Weighted stacking plus traveltime inversion: a proposed prestack P-S inversion

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ABSTRACT

This paper outlines an alternative method for obtaining P- and S-wave velocity models and V_S/V_P ratio, from P-S data. The method interactively minimizes an error function (observed - modeled P-S amplitudes) by using a trial V_S/V_P value as input. This minimization process consists of a modified least-squares method, which generates constraints on $\Delta V_P/V_P$, $\Delta V_S/V_S$ and V_S/V_P ratios. A P-S traveltime tomography scheme is followed for fitting the observed P-S traveltimes under the assumption that above constraints are honored. This traveltime inversion provides the final P- and S-wave interval velocity fields.

Although, this method could be more expensive than other published methods (Smith and Gidlow,1987; Stewart, 1990; Vestrum and Stewart, 1993; Ferguson, 1996, and Ferguson and Margrave, 1996), it is possible to gain a more robust P-S inversion algorithm combining amplitude (post-stack) and traveltime (pre-stack) informations at once, for getting a more realistic P- and S-wave velocity field with vertical and horizontal gradients, while P-S amplitudes are well represented. Additionally, this new approach does not need, in principle, any shear sonic log information or P-wave and S-wave velocity models in P-S time to get elastic parameters.

INTRODUCTION

Smith and Gidlow (1987) developed a method for estimating rock properties by using weighted CMP stacking. In this way, the normal CMP stacking process is modified through weights applied on the NMO-corrected CDP gathers. This weighted stacking process provides P-wave velocity reflectivity, S-wave velocity reflectivity, pseudo-Poisson's ratio reflectivity and fluid factor traces. These weights are obtained from a least-squares fit of the observed P-P reflectivity by means of a P-P reflectivity model given by Aki and Richards (1979). Stewart (1990) extended this method to P-S seismic data through a joint P and P-S inversion, which combines P-P and P-S reflectivities for obtaining S-wave velocity reflectivity in P-S time. Vestrum and Stewart (1993) applied this approach on synthetic P-S data, showing its robustness for representing the input P- and S-wave velocity models. Ferguson (1996) modified this approach to involve a simplification which cast the inversion, not as a joint inversion, but, as an inversion of P-S data alone. This new approach achieve two advantages: (i) the potentially awkward step of correlating P-P and P-S gathers in both time and offset is reduced to a simple correlation in time, and (ii) a method by which standard post-stack inversion methods can be used to yield S-wave interval velocity. However, this P-S inversion method has two problems. First, it is necessary to get an input P- and S-wave velocity model in P-S time, which requires shear sonic logs or V_S/V_P values. Second, the inverted S-wave velocity depends on the bandwidth signal, particularly the generally absent low frequencies. Although, Ferguson (1996) shows a procedure to recover the absent low frequency band, the resolution of the inverted S-wave velocity is effected. The present paper attempts to establish an alternative methodology for overcoming the above problems. It uses a modification of the least-squares method to minimize an error function between observed and modeled P-S reflectivities. This approach provides V_S/V_P values, which can be used for improving CCP binning or related to lithology and/or fluid properties. Additionally, no shear sonic log data or S-wave background velocity model is required prior to inversion. Finally, a P-S traveltime tomography scheme is used to obtain P- and S-wave interval velocities. This is accomplished under the assumption that the final P- and S-wave velocity models honor the constraints on $\Delta V_P/V_P$, $\Delta V_S/V_S$, and V_S/V_P ratios, obtained during the preceding P-S AVO inversion step.

P-S AVO INVERSION

The P-S amplitude reflection for an event on a NMO-corrected CCP gather can be expressed as:

$$R_{ps}(\theta) = a\frac{\Delta V_p}{V_p} + b\frac{\Delta V_s}{V_s} \tag{1}$$

where $a = f(\theta, V_S/V_P)$ and $b = g(\theta, V_S/V_P)$ are functions of the incidence angle θ, V_P/V_S, V_P is the P-wave velocity and V_S is the S-wave velocity (Aki and Richards, 1980; Smith and Gidlow,1987).

In this paper, we use an approximation of R_{PS} (Donati and Martin, 1998), valid for incidence angles less than 60 degrees:

$$R_{ps}(\theta) = -\frac{B_o}{2}\sin\theta + \frac{1}{4}(B_1 - B_o)\frac{V_s^{\,2}}{V_p^{\,2}}\sin^3\theta + \frac{1}{8}\frac{V_s^{\,2}}{V_p^{\,2}}B_1\sin^5\theta \tag{2}$$

where the coefficients B_0 and B_1 are functions of $\Delta Vp/Vp$ and $\Delta Vs/Vs$:

$$B_o = \left(1 - 2\frac{V_s}{V_p}\right)\frac{\Delta\rho}{\rho} + 4\frac{V_s}{V_p}\frac{\Delta V_s}{V_s}\ , \tag{3}$$

and

$$B_1 = 2\frac{V_s}{V_p}\left(1 + \frac{V_s^{\,2}}{V_p^{\,2}} + 2\frac{V_s}{V_p}\right)\left[\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_s}{V_s}\right]. \tag{4}$$

The incidence angles can be estimated by using an iterative ray-tracing algorithm through an initial P- and S-wave velocity model, which defines the initial trial V_S/V_P ratio. Equation (2) can be regrouped as:

$$R_{ps}\left(\theta, V_p, V_s\right) = F\left(\theta, {}^{V_s}\!/\!{}_{V_p}\right)\frac{\Delta V_p}{V_p} + G\left(\theta, {}^{V_s}\!/\!{}_{V_p}\right)\frac{\Delta V_s}{V_s} \qquad (5)$$

where Gardner's rule (ρ.=k $V_p^{1/4}$) is used to replace density by P-wave velocity.

Because the factors V_S/V_p, $\Delta V_p/V_p$, and $\Delta V_S/V_S$ are intrinsically related, the ordinary, weighted or even generalized least-squares methods cannot be used in a straight forward way.

An alternative way to fit them by least-squares involves minimizing an error function $\varepsilon=\varepsilon(V_S/V_p, \Delta V_p/V_p, \Delta V_S/V_S)$ by two steps:

$$\min_{\frac{\Delta V_p}{V_p}, \frac{\Delta V_s}{V_s}, \frac{V_s}{V_p}} \varepsilon = \min_{\frac{V_s}{V_p}} \varepsilon^*\left({}^{V_s}\!/\!{}_{V_p}\right) \qquad (6)$$

where

$$\varepsilon = \left(R_{ps}^{obs} - R_{ps}^{\mathrm{mod}}\right)^2 = \left(R_{ps}^{obs} - F\left({}^{V_s}\!/\!{}_{V_p}\right)\frac{\Delta V_p}{V_p} - G\left({}^{V_s}\!/\!{}_{V_p}\right)\frac{\Delta V_s}{V_s}\right)^2 \qquad (7)$$

and

$$\varepsilon^*\left({}^{V_s}\!/\!{}_{V_p}\right) = \min_{\frac{\Delta V_p}{V_p}, \frac{\Delta V_s}{V_s}} \varepsilon \quad , \qquad (8)$$

where n is the number of traces (or offsets) contributing to the NMO-corrected CCP gathers at the particular time sample under analysis.

In other words, the suggested approach to fit R_{ps}^{obs} (observed P-S amplitude reflection) by using eq. 5 is: (i) use an ordinary, weighted or generalized least-squares method which accepts as input the trial value of V_S/V_p and returns function error ε*, $\Delta V_p/V_p$, and $\Delta V_S/V_S$ ratios evaluated at this trial V_S/V_p ratio. The inverted $\Delta V_S/V_p$ and $\Delta V_S/V_S$ ratios, which minimize $\varepsilon^*(V_S/V_p)$ in a least-squares sense, are given by Smith and Gidlow (1987) but using different weights a and b (eq. 1). Then, (ii) search for a minimum of $\varepsilon^*(V_S/V_p)$ using any convenient method. When it is obtained, the value $(V_S/V_p)^*$ minimizing $\varepsilon^*(V_S/V_p)$ is output, along with the associated $\Delta V_p/V_p$ and $\Delta V_S/V_S$ values.

Figure 1 shows a flowchart of the process.

PRACTICAL CONSIDERATIONS

It is necessary here to clarify the practicalities of this approach. Formally speaking, we should apply ray-tracing to obtain the functions F and G (eq. 5), for each trial V_S/V_P ratio. This procedure is time consuming and expensive. But, it is possible to avoid it, if the effect of the errors in the estimation of the incidence angle on modeled R_{ps} are considered less significant than errors in V_S/V_P ratio.

Ferguson (1996) showed that the large errors in S-wave inverted velocities in comparison to real shear-wave logs, are mainly due to violation of the small changes in rock properties assumption (Aki and Richards, 1980). In other words, $\Delta V_P/V_P$ and $\Delta.V_S/V_S$ ratios must be much less than unity. In contrast, the smaller errors at interfaces, which obey this assumption on $\Delta V_P/V_P$ and $\Delta V_S/V_S$ ratios, are mainly due to the approximation of the incidence angles. Additionally, Ferguson (1996) concluded that the error values in estimating S-wave velocity, V_S, due to error in background V_P and V_S velocities (or errors in V_S/V_P ratio), show a greater dependence on accuracy in P-wave background velocity than S-wave background velocity.

Based on these results, it is clear that V_S/V_P ratio mainly controls inverted S-wave velocity, through the modeled R_{ps} Then, variations on R_{ps} due to incidence angle error has a small effect on inverted S-wave velocity. The proposed P-S inversion assumes the same initial the incidence angles model during the estimation of the functions F and G eq. 5, for each trial V_S/V_P ratio. Then, the initial (or background) incidence angle model can be generated, by using an initial P-P velocity model and assuming an initial V_S/V_P ratio. This initial incidence angle model will be used without changes during minimization process of $\varepsilon.^*(V_S/V_P)$ for each trial V_S/V_P ratio.

P- AND S-WAVE VELOCITY MODELS ESTIMATION

The ratios $\Delta V_P/V_P$, $\Delta V_S/V_S$,and V_S/V_P are obtained from inverting the P-S amplitudes of a NMO-corrected CCP gather through P-S weighted stacks (Smith and Gidlow, 1987). But, the Aki and Richards (1980) approximation for P-S reflection coefficients, considers only mean values of density, P-wave velocity and S-wave velocity through an interface. This means that, there are different possible P- and S-wave velocity models which fit the obtained constraints on $\Delta V_P/V_P$, $\Delta V_S/V_S$, and V_S/V_P after AVO P-S inversion (Figure 2).

Ferguson (1996) obtains a S-wave velocity model from $\Delta V_S/V_S$ values by applying a recursive P-S post-stack inversion. This scheme requires an initial S-wave velocity value (seed) at some P-S time, before applying P-S recursive inversion. This seed value may not be accurate or even available. Additionally, the constrained linear inversion scheme requires shear-wave logs at P-S time, as an initial guess, which may be unavailable. Other limiting factors include the problem of missing low frequencies. Ferguson (1996) proposed some techniques to overcome this problem with some limitations in the resolution of the S-wave sections.

To avoid the above, we suggest an additional constraint for getting V_p and V_s interval velocities: fit observed P-P and P-S traveltimes, associated with an horizon of interest, through combined P-P and P-S traveltime inversions. In other words, the traveltime fit is considered together with inverted $\Delta V_p/V_p$, $\Delta V_s/V_s$, and V_s/V_p ratios as constraints of the final P- and S-wave velocity models. In the case of an accurate P-wave velocity model, we will need only to perform P-S tomography inversion to fit the observed P-S traveltimes. Here, we must include the inverted constraints on $\Delta V_p/V_p$, $\Delta V_s/V_s$, and V_s/V_p,obtained from P-S AVO inversion, for reducing the number of possible S-wave velocity models. This implies working CCP gathers without NMO correction and P-P and P-S correlations for picking target horizons in time.

Although, this method could be more expensive than other published methods (Smith and Gidlow, 1987; Stewart, 1990; Vestrum and Stewart, 1993, and Ferguson, 1996), it is possible to gain a more robust P-S inversion algorithm combining amplitude (post-stack) and traveltime (pre-stack) information at once, to get a more realistic P- and S-wave velocity field with vertical and horizontal gradients, while P-S amplitudes are well represented.

If, we call $\Delta V_p/V_p$=A, $\Delta V_s/V_s$=B, and V_s/V_p=C, then the above constraints on $\Delta V_p/V_p$, $\Delta V_s/V_s$, and V_s/V_p ratios can be formulated as follows:

$$(2-C)V_{p,i+1} = (2+C)V_{p,i} \tag{9}$$

$$2V_{s,i+1} - ABV_{p,i+1} = 2V_{s,i} + ABV_{p,i} \tag{10}$$

This means that, the P- and S-wave interval velocities are obtained in a layer stripping way. In other words, if we fit the P-P and/or P-S traveltimes for the first layer, on CMP and/or CCP gathers without NMO correction, then the P- and S-wave velocities for deeper layers can be obtained through eqs. (9) and (l0) P- and S-wave velocity models, which fit the observed P-P and P-S traveltimes, both velocity fields should obey both conditions for each interpreted horizon in time.

In principle, if these conditions are honored, any residual P-P and/or P-S traveltime errors could be associated with a structural effect, affecting the raypaths (and incidence angles). In this case, the tomographic algorithm could be applied to accept P- and S-wave velocity models as input, honoring conditionals eqs. 9-10, and modeling residual traveltimes only as due to layer thickness variations, keeping these P- and S-wave velocity fields fixed. D’Agosto and Michelena (1997) showed a tomographic traveltime inversion for P-S data which could be tested here.

Another alternative method is to use pre-stack depth migration for structural imaging, where the migration velocities are obtained by perturbing P- and S-wave velocity field obtained from traveltime inversion. In this way, the final P- and S-wave velocity model will address traveltime and structural constraints at once, giving a more robust P-S inverted velocity model.

Figure 3 shows the flowchart of P-S traveltime tomographic inversion.

CONCLUSIONS

An alternative pre-stack P-S inversion for obtaining a P-S velocity field from P-S data is addressed. This proposed method lets us obtain an estimation of V_S/V_P ratio as well.

Including constraints on inverted $\Delta V_P/V_P$, $\Delta V_S/V_S$, and V_S/V_P ratios, a traveltime inversion method will provide realistic P- and S-wave interval velocities. A prestack depth migration could improve the final P-S velocity model if large structures are present.

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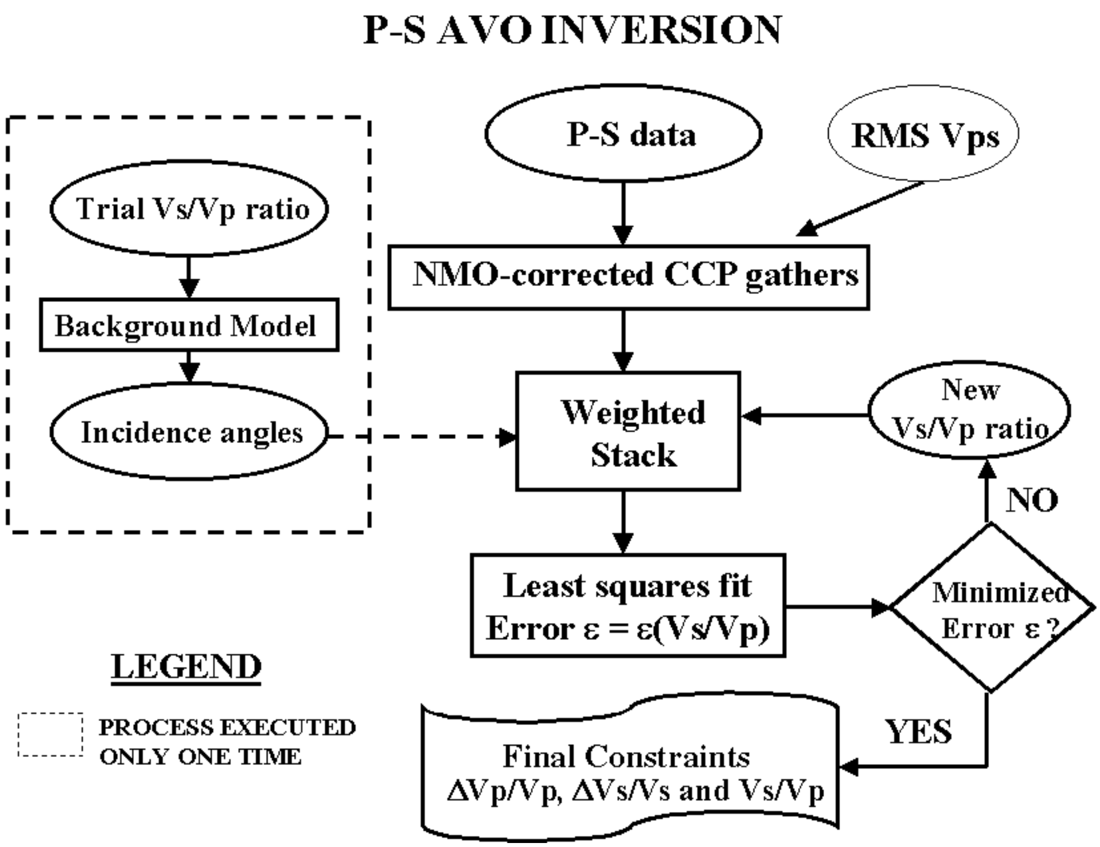

Figure 1. P-S AVO inversion

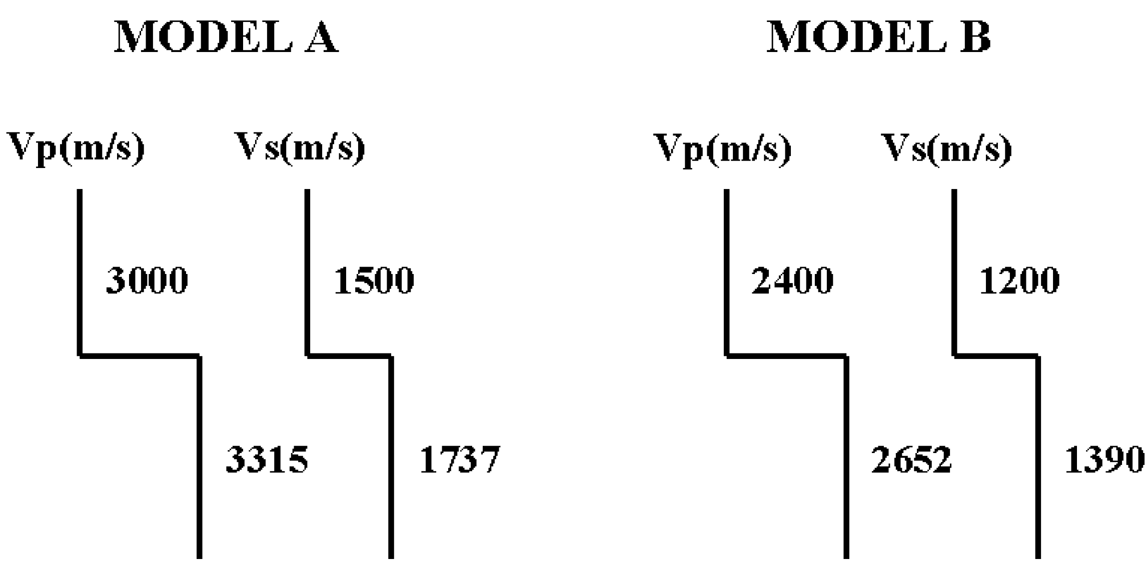

Figure 2. Different P- and S-wave velocity models fitting the same ΔVp/Vp, ΔVs/Vs, and Vs/Vp constraints. Constraints used: ΔVp/Vp = 0.1, ΔVs/Vs = 0.14, and Vs/Vp = 0.51.

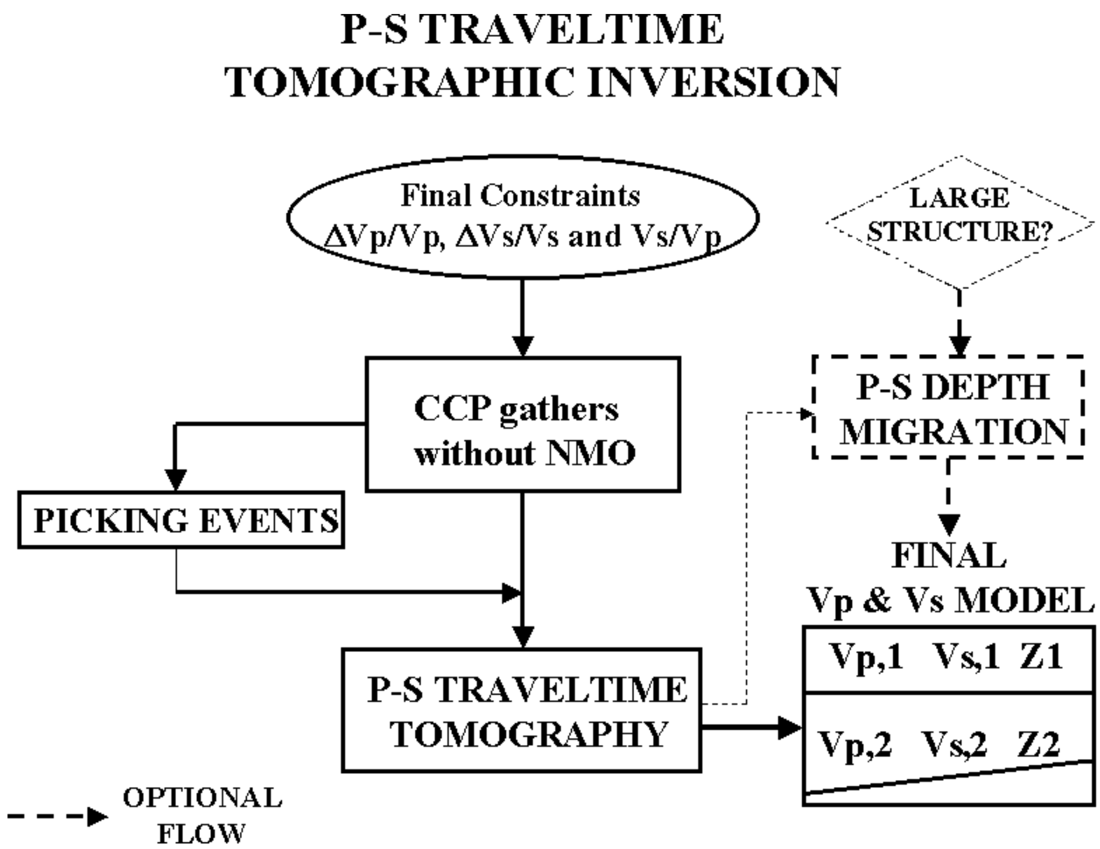

Figure 3. P-S traveltime tomographic inversion