Past, present and future of geophysical inversion – a Y2K analysis

Sven Treitel[1] and Laurence R. Lines

ABSTRACT

The estimation of subsurface properties of the earth by use of observed geophysical data is identified as "geophysical inversion", and can be defined to include nearly everything that geophysicists do. As we approach the new millennium, it is interesting to contrast past and present inversion methods with possibilities for the future. Although the earliest inversions were done entirely within the geophysicist's brain, inversion became more quantitative and computational with the advent of high performance computing. Geophysicists have recently invoked global optimization through the use of exhaustive search techniques. Three-dimensional elastic models have been included as inversion drivers. Constraints and Bayesian statistics are used in the appraisal of inversion solutions. Although inversion progress has advanced with concomitant progress in computing, automated inversion, without a human arbiter, is still a fantasy.

INTRODUCTION

Geophysicists have been working on solutions to the inverse problem since the dawn of our profession. An interpreter infers subsurface properties on the basis of observed data sets, such as seismograms or potential field recordings. A rough model of the process that produces the recorded data resides within the interpreter's brain; the interpreter then uses this rough mental model to reconstruct subsurface properties from the observed data. In modern parlance, the inference of subsurface properties from observed data is identified with the solution of a so-called "inverse problem". In contrast, the "forward problem" consists of the determination of the data that would be recorded for a given subsurface configuration and under the assumption that given laws of physics hold. Until the early sixties geophysical inversion was carried out almost exclusively within the geophysicist's brain. Since then we have learned to make the geophysical inversion process much more quantitative and versatile by recourse to a growing body of theory, along with the computer power to reduce this theory to practice. We should point out the obvious, however, namely that no theory and no computer algorithm can presumably replace the ultimate arbiter who decides whether the results of an inversion make sense or represent nonsense: the geophysical interpreter. Perhaps our descendants writing a future Y3K review article can report that a machine has been solving the inverse problem without a human arbiter---for the time being, however, what might be called "unsupervised geophysical inversion" remains but a dream.

Let us now adhere to the above very broad definition of the inverse problem: then the familiar algorithms we apply in the processing center can all be viewed as procedures to invert geophysical data. For example, seismic migration tries to

[1] TriDekon Inc.

reconstruct actual subsurface geometry from seismic recordings (Gardner, 1985). Inversion for the earth's reflectivity can be achieved by attenuating multiple reflections through predictive deconvolution (Peacock and Treitel, 1969) or by modeling the primaries and multiples in the earth's impulse response (Lines and Treitel, 1984). Amplitude variation with offset (AVO) (Castagna and Backus, 1993) processing involves inversion for rock properties from surface amplitude measurements. Inversion can handle different types of geophysical data. One can simultaneously or sequentially fit different geophysical data sets such as seismic, potential field, and borehole data with the same earth model (e.g., Lines, Schultz, and Treitel, 1988). Many other inversion examples abound. In each case we assume that a physical law holds. In most seismic inversions, for example, these laws are described by the seismic wave equation. Then algorithms based on this physical law enable us to invert the observed data for the subsurface characteristics, which gave rise to these observations in the first place.

Ours is a very broad definition of inversion, one that is (regrettably) not followed in the general literature, and most definitely not in the literature of exploration geophysics. After outlining the theoretical background of the inverse method, we will discuss some of the more popular procedures in exploration geophysics that are commonly identified as "inverse methods". We need to reiterate, however, that these identifications have often been arbitrary, and that much of what we do routinely falls under the umbrella of geophysical inversion.

THEORETICAL BACKGROUND

Inversion can be defined as a procedure for obtaining subsurface models that may adequately describe an observed data set. In the case of geophysical data, our observations consist of what might be called the physical signature of a subsurface structure: the structure's reflected (or scattered) wave field due to excitation by a seismic or an electromagnetic source; a structure's anomalous gravity or magnetic field, etc. The theoretical foundations for modern inverse theory can be found in the early work of Backus and Gilbert (1967, 1968, and 1970).

The inversion process is closely related to forward modeling. Forward modeling makes use of a mathematical relationship, such as the wave equation, to synthesize an earth model's response to an excitation, for example a pulse of seismic energy. Such models are defined in terms of a set of parameters, such as layer velocities and layer densities. It is of course crucial to choose a forward modeling procedure which can adequately describe the observations. In the seismic case, forward modeling is implemented with an algorithm that produces a synthetic seismogram, such as a seismic ray tracer, a finite difference or a finite element wave equation solver. In the case of gravity data, for example, the forward modeling procedure consists of a code that computes gravity fields from an assumed subsurface density distribution. In addition to the choice of an appropriate mathematical model, it is also important to know how many model parameters should be used and which parameters are the most significant. The choice of the "right" model depends on the exploration problem at hand. For example, a model of horizontal layers may be appropriate for geology in central Kansas, but would not be suitable for the Overthrust Belt of Wyoming or the Alberta foothills.

Inversion or "inverse modeling" attempts to reconstruct subsurface features from a given set of geophysical measurements, and to do so in a manner that the model response "fits" the observations according to some measure of error. Hence, the choice of a "good" model is crucial. Even assuming that our choice of model is adequate, numerous problems still remain. In fact, Jackson (1972) has aptly described inversion as the "interpretation of inaccurate, insufficient, and inconsistent data."

In addressing some of these issues, it is convenient to adopt symbolic notation. We denote the forward modeling process as a transformation $f = T(x)$, where f is the model response, x is a vector containing the set of subsurface model parameters, and T is some transformation (linear or nonlinear) which we assume will mathematically describe an observed physical process. In the seismic case, T produces a model response in the form of a synthetic seismogram. The inverse procedure can then be written as $x' = T^{-1}(y)$, where x' is now a vector containing the set of estimated subsurface model parameters (the model space) derived from the data vector y (the data space). The operator T^{-1} then denotes the inverse transformation from data space to model space.

Even if the model choice (or the choice of T) is physically meaningful, numerous problems remain. Firstly, it is possible that T^{-1} may not be determinable. The acquired data could have "blind spots". For example, a seismic source may fail to illuminate a given portion of the subsurface, so there will be no way to reconstruct it from the recorded data. In addition, real data is always corrupted by noise, and except for cases of theoretical interest, more than one subsurface model will inevitably satisfy the observed data within a specified measure of error---in other words, inversion is not unique. These problems have been studied intensely by the theoreticians. In their language, the inverse problem is "ill-posed"---small variations in the solution vector x' can produce large fluctuations in the model response f, and small fluctuations in the observed data y can produce large fluctuations in the solution vector x'.

The observed and the theoretical geophysical responses are matched by the use of a suitable optimization algorithm. All such algorithms are designed to minimize some measure of the difference between the observed and the computed data. Most schemes start out with an initial guess of the model parameters, from which an initial model response. In the next step, the optimization algorithm yields a set of adjusted or updated parameter estimates. These updated parameters are then "plugged" into the theoretical model, and the resulting new theoretical response should produce (hopefully) an improved match to the data. If this happens, the inversion is said to converge; if not, there are numerous means to achieve convergence, although none of the known methods is foolproof. Because the model response is generally a non-linear function of the model parameters, it is necessary to do these calculations iteratively; that is, the above procedure must be applied many times in succession until a satisfactory degree of agreement between the theoretical and the recorded seismic responses has been achieved.

A good match between the two responses provides us with a necessary, but by no means sufficient condition for the calculation to converge to the “ground truth” below us. As was remarked earlier, the solutions we obtain are not unique; in fact, it can be shown that an infinite number of solutions satisfy the data within prescribed error bounds (Cary and Chapman, 1988). Fortunately, we can constrain these solutions to be biased toward "a priori" knowledge about the subsurface parameters. Such constraints can be either "hard", say density and velocity are positive and lie between given lower and upper bounds, or they can be "soft" and expressible in the form of multidimensional probability density functions, whose dimension is the number of parameters describing a given model. Prior probability densities describing prior knowledge (or prejudices) about the model parameters can be combined with a so-called "likelihood function" which depends on the misfit between the model response and the observed data. We then end up with a so-called “a posteriori” probability density distribution of solutions to our given inverse problem. The peak (or peaks) in the resulting multidimensional "a posteriori" probability distribution should then reveal the most likely set of model parameter values. These values, in turn, should produce a model response satisfying the observed data within prescribed error bounds. Tarantola (1987) is one of the earliest proponents of this inversion philosophy. He based his ideas on the classic work of the British clergyman and statistician Thomas Bayes (1763). The more recent literature refers to the approach as "Bayesian Inversion". Excellent discussions can be found in papers by Duijndam (1988a, 1988b) and by Gouveia and Scales (1997, 1998). The broader implications of Bayesian inversion have been discussed in an incisive essay by Scales and Snieder (1997). It must be said that Bayesian inversion has yet to gain widespread use in the exploration geophysical world, but it shows much promise for the future.

The result of an inverse calculation depends on both the choice of the forward model whose response should match the observed data, as well as on the selection of an appropriate error criterion for minimization. The conventional approaches are based on the cumulative least squared error (LSE) and the cumulative least absolute deviation (LAD) principles. In addition to error criteria, it is usually advisable to employ smoothness constraints in order to avoid spurious oscillations in the solution vector (Constable, Parker, and Constable, 1987). More generally, there is a tradeoff between resolution and noise suppression: a better resolved (sharper) solution is achievable only at the price of poorer noise suppression, and vice-versa (Treitel and Lines, 1982).

The (general) nonlinear problem is typically solved by an iterative application of a given optimization algorithm. The problem is that in order for convergence to the “correct” subsurface model to take place, the initial guess must be “close” to the true solution. There has therefore been much recent interest in the development of so-called “global optimization” algorithms, which, in theory at least, can produce models whose responses fit the observed data well. Among such methods we mention genetic algorithms and simulated annealing (Smith, Scales, and Fischer, 1992; Sen and Stoffa, 1995), as well as Monte Carlo search (Cary and Chapman, 1988). More recently, there has been growing interest in the use of artificial neural networks to solve inverse problems (Calderon-Macias, Sen and Stoffa, 1998)

SOME POPULAR OLDER INVERSE METHODS IN EXPLORATION GEOPHYSICS

Much of seismic data processing is based on the assumption that local geology can be approximated by a one dimensional (1-D) layer-cake model consisting of a stack of horizontal, homogeneous plane parallel layers. Each layer possesses a characteristic density, velocity, and thickness. This simple earth model allowed Dix (1955) to estimate layer velocities from observed seismic reflection times and from the known seismic source-receiver distances. In other words, Dix was solving a geophysical inverse problem by determining layer velocities from observed travel-time data. The fact that Dix's method continues to enjoy such widespread application to this day illustrates the power and versatility of the simple 1-D subsurface model.

The layer-cake model also forms the basis for the familiar Common Midpoint (CMP) stacking procedure, for which the normal moveout corrected summation of seismic traces sharing a common source-receiver midpoint produces a sum trace that approximates the response of the layered 1-D earth to normally incident plane waves. A popular and highly successful seismic trace model for such data is given by the convolution of a source wavelet with the reflectivity (i.e., the series of subsurface normal incidence reflection coefficients) of the medium. In this case the objective of a 1-D inversion procedure is to recover estimates of the reflectivity from the CMP trace, along with layer thicknesses and impedance contrasts at each interface.

The estimation of normal incidence reflection coefficients for a layered medium is almost always based on the Goupillaud model (Goupillaud, 1961). This model consists of a stratified system in which all layers have equal two-way traveltime intervals. Subsequently Kunetz (1964) used the Goupillaud model to formulate an inversion procedure, which produces reflection coefficient estimates from knowledge of the layered medium's impulsive synthetic seismogram. Unfortunately, this approach turned out to be quite unstable in practice, and indeed was an early victim of the ill-posedness inherent in most inverse problem formulations. In current practice, reflection coefficient estimates are obtained with much more involved inversion algorithms. The stacked field trace is first dereverberated to attenuate multiply reflected energy, then it is submitted to signature deconvolution in order to obtain an estimate of normal incidence reflectivity. To the extent that this process works at all, an impedance estimation technique due to Lindseth (1972, 1979) and to Lavergne and Willm (1977) continues to be a popular and often-used seismic trace inversion tool. Lindseth gave the name "Seislog" to this method, since it can produce estimates of the continuous velocity log from the observed CMP trace. A "blocky" or parametric version of Seislog was introduced by Oldenburg et al. (1983). In fact, to many applied geophysicists the Seislog method is synonymous with seismic inversion in general. This is clearly not so, as we have already tried to indicate earlier. Because real seismic data are bandlimited and noisy, the Seislog method often breaks down, but it is also true that it has scored many successes.

SOME NEWER INVERSE METHODS IN EXPLORATION GEOPHYSICS

The past few decades have seen countless successful applications of inverse theory to global geophysics, but acceptance of these new technologies in our own field of exploration geophysics has not been quite as spectacular. This statement would clearly be false if seismic migration were considered part of geophysical inverse theory (as it ought to be!). Excluding seismic migration methods, then, we need next to discuss seismic travel-time inversion, more commonly known as seismic travel-time tomography, as well as seismic full-wave form inversion. In travel-time inversion a set of observed (picked) travel times are iteratively fitted to travel times obtained from a suitable forward modeling algorithm until a satisfactory degree of agreement between the observed and the computed travel times has been achieved. The preferred forward modeling algorithms for such calculations are 2-D or 3-D ray-tracers. These algorithms now exist for both acoustic as well as elastic media, and many can handle seismic anisotropy as well. Travel-time tomography has found an important niche in borehole-to-borehole surveys, where transmission velocity tomograms showing the detailed variation of the material velocity in profile planes joining two or more boreholes can be monitored over time if the surveys are repeated at given intervals. These tomograms thus constitute a crucial part of what has come to be known as "time-lapse", or 4-D reservoir monitoring. Travel-time reflection tomography has also come into its own in conjunction with seismic migration methods in order to produce iterative estimates of seismic migration velocities.

Full-wave form inversion is then the obvious generalization of travel-time inversion: rather than fitting observed picked travel times to computed travel times, full-wave form synthetic seismograms are now fitted to full wave-form recorded data, without the tedium of picking individual events, as is the case for travel-time inversion. For real-life problems, the computational demands for full waveform inversion continue to remain too vast for even present generation computers; this exciting technology has yet to see routine use in exploration geophysics. But recent work by Gouveia and Scales (1997, 1998) clearly suggests that impressive results will be achievable with full-wave form inversion once the computational hurdles have been overcome. One of the difficulties facing full waveform inversion is the danger of fitting a given model response to the noise components of the data. While this is a problem for all inverse schemes, it is particularly severe for full waveform inversion (see, e.g., Cary and Chapman, 1988).

Thus far the implication has been that each geophysical data set is inverted with a forward model selected to simulate the particular physical process producing the recordings. Thus a gravity simulation algorithm might produce a synthetic gravity field that is to be matched to a set of observed gravity readings, while a seismic wave propagation simulator might produce synthetic seismograms to be matched to a set of seismic field traces, and so on. Clearly it makes sense to invert combinations of geophysical data sets in an attempt to gain additional subsurface information. The question then arises whether to invert the data jointly or sequentially. In the former case, seismic data and gravity data or simultaneously fitted to their respective observed data sets; in the latter, the structural information obtained from say an initial seismic inverse calculation is used as input for the simulation of a structure for which the gravity field is to be determined, and so on. A big unsolved problem in the case of joint inversion is the relative weight to be given to each data set. There is no known objective way to do this, so this weighting is perforce subjective. Lines, Schultz and Treitel (1988) discuss these issues in some detail, and illustrate both approaches with examples.

TOWARD THE FUTURE

Earlier on we remarked that most of the currently used geophysical processing techniques can be viewed as attempts to solve the ubiquitous inverse problem: we have geophysical data, we have an abstract model of the process that produces the data, and we seek algorithms that allow us to invert for the model parameters. The theoretical and computational aspects of inverse theory will gain importance as geophysical processing technology continues to evolve. Iterative geophysical inversion is not yet in widespread use in the exploration industry today because the computing resources are barely adequate for the purpose. After all, it is only now that 3-D prestack depth migration has become economically feasible, and the day will surely not be far off when the inversion algorithms described above will come into their own, enabling the geophysicist to invert observations not only for a structure's subsurface geometry, but also for a growing number of detailed physical, chemical, and geological features. The day that such operations become routine will also be the day that geophysical inverse theory has come into its own in both mineral exploration and petroleum exploration.

REFERENCES

Backus, G.E., and Gilbert, J.F., 1967, Numerical applications of a formalism for geophysical inverse problems: Geophy. J. Roy. Astr. Soc., 13, 247-276.

Backus, G.E., and Gilbert, J.F., 1968, The resolving power of gross earth data: Geophy. J. Roy. Astr. Soc., 16, 169-205.

Backus, G.E., and Gilbert, J.F., 1970, Uniqueness in the inversion of inaccurate gross earth data: Phil. Trans. Roy. Soc. London, 266, 123-192.

Bayes, T., 1763, Essay towards solving a problem in the doctrine of changes: republished in 1958 in Biometrika, 45, 298-315.

Cary, P., and Chapman, C.H., 1988, Automatic 1-D waveform inversion of marine seismic retraction data: Geophys. Journal, 105, 289-294.

Calderon-Macias, C., Sen, M.K., and Stoffa, P., 1998, Automatic NMO correction and velocity estimation by a feedforward neural network: Geophysics, 63, 1696-1707.

Castagna, J.P., and Backus, M.M., 1993, Offset-dependent reflectivity – Theory and practice of AVO analysis: SEG publication, Tulsa

Constable, S.C., Parker, R.L., and Constable, C.G., 1987, Occam's inversion: A practical algorithm for generating smooth models from electromagnetic sounding data: Geophysics, 52, 289-300.

Duijndam, A.J.W., 1998a, Bayesian estimation in seismic inversion. Part I: Principles: Geophys. Prosp., 36, 857-989.

Duijndam, A.J.W., 1998b, Bayesian estimation in seismic inversion. Part II: Uncertainty Analysis: Geophys. Props., 36, 899-918.

Gardner, G.H.F., 1985, Migration of seismic data: SEG Publication, Tulsa

Gouveia, W.P. and Scales, J.A., 1997, Resolution of seismic waveform inversion: Bayes versus Occam: Inverse Problems, 13, 323-349.

Gouveia, W.P. and Scales, J.A., 1998, Bayesian seismic waveform inversion: parameter estimation and uncertainty analysis: Jl. Geoph. Res., 103, 2759-2779.

Goupillaud, P., 1961, An approach to inverse filtering of near-surface layer effects from seismic records: Geophysics, 46, 754-760.

Jackson, D.D., 1972, Interpretation of inaccurate, insufficient and inconsistent data: Geophys. J. Roy. Astr. Soc., 28, 97-109.

Kunetz, G., 1964, Generalization des operateurs d'antiresonance a un nombre quelconque de eflecteurs: Geophys. Prosp., 12, 283-289.

Lavergne, M., and Willm, C., 1977, Inversion of seismograms and pseudo velocity logs: Geophys. Prosp., 25, 231-250.

Lindseth, R.O., 1972, Approximation of acoustic logs from seismic traces: J. Canadian Well Logging Society, 5, 13-26.

Lindseth, R.O., 1979, Synthetic sonic logs – a process for stratigraphic interpretation: Geophysics, 44, 3-26.

Lines, L.R., and Treitel, S., 1984, Tutorial: A review of least-squares inversion and its application to geophysical problems: Geophys. Prosp., 32, 159-186.

Lines, L.R., Schultz, A., and Treitel, S., 1988, Cooperative inversion of geophysical data: Geophysics, 53, 8-20.

Oldenburg, D.W., Scheuer, T., and Levy, S., 1983, Recovery of the acoustic impedance from reflection seismograms: Geophysics, 48, 1318-1337.

Peacock, K., and Treitel, S., 1969, Predictive deconvolution – theory and practice: Geophysics, 34, 155-169.

Sen, M. and Stoffa, P.L., 1995, Global optimization methods in geophysical inversion: Elsevier Science Publishing Co.

Smith, M., Scales, J.A., and Fischer, T., 1992, Global search and genetic algorithms: The Leading Edge, 11, 22-26.

Scales, J.A., and Sneider, R., 1997, To Bayes or not to Bayes: Geophysics, 62, 1045-1046.

Tarantola, A., 1987, Inverse problem theory – methods for data fitting and model parameter estimation: Elsevier Science Publishing Co.

Treitel, S., and Lines, L.R., 1982, Linear inverse theory and deconvolution: Geophysics, 47, 1153-1159.