Optimal time-delay spiking deconvolution and its application in the physical model measurement

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ABSTRACT

Spike deconvolution based on Wiener filter theory usually has the desired output as a zero-lag spike, which works well for wavelets of minimum phase. In this paper, the algorithm that extends the standard spiking deconvolution to mixed-phase wavelet is presented. The optimal output spike has a time delay, determined from the projection matrix, and forms an optimal time-delay spiking deconvolution. The application of this algorithm to physical model measurements shows that this technique is efficient for the case when the wavelet is well determined.

INTRODUCTION

In seismic data processing, spiking deconvolution is routinely applied to compress the effective source wavelet contained in the seismic traces to improve temporal resolution. For the case of a normal incidence plane wave (and neglecting multiples), a seismic trace, the seismic recorded seismogram *x(t)* is a convolution between the primary reflectivity *r(t)* and the seismic wavelet w(t), *i.e.*

$$x(t) = r(t) * w(t) \tag{1}$$

where "*" denotes convolution (Yilmaz, 1987). In equation (1) noise is not considered. The goal of the spiking deconvolution process is to recover *r(t)* by removing the effects of *w(t)* from *x(t)*. If a filter operator *f(t)* were defined such that convolution of *f(t)* with the known seismogram *x(t)* produces an estimate $\hat{r}(t)$, then

$$\hat{r}(t) = x(t) * f(t) \tag{2}$$

By substituting equation (2) into equation (1), we have

$$\hat{r}(t) = r(t) * w(t) * f(t) \tag{3}$$

therefore, in order to have a best estimate of *r(t)*, we need

$$\delta_0(t) = w(t) * f(t) \tag{4}$$

where $\delta_0(t)$ represents the zero time delay delta function, i.e. $\delta_0(t)=1$ when $t=0$ and $\delta(t)=0$ when $t \neq 0$. When the wavelet *w(t)* is well estimated, the problem of deconvolution defined in equation (2) becomes that of solving equation (4) to obtain the inverse filter operator *f(t)*.

In real applications, both *w(t)* and *f(t)* can be represented as discrete forms with discrete samples *l* and *n*, respectively, and equation (4) can be written in a matrix form (e.g. Hatton et al., 1988)

$$\underline{\underline{W}}\,\underline{F} = \underline{Y} \tag{5}$$

where

$$\underline{\underline{W}} = \begin{bmatrix} w_0 & 0 & 0 & \cdot & \cdot & 0 \\ w_1 & w_0 & \cdot & \cdot & \cdot & \cdot \\ w_2 & w_1 & w_0 & \cdot & \cdot & \cdot \\ w_3 & w_2 & w_1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & \cdot & w_{m-1} \end{bmatrix}_{m\times n}$$

and

$$\underline{F} = (f_1, f_2, \ldots\ldots f_{n-1})^T, \qquad \underline{Y} = (y_0, y_1, \ldots\ldots, y_{m-1})^T.$$

where $m=n+l$. Generally, equation (5) does not have an exact solution because m is always larger than n and $w(t)$ is bandlimited. The inverse filter, $\underline{F}$, which satisfies equation (5) in the least-squares sense, is written in the form of

$$\underline{F} = (\underline{\underline{W}}^T \underline{\underline{W}})^{-1} \underline{\underline{W}}^T \underline{Y} \tag{6}$$

is called a Wiener filter. The operator $\underline{F}$ corresponding to a zero time-delay spike delta function forms the standard spiking deconvolution operator, which is routinely applied to seismic data processing.

It is well known that standard spiking deconvolution works well when the wavelet is minimum phase (e.g. Yilmaz, 1987). The physical explanation of this design is that the distribution of energy of minimum phase wavelet is concentrated at the beginning. Therefore, the standard spiking deconvolution may not work optimally if the wavelet is not a minimum phase. For example, if the wavelet is a maximum phase, the desired output should be a spike with a maximum phase delay, i.e. (e.g. Karl, 1989)

$$\underline{Y} = \delta_{\max}(t) = (0,0,\ldots\ldots,0,y_{m-1})^T \tag{7}$$

As increasing knowledge of the mixed delay nature of seismic wavelets is obtained, more work is done on the spiking of mixed delay wavelets (e.g. Ziolkowski, 1970; Eisner & Hampson, 1990). In this paper, we present a simpler algorithm for the case that the mixed phase wavelet is known. This method can be combined with methods of seismic wavelet estimation technique for a complete deconvolution. The example of spiking deconvolution of the physical model data illuminates the efficiency of our algorithm.

OPTIMAL TIME DELAY SPIKE DECONVOLUTION

Equation (6), which is derived by a least squares method, often gives an impression that the inverse filter operator obtained from this equation is always the best because object function $\| \underline{\underline{W}}\,\underline{F} - \underline{Y} \|^2$ is minimum. However, this impression is sometime misleading. Taking a thought experiment as an example: assume we have a maximum phase wavelet and solve equation (6) for the desired outputs as the zero- and maximum-lag spikes, respectively. Then, comparing the values of $\| \underline{\underline{W}}\,\underline{F} - \underline{Y} \|^2$ for these two different cases, we will find that even though in each case the object function is minimum, the values are different. Therefore, as Ford (1978) noticed, in spiking deconvolution, the object function depends on the different delays of the desired output spike. The optimal time delay spike deconvolution is the one where the object function is the smallest among the all the possible time delays (Robinson and Treitel, 1980).

Substituting equation (6) into (5), the predicted output with $\underline{F}$ is given by

$$\hat{Y} = \underline{\underline{W}}(\underline{\underline{W}}^T \underline{\underline{W}})^{-1} \underline{\underline{W}}^T Y \quad . \tag{8}$$

Defining

$$\underline{\underline{P}} = \underline{\underline{W}}(\underline{\underline{W}}^T \underline{\underline{W}})^{-1} \underline{\underline{W}}^T \tag{9}$$

as a projection matrix, this matrix shows how the designed output Y can be mapped into the predicted output. Since the designed output of spiking deconvolution is a spike with a time delay, the predicted output is actually one of the column vectors in the projection matrix that corresponds to the specific time delay. For example, if the time delay is the j-th sample in Y, then is the j-th column in the projection matrix. Therefore, the matrix $\underline{\underline{P}}$ gives total information for all possible effects of the deconvolution. Ideally, if the matrix P is an identity matrix, then $\hat{Y}$ is the same as Y. However, $\underline{\underline{P}}$ is often not the identity. Therefore, we need to find the one column vector that is most close to a spike, which is equivalent to finding the one that makes the objective function minimal. The optimal time delay can be obtained by visual inspection of the projection matrix. It can be also performed by minimizing the following function $\phi(t)$

$$\Phi_j = \sum_i [p_{i,j} / \max(p_{i,j})]^2, \qquad \mathrm{i} \neq \mathrm{j} \pm \mathrm{q}, \quad j = 1,2,...n \tag{10}$$

where q is the resolution width. If $\hat{Y}$ is a spike at the j time sample delay, then σ_j is zero.

APPLICATION IN THE PHYSICAL MODEL MEASUREMENT

The seismic measurement was carried out on the physical made of acrylics with its volume of $5700 \times 3550 \times 500$ (length, width, height) mm^3 (figure 1). The velocity of the material is 2750 m/s for acoustic waves. Grooves were cut at the bottom with the widths increasing from 1 mm to 10 mm. The height of the grooves is 5mm. Nearly zero offset seismic data were taken at the middle of the top along the line that is perpendicular to the grooves.

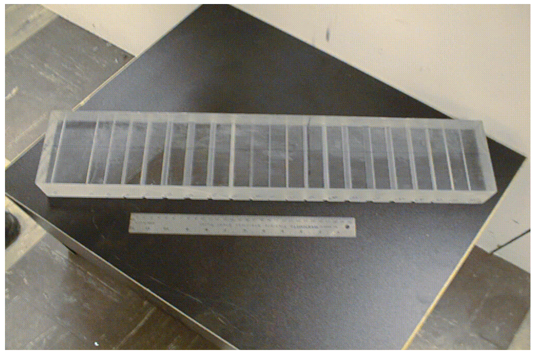

Figure 1: Physical model for seismic experiments.

Figure 2a is the image of the measured data set. It shows that the main signals are the waves reflected from the bottom of the model and the tops of the groves. The traces from 40 to 80 are plotted in figure 2b.

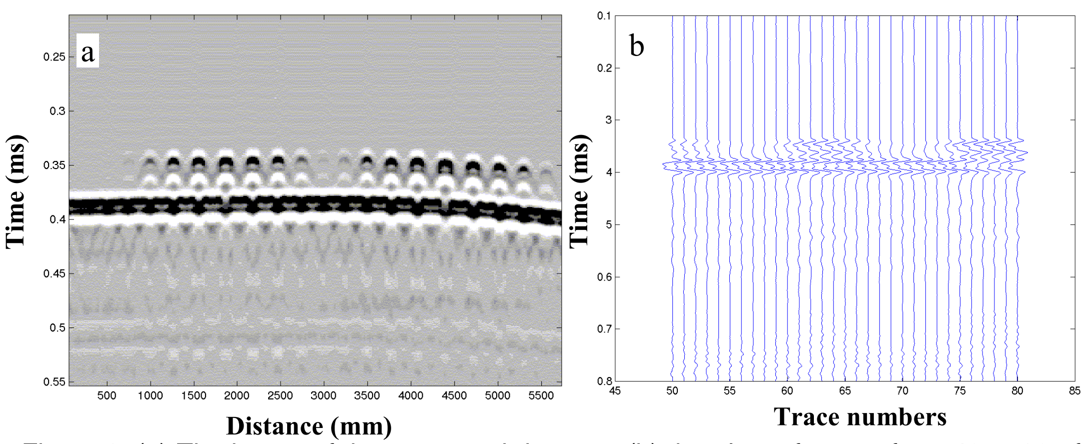

Figure 2. (a) The image of the measured data set; (b) the plots of traces from 40 to 80 of the measured data.

The source wavelet as estimated from reflections at the bottom on the left is shown in figure 3.

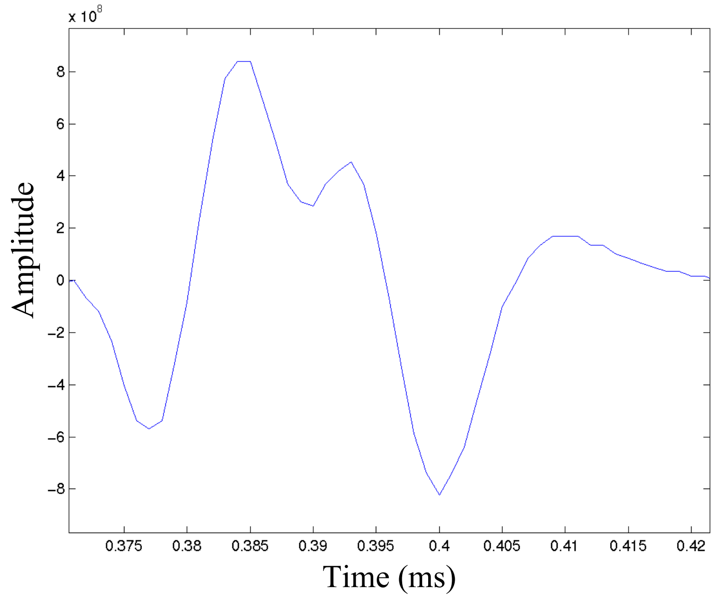

Figure 3. The estimated wavelet from reflections.

The image of project matrix is shown in figure 4a, which gives all the information of the possible effects of different time lag spike deconvolutions. Figure 4b is the plots of the first 20 columns of the matrix as the traces display. From the figure we can see that the standard spiking deconvolution does not work very well in this particular case.

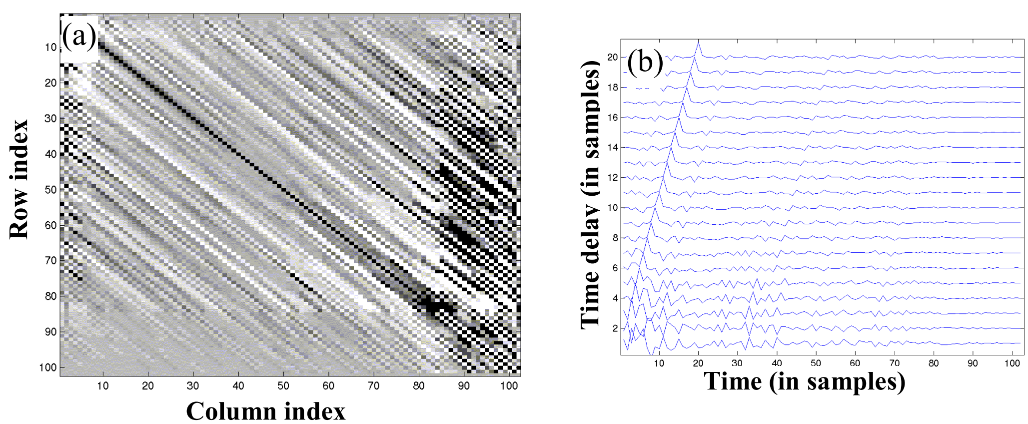

Figure 4. (a). The image of the projection matrix and (b) first 20 columns plotted as the seismic traces.

Figure 5 is the plot of the objective function Φ, which shows that the optimal time delay is about 42 samples time delay. The results of the wavelet deconvolution with zero- and 42- sample delays are shown in figure 6 indicating that the 42 samples time delay operator produces more spiking result. The zero- and 42 time samples delay spiking deconvolution operators are applied to the measured data set, respectively and the results are shown images in figure 7 and their parts of the traces are in figure 8.

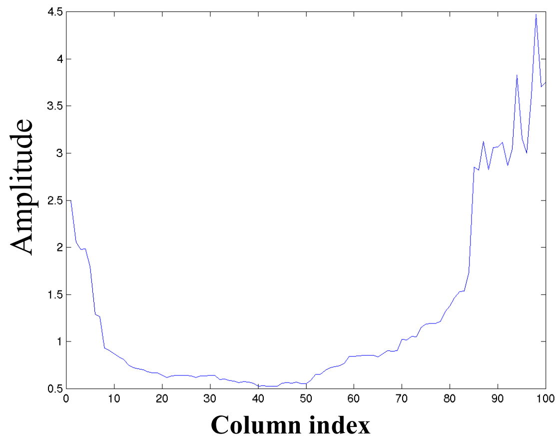

Figure 5. The plot of Φ versus the lag of the desired output spike.

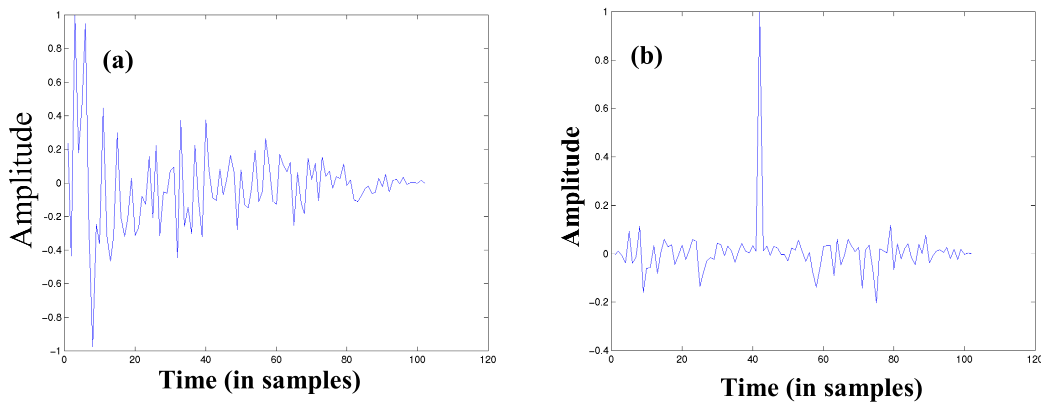

Figure 6. The results of the wavelet deconvolution with (a) zero lag and (b) 42 time sample delays spiking operators.

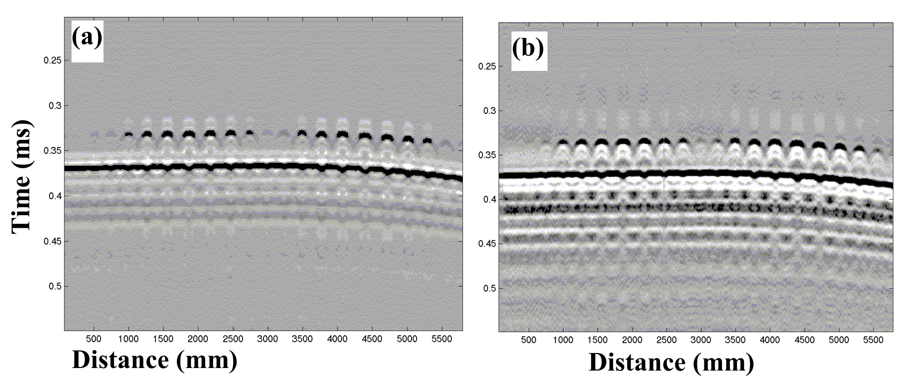

Figure 7. The images of the measured data after deconvolution with (a) zero- and (b) 42 time samples time delay spiking deconvolution operators.

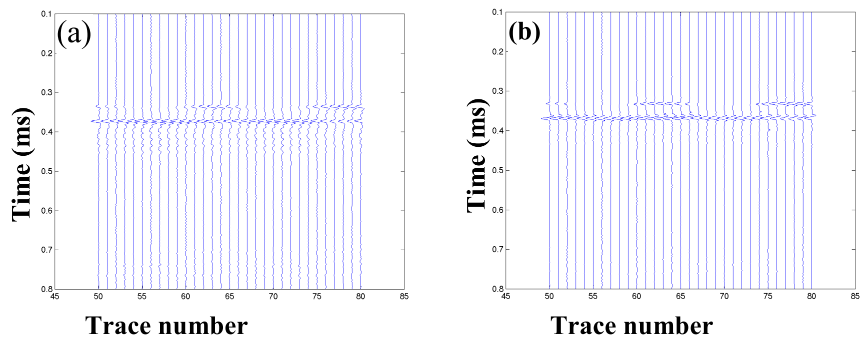

Figure 8. The traces plots (from 40 to 80) of the measured data after spiking deconvolution with (a) zero- and (b) 42 time samples time delay spiking deconvolution operators.

DISCUSSION AND CONCLUSIONS

The projection matrix provides complete information about the possible effects of different time lag spiking deconvolutions. The time delay determined via the project ion matrix leads to an optimal time-delay spiking deconvolution. The application of this technique to a physical model measurement shows that, if source wavelet is well known, this technique is effective. Based on the same idea, finding the smallest value of the function Φ(t), Ford (1978) developed an algorithm using z-transform and Robinson and Treitel (1980) developed an algorithm by comparing the results of all possible time-day spiking deconvolutions. However, ours is much simpler than these algorithms. Because the process of the calculation in our algorithm is exactly the same as Wiener filter, our algorithm should have all of the advantages of a Wiener filter. It should mentioned that the inverse of ($\underline{\underline{W}}^T \underline{\underline{W}}$) may not exist and therefore, the general inverse may need to be applied.

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