

## Density contrast is difficult to determine from AVO

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### INTRODUCTION

A recent analysis of seismic amplitude variation with offset (AVO) by Fatti et al. (1994) recasts the AVO equations of Aki and Richards (1979) into a form that is preferable for analysis of rock properties. Fatti et al. showed that the AVO equation could be expressed in terms of P-wave reflectivity, S-wave reflectivity and fractional density change. In an interesting paper, Goodway et al. (1997) then utilized Fatti's equation to deal with Lamé parameters, and such parameters were used to effectively detect hydrocarbons. In this short note, I use sensitivity analysis of Fatti's equation to formally show that direct detection of rock density is difficult for limited apertures and typical seismic velocities. Sensitivity analysis allows us to estimate fractional changes of P-wave and S-wave impedance, but shows that AVO effects are not sensitive to density changes for normal seismic apertures.

### METHODOLOGY

In order to derive Fatti's equation, start with the linearized approximation to the Knott-Zoeppritz equations as given by Goodway et al. (1997).

$$R_{pp}(\theta) = \frac{1}{4} \left( 1 + \tan^2 \theta \right) \frac{\Delta(\lambda + 2\mu)}{\lambda + 2\mu} - 2 \left( \frac{\beta}{\alpha} \right)^2 \sin^2 \theta \frac{\Delta\mu}{\mu} + \frac{1}{4} \left( 1 - \tan^2 \theta \right) \frac{\Delta\rho}{\rho} \quad (1)$$

Here  $\lambda, \mu$  are Lamé parameters,  $\rho$  = density,  $\alpha$  = P-wave velocity,  $\beta$  = shear wave velocity, and  $\theta$  = angle of incidence.

One can recognize this equation in the form given by Fatti et al. (1994) if we rewrite the equation in terms of impedance and density. This can be done by recalling that  $(\lambda + 2\mu) = \rho\alpha^2 = I_p^2/\rho$ , where  $I_p$  = P-wave impedance. Similarly, we recall that  $\mu = \rho\beta^2 = I_s^2/\rho$  where  $I_s$  = shear wave impedance. Therefore, we can rewrite the terms in equation (1) by using:

$$\frac{\Delta(\lambda + 2\mu)}{(\lambda + 2\mu)} = \frac{\Delta(I_p^2/\rho)}{I_p^2/\rho} = \frac{2\Delta I_p}{I_p} - \frac{\Delta\rho}{\rho} \quad (2)$$

and

$$\frac{\Delta\mu}{\mu} = \frac{\Delta(I_s^2/\rho)}{I_s^2/\rho} = \frac{2\Delta I_s}{I_s} - \frac{\Delta\rho}{\rho} \quad (3)$$

If we substitute equations (2) and (3) into (1) and denote the normal incidence P-wave and shear wave reflectivities by  $r_{pp} = \Delta I_p / 2I_p$  and  $r_{ss} = \Delta I_s / 2I_s$  respectively, we obtain Fatti's equation for P-P reflections given by equation (4).

$$R(\theta) = \left(1 + \tan^2 \theta\right) r_{pp} - 8 \left(\frac{\beta}{\alpha}\right)^2 \sin^2 \theta r_{ss} - \left(\frac{1}{2} \tan^2 \theta - 2 \left(\frac{\beta}{\alpha}\right)^2 \sin^2 \theta\right) \frac{\Delta \rho}{\rho} \quad (4)$$

Equation (4) has terms containing the zero offset P-wave reflection coefficient,  $r_{pp}$ , the zero-offset S-wave reflection coefficient,  $r_{ss}$ , and the fractional change in density,  $\Delta \rho / \rho$ . The coefficient multiplying  $\Delta \rho / \rho$  is given by  $(1/2 \tan^2 \theta - 2(\beta/\alpha)^2 \sin^2 \theta)$ .

For small values of  $\theta$ , this coefficient is often nearly zero, since  $\beta/\alpha$  is usually nearly  $1/2$  and for small values of  $\theta$ ,  $\tan \theta \cong \sin \theta$ . The fact that this coefficient is small generally means that we can have large variations in our estimates of  $\Delta \rho / \rho$  without having a large effect on values of  $R(\theta)$ . This means that it is often difficult to invert for values of fractional density contrast by using AVO values.

A quantitative measure of this inversion difficulty is found by using the sensitivity analysis methods of Jackson (1976), as described by Lines and Treitel (1985). As an example of density estimation from AVO, we apply sensitivity analysis to a model from Goodway et al. (1997). Their model of a shale layer over a gas sand layer can be described in tabular form by Table 1.

**Table 1.** Model from paper by Goodway et al. (1979).

Layer	P-wave velocity (in m/s)	S-wave velocity (in m/s)	Density (gm/cm <sup>3</sup> )
Shale	2898	1290	2.425
Gas-Sand	2857	1666	2.275

This model produces a negligible reflection at normal incidence and increases in amplitude with offset (ref. Lines and Treitel, 1985). Figure 1 shows the AVO response,  $R_{pp}(\theta)$ , as a function of angle from  $\theta = 1^\circ$  to  $\theta = 45^\circ$ .

In least-square inversion, we solve for a parameter change vector,  $\underline{x}$ , which fits a model response,  $\underline{f}$ , to data,  $\underline{y}$ , in a least squares sense. As shown by Lines and Treitel (1985), the least squares equations can be written as:

$$\underline{Ax} = \underline{y} \quad (5)$$

for the case where the initial model guess and its model response are assumed to be zero. Here A is the Jacobian or sensitivity matrix containing partial derivatives of the model response with respect to the model parameters. That is,

$$A_{ij} = \frac{\partial f_i}{\partial x_j} \quad (6)$$

In our case we set  $\underline{x}^T = (x_1, x_2, x_3) = (r_{pp}, r_{ss}, \frac{\Delta\rho}{\rho})$ . Therefore, the Jacobian coefficients are given by:

$$A_{i1} = (1 + \tan^2 \theta_i) \quad (7a)$$

$$A_{i2} = -8 \left( \frac{\beta}{\alpha} \right)^2 \sin^2 \theta_i \quad (7b)$$

$$A_{i3} = - \left( \frac{1}{2} \tan^2 \theta - 2 \left( \frac{\beta}{\alpha} \right)^2 \sin^2 \theta \right) \frac{\Delta\rho}{\rho} \quad (7c)$$

where  $\theta_i$  is the angle of incidence for the *i*th trace.

In the sensitivity analysis due to Jackson (1976), one first finds a solution which gives a satisfactory fit from a least squares perspective and has a mean squared error of  $\hat{r}^2$ . The next step involves computing a set of “edge models” that are barely acceptable from a least squares perspective. That is, these “edge models” would fit the data to within some relaxed error criterion, given by  $\hat{\sigma}^2$ , which is somewhat greater than the original error criterion - but which would be considered an acceptable error. As shown by Jackson (1976) and Lines and Treitel (1985), the difference between the edge model and the original solution is given by:

$$\Delta \mathbf{x} = \left[ \frac{n(\hat{\sigma}^2 - \hat{r}^2)}{\lambda_k} \right]^{\frac{1}{2}} \mathbf{v}_k \quad (8)$$

where  $\mathbf{v}_k$  is a parameter eigenvector of the Jacobian matrix,  $\lambda_k$  is its associated singular value, and *n* is the number of data points. As can be seen by equation (8), the variation in the parameters is determined by the size of the allowable error and the size of the singular values for a particular eigenvector (with the smallest singular value giving the largest variation in the model parameters). Since it is somewhat difficult to get an intuitive grasp of the size of edge model variation from inspection of (8), we shall compute a few simple examples to show how much variation we can expect in our parameters. The sensitivity analysis due to Jackson (1976) computes models that are barely acceptable from a least-squares sense, once we have found a solution that satisfies the convergence criteria.

## RESULTS

The synthetic seismogram for the model in Table 1 is shown in Figure 1 for angles from 0 to 45 degrees. If we treat these seismic traces as data and start with an initial

guess of  $\underline{x} = 0$ , we can invert to produce values of  $\underline{x}$  which produce nearly a perfect fit to the data.

In our example, the least squares solution is given by  $r_{pp} = -0.037564$ ,  $r_{ss} = 0.105794$  and  $\Delta\rho/\rho = -0.061857$ . This is close to the desired result of  $r_{pp} = -0.39030$ ,  $r_{ss} = 0.095672$  and  $\Delta\rho/\rho = -0.061856$

However, as we relax our error criterion slightly, and use  $\hat{\sigma}^2 = 10^{-6}$ , with  $\hat{r}^2 = 10^{-7}$ , we get the edge models summarized in Table 2.

**Table 2.** Edge solutions calculated from most squares inversion.

	$r_{pp}$	$r_{ss}$	$\Delta\rho/\rho$
Solution	-0.037564	0.105794	-0.061857
Edge Model 1	-0.0421833	0.106970	-0.061580
Edge Model 2	-0.028699	0.138118	-0.051272
Edge Model 3	-0.031749	0.191529	-0.328538

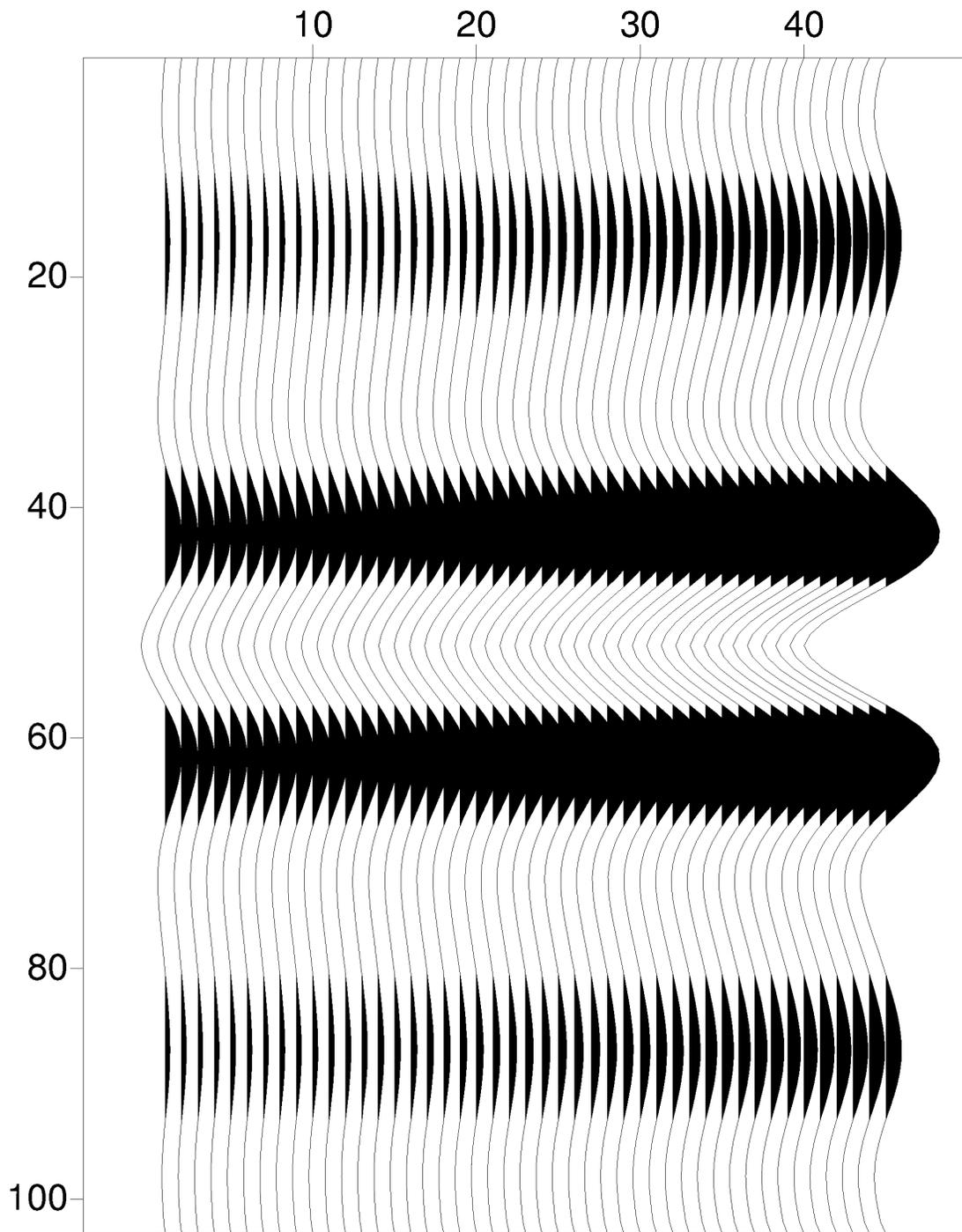
We note that the largest variation in the variables is in the estimate of density contrasts – especially for Edge Model 3. Although the variation in density estimation varies by about a factor of 5 from Edge Model 3 compared to Edge Model 1, the model responses shown in Figures 2-4 show reasonably good fits to the data.

## CONCLUSIONS

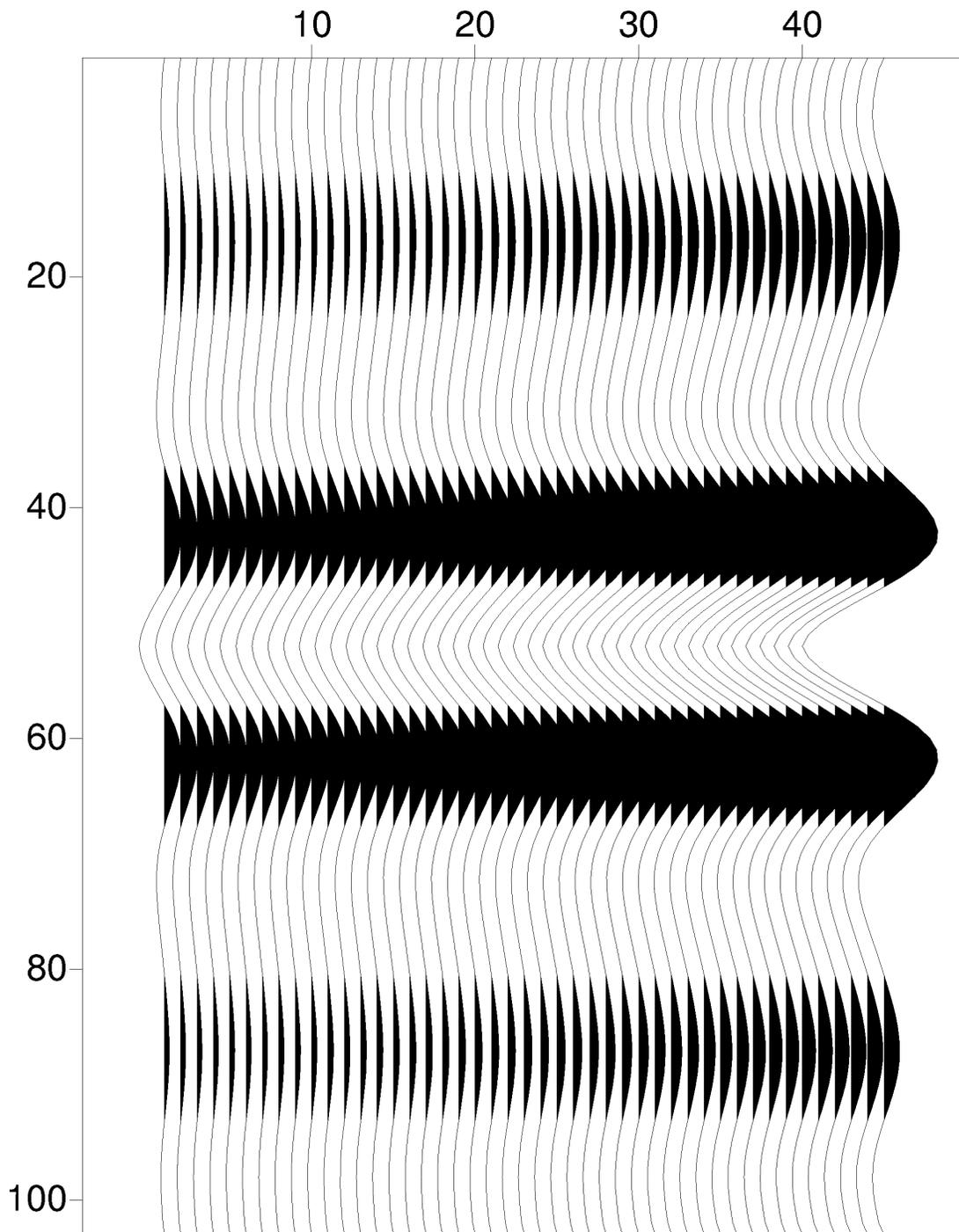
Fatti's equation gives a good representation of AVO in terms of P-wave impedance contrast, shear wave impedance contrast, and density contrast. From this form of the AVO equation, it can be seen that AVO would usually be least sensitive to density variation. This is confirmed for a simple model using sensitivity analysis.

## REFERENCES

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**Figure 1.** Desired model response for the Goodway model.



**Figure 2.** Edge model 1 from most squares inversion.

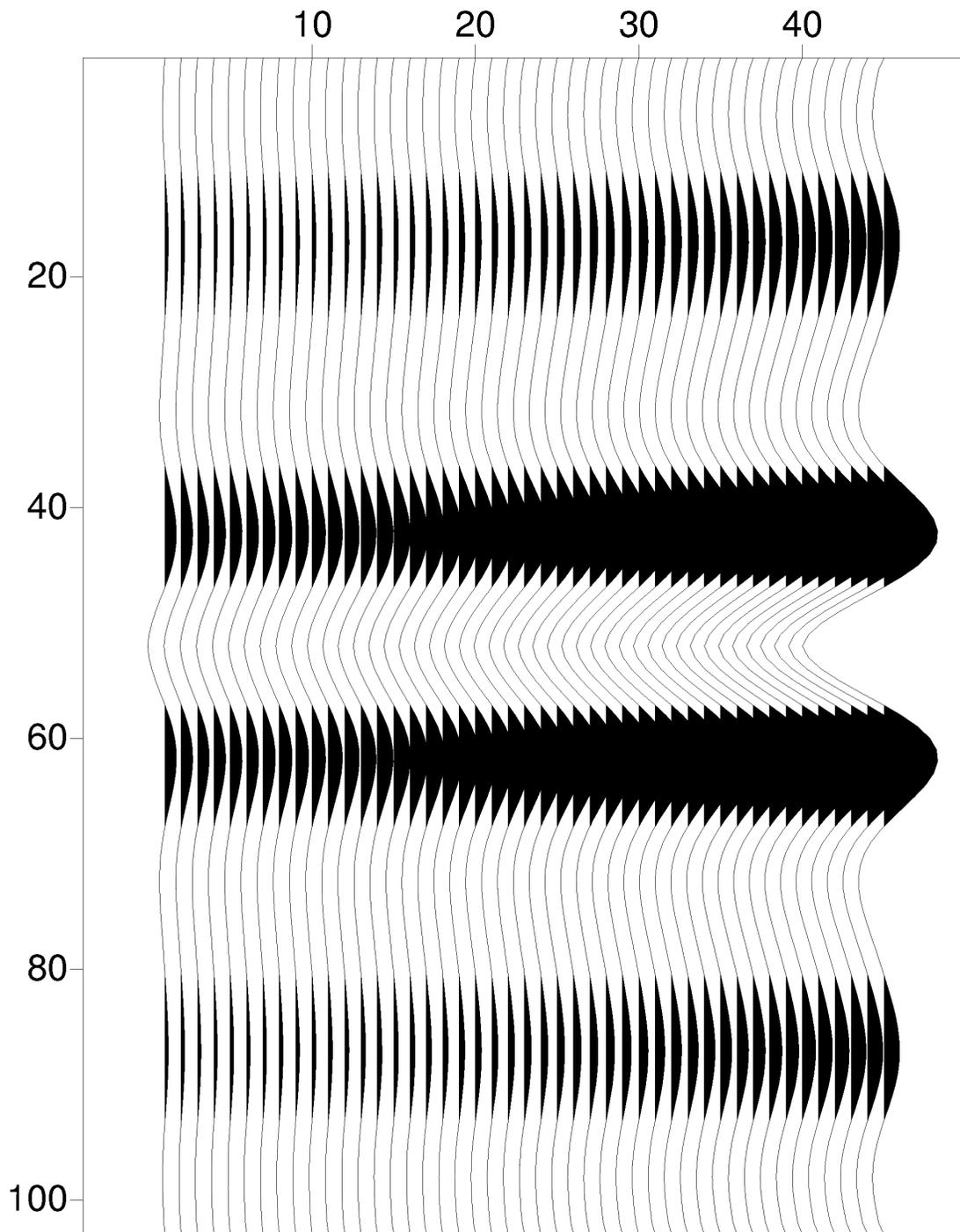
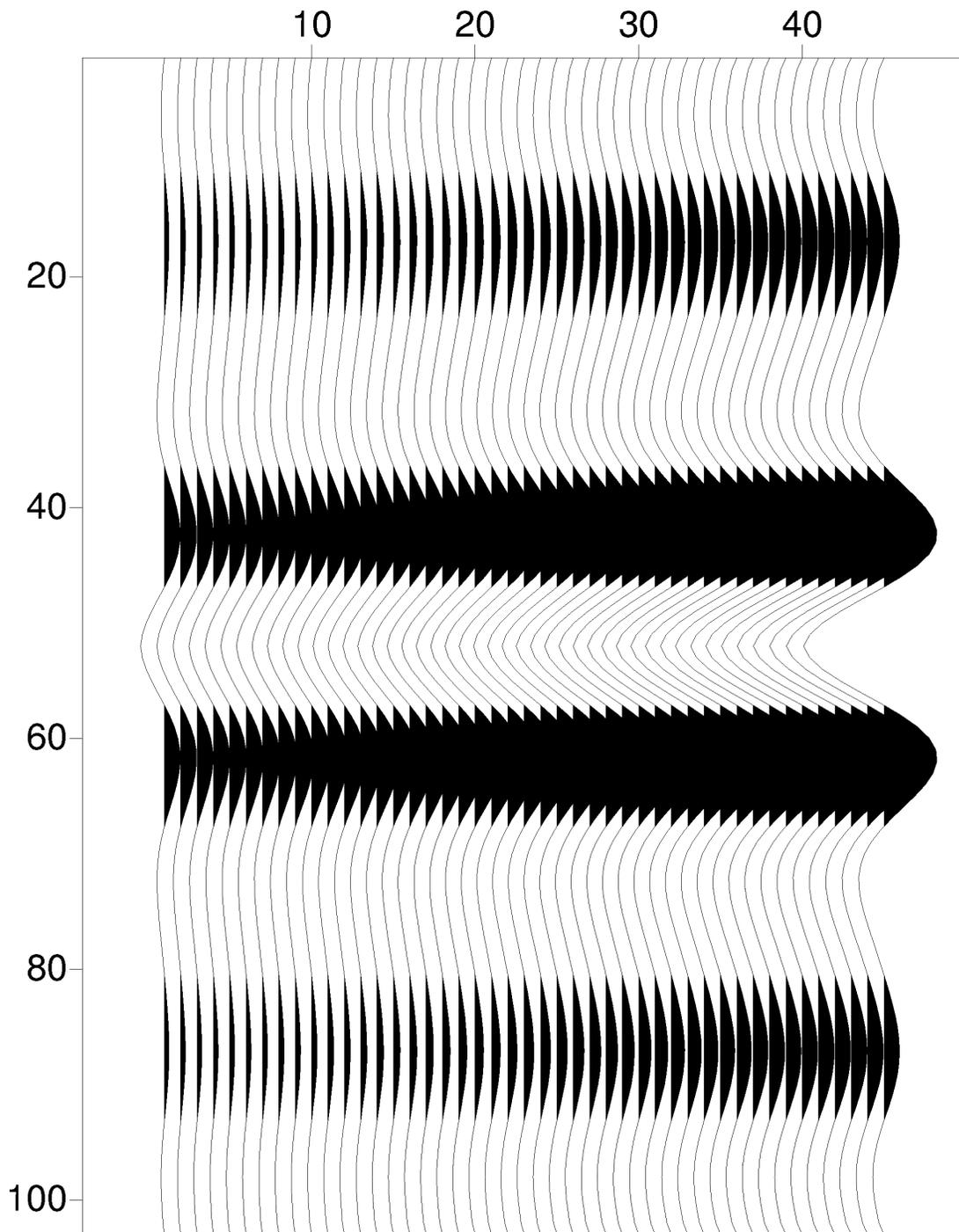


Figure 3. Edge model 2 from most squares inversion.



**Figure 4.** Edge model 3 from most squares inversion.