., Geiger, H. D., and Margrave, G. F., 1998, The equivalent offset method of prestack migration, Geophysics, in press for Nov. - Dec. 1998 issue.