A proposal for suppression of downcoming waves at the ocean bottom with multicomponent data

Chanpen Silawongsawat and Gary F. Margrave

ABSTRACT

A new method to combine pressure and vertical and horizontal particle velocities at the ocean bottom is proposed to separate up- and downcoming waves. The method treats the problem in a manner that the total wavefield is accounted for, instead of only in vertical direction as the Barr and Sanders method. In order to support the theory, the scalar relationship between the vector wavefield of particle velocity and the scalar wavefield of pressure is constructed based on Hooke's law. A relationship between the radial component below and above seafloor is constructed accounting for the discontinuity of displacement at a liquid-solid interface. Then the expressions to combine the ocean bottom multicomponent data for the upcoming waves in each component are introduced.

INTRODUCTION

In the beginning, there were only two components, hydrophone and vertical geophone, combined in the Ocean Bottom Cable (OBC) measurement. Barr and Sanders (1989) introduced the pressure and vertical particle velocity (*P*-*Z*) combination procedure to attenuate water-column reverberations under a zero-offset assumption. Since then, the procedure has been developed using only these two components (2C).

Osen et al (1996) presented a possibility to remove the downcoming waves in particle velocity data by combining the vertical and radial components. Matching vertical and horizontal components can be difficult because the vertical's signals are strong where the radial's are weak, and vice versa, as shown in study of Amundsen and Reitan (1995). Another method has been presented by Ikelle (1999) with a need to include the towed streamer data as an extra component. It offers many more possibilities to remove almost all surface- and seabed-related multiples, and their consequences. Another advantage is that the knowledge of the seafloor properties can be totally omitted. However the towed streamer is often impractical in the congested area where the OBC is used.

Some experiments with a two-component (2C) method based on Barr and Sanders show unmatched amplitude at far-offset between pressure and vertical particle-velocity in both *x-t* and *tau-p* domain (Bale, 1998). Other than the angle variation of the ocean-bottom reflection coefficient, this can be because the pressure is a measurement of the total of a scalar wavefield whereas the vertical component, V_z, is only one component of the complete vector of magnitude $|V|$. The magnitude of $V_z/|V|$ usually becomes smaller as the offset increases. Also some events dominate in horizontal components and present very little in the vertical. In consequence, the errors from noise in real data when we try to match their amplitudes (vertical to pressure) can be significantly amplified, compared to the signal. On the contrary in the horizontal components, signals grow with the offset as the vertical decreases. Today, the horizontal measurements, radial and transverse, are always included in the OBC, considering how much more information gained at slight extra cost. Thus the downcoming wave elimination, which sometimes is called multiple attenuation or deghosting, should also include and be available to the horizontal components.

Conventional surface seismic usually deals only with upcoming and downgoing waves at the receivers. In the subsurface, the data is a combination of all possible waves: upcoming, downcoming, upgoing and downgoing, as the diagram shown in Figure 1(c).

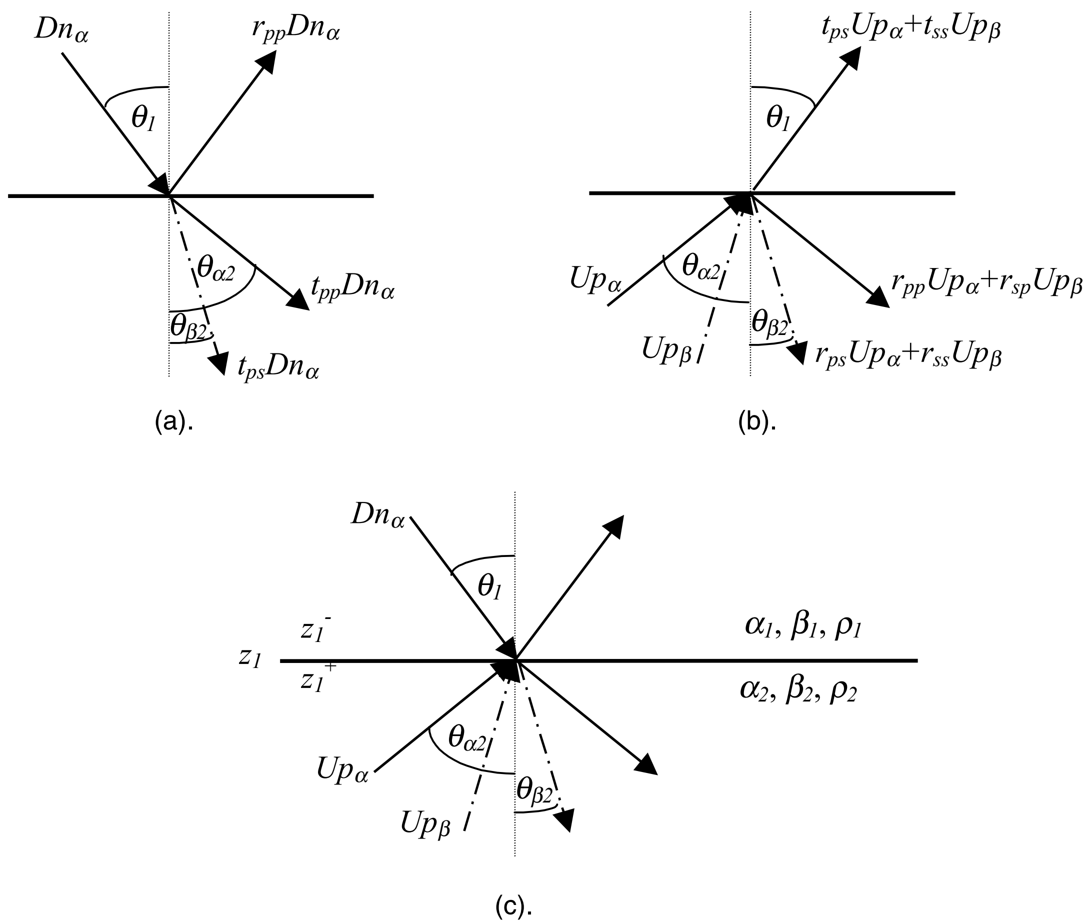

Figure 1. (a) A downcoming wave and its scattered waves. (b) Upcoming, P and S denoted by α and β respectively, and their scattered waves. (c) Plane wave arrivals at ocean bottom interface, z_1, which are the combination of (a) and (b). All waves have the same ray parameter.

However, the resultant waves, up- and downgoing, at an interface can be calculated from their incident waves, up- and downcoming, Figure 1(a) and (b). For surface seismic, this inclusion of reflected waves (Dankbaar,1985) at the geophone for the upcoming arrivals is called the free-surface effect. For subsurface seismic, we shall call the resultant wave inclusions at interface receivers an interface effect. Geophone responses are usually accounted for such effects prior to the decomposition into vertical and horizontal components. The up- and downcomings, which arrive at the receivers at the same time, are independent of each other and have different interface effects, Figure 1(a)-(c). In order to obtain the subsurface information, the up arrivals as in Figure 1(b) are desired. The method of this paper to separate the up- and down-coming waves is also a multiple suppression in water column. Additionally, it operates as the first step for a P-S mode separation at the ocean bottom, for instance the method presented by Donati (1996).

METHODOLOGY

Particle velocity and pressure relationship

There are two possible ways to draw a relationship between the scalar wavefield pressure, P, and the vector wavefield particle velocity, $\vec{v}$. One of them is through the equation of motion, which is

$$\vec{\nabla} P = \rho \frac{\partial \vec{v}}{\partial t} \tag{1}$$

where ρ is density. Noticeably, this is a dynamic vector equation. Its vertical component gives the expression for the vertical velocity as a function of the pressure (Amundsen, 1993). Barr and Sanders (1989) used this result in their method for the elimination of water column reverberations. However, in a general case, according to Sheriff and Geldart (1995, p.38), pressure is proportional to the fractional volume change, or dilatation, through

$$P = -\kappa \vec{\nabla} \cdot \vec{u} \tag{2}$$

where κ is the bulk modulus given for 2D by

$$\kappa = \rho(\alpha^2 - \beta^2), \tag{3}$$

or for 3D

$$\kappa = \rho(\alpha^2 - \frac{4}{3}\beta^2). \tag{4}$$

Thus we take a time derivative of equation (2) to get particle velocity, this equation becomes

$$\frac{\partial P}{\partial t} = \kappa \vec{\nabla} \cdot \vec{v}. \tag{5}$$

This is a scalar equation, which shows how a combination of all velocity components relates to the pressure. Consider the relationships between P and $\vec{v}$ expressed in equations (1) and (5). They are derived from two independent relations which, taken together, lead to the elastic wave equation. So if they are combined, the wave equation (or the dispersion relation) will result. They describe the relationships between the scalar wavefield pressure and the vector wavefield particle velocity in two different ways. Equation (1) states how the pressure individually relates to each velocity component. Alternatively, equation (6) is obtained from Hooke's law and relates pressure with a combination of all velocity components. In brief, the equation of motion connects the pressure and particle velocity as vectors whereas Hooke's law leads to a scalar relationship.

We can expand (5), for 2D, into horizontal and vertical (x and z) components as

$$\frac{\partial P}{\partial t} = -\kappa\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z}\right) . \tag{6}$$

Taking a double-spatial Fourier transform over x and z on equation (6) and separating upward and downward traveling waves, we have

$$i\omega\mathscr{P} = -\kappa(ik_x V_x + ik_z V_{z+} - ik_z V_{z-}) \tag{7}$$

or

$$\mathscr{P} = -\kappa(pV_x + qV_{z+} - qV_{z-}) , \tag{8}$$

where subscripts z+ denotes a wavefield propagates in the increasing z direction, downward traveling in depth, and z- denotes upward traveling wavefield. Also p and q are horizontal and vertical slowness given by

$$p = \frac{k_x}{\omega} = \frac{\sin\theta}{\alpha} \quad \text{and} \quad q = \frac{k_z}{\omega} = \frac{\cos\theta}{\alpha} , \tag{9}$$

ω is the temporal frequency and θ is the wave propagation angle. The spatial derivative depends on the direction of wave propagation in relation to the direction of the derivative. Accordingly, the vertical component, $\frac{\partial v_z}{\partial z}$, is split into two separate terms for upward and downward traveling waves as shown. For simplification, we consider only the positive offset of a record.

This equation (8) is the starting point for a method that includes the horizontal and vertical components for water-layer reverberation suppression.

Interface effects for downcoming waves

The receivers, located at the ocean bottom which is a liquid-solid interface, detect not only the incident wavefield arriving at the interface but also the resultant waves scattered from there. Let r_{pp}, t_{pp} and t_{ps} be P-P reflection, P-P and P-S transmission coefficients of the ocean bottom, respectively. Considering the reaction just above the ocean bottom in Figure 1(a), only P-waves exist in the water. Thus in the water layer, there is only a reflected P as a scattered wave from a downcoming P as an incident wave. A subscript *Dn* denotes a downcoming incident wave with its interface effects.

Referring to Figure 2, while the hydrophones and the horizontal geophones would detect ($1+r_{pp}$) as the sum of incident and scattered waves, the vertical geophone would record ($1-r_{pp}$) because of the directional nature of particle displacement. Let z_1 be the depth of the seafloor and the superscripts – and + indicate levels just above and just below the seafloor, respectively. Mathematical statements for the content above are

$$\mathscr{P}_{Dn}(z_1^-) = (1 + r_{pp})\mathscr{P}_{Inc}(z_1^-) , \tag{10}$$

$$V_{xDn}(z_1^-) = (1 + r_{pp})V_{xInc}(z_1^-) , \tag{11}$$

$$V_{zDn}(z_1^-) = (1 - r_{pp})V_{zInc}(z_1^-) , \tag{12}$$

where the *Inc* subscripts an incident downcoming wave.

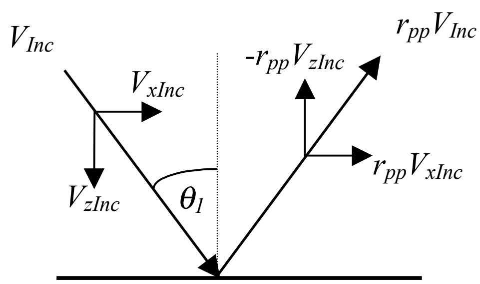

Figure 2. Figurative explanation for equation (11) and (12)

Equation (8) is valid for the total pressure and velocity wavefields at some position and time. However, it is simpler to work with the incident wavefields before reflection. This gives

$$\mathscr{P}_{Inc}(z_1^-) = -\kappa(pV_{xInc}(z_1^-) + qV_{zInc}(z_1^-)) \tag{13}$$

Then, substitute into equation (13) the expressions for the incident wave as a function of the scattered waves from (10)-(12). This becomes

$$\frac{1}{1+r_{pp}}\mathscr{P}_{Dn}(z_1^-) = -\kappa[\frac{p}{1+r_{pp}}V_{xDn}(z_1^-) + \frac{q}{1-r_{pp}}V_{zDn}(z_1^-)] . \tag{14}$$

This expression relates the pressure field measured by the hydrophones in the water layer above the seafloor, z_1^-, to the expected velocity components. Pressure is also a combination of normal stresses that are continuous across a liquid-solid interface (Sheriff and Geldart, 1995, p.70). Thus the pressure term can be either above or below z_1.

$$\mathscr{P}_{Dn}(z_1^+) = \mathscr{P}_{Dn}(z_1^-) \tag{15}$$

However the velocity field is measured, by the geophones that couple with the earth layer, below the interface. Even though the horizontal velocity is discontinuous, the relation between $V_x(z_1^-)$ above and $V_x(z_1^+)$ underneath the liquid-solid interface for the horizontal component of downcoming waves can be deduced. Consider the particle velocity below z_1 in Figure 1(a),

$$V_{xDn}(z_1^+) = (\frac{\alpha_2}{\alpha_1} t_{pp} + \frac{\beta_2 q_{\beta 2}}{\alpha_1 p} t_{ps}) V_{xInc}(z_1^-) \quad . \tag{16}$$

The vertical component of the particle velocity is continuous across the liquid-solid interface.

$$V_{zDn}(z_1^+) = (\alpha_2 q_{\alpha 2} t_{pp} - \alpha_2 p t_{ps}) \frac{V_{zInc}(z_1^-)}{\alpha_1 q_1} = V_{zDn}(z_1^-) \quad . \tag{17}$$

Combining equations (12) and (16) through the horizontal component of the incident wave V_{xInc} gives

$$V_{xDn}(z_1^-) = \left(\frac{\alpha_1 p(1 + r_{pp})}{\alpha_2 p t_{pp} + \beta_2 q_{\beta 2} t_{ps}} \right) V_{xDn}(z_1^+) \quad . \tag{18}$$

Therefore, together with the implications of (15) and (17), the substitution of $V_x(z_1^-)$ in equation (18) into (14) gives the multicomponent connection of the downcoming incident wave for the OBC data as

$$\frac{1}{1 + r_{pp}} \mathscr{P}_{Dn}(z_1^{\pm}) = -\kappa[\varepsilon p V_{xDn}(z_1^+) + \frac{q}{1 - r_{pp}} V_{zDn}(z_1^+)] \quad , \tag{19}$$

where ε is

$$\varepsilon = \frac{\alpha_1 \; p}{\alpha_2 p t_{pp} + \beta_2 q_{\beta 2} t_{ps}} \quad . \tag{20}$$

Elimination of downcoming and the normalization of upcoming wave

At the liquid-solid interface in Figure 1(c), all receiver responses can be described in terms of their up and down arrivals including their scatterings from Figure 1(a) and (b), respectively, as

$$\mathscr{P}(z_1) = \mathscr{P}_{Up}(z_1^+) + \mathscr{P}_{Dn}(z_1^+) \quad , \tag{21}$$

$$V_x(z_1) = V_{xUp}(z_1^+) + V_{xDn}(z_1^+), \tag{22}$$

$$V_z(z_1) = -V_{zUp}(z_1^+) + V_{zDn}(z_1^+). \tag{23}$$

The up and down terms in (24)-(24) are independent of each other. However, only the vertical component is sensitive to the up-down direction of propagation. (Mathematically, note the convention that the upward traveling waves are negative and downward traveling waves are positive, corresponding to the depth axis. Yet the desired response is the upcoming waves. Thus the geophone has been set up to detect the upward traveling as a positive signals in seismic record, then the down traveling as a negative.) Due to this difference in sign conventions for up and down travelling waves in vertical component, the elimination of downcoming arrivals for pressure or vertical component can be eliminated. The last terms in (22)-(24), of downcoming waves with their interface effects, are related in equations (14) and (19) above and below a liquid-solid interface, respectively. Thus, substituting a downcoming response from one of (22)-(24), corresponding to the desired upcoming-response component, into (20) and rearrange the equation, yields

$$\mathscr{P}_{Up}(z_1^+) = \mathscr{P}(z_1^+) + \kappa(1+r_{pp})[\varepsilon p V_{xDn}(z_1^+) + \frac{q}{(1-r_{pp})} V_{zDn}(z_1^+)], \tag{24}$$

$$V_{xUp}(z_1^+) = V_x(z_1^+) + \frac{1}{\varepsilon p}[\frac{\mathscr{P}_{Dn}(z_1^+)}{\kappa(1+r_{pp})} + \frac{q}{1-r_{pp}} V_{zDn}(z_1^+)], \tag{25}$$

$$V_{zUp}(z_1^+) = -\{V_z(z_1^+) + \frac{1-r_{pp}}{q}[\frac{\mathscr{P}_{Dn}(z_1^+)}{\kappa(1+r_{pp})} + \varepsilon p V_{xDn}(z_1^+)]\}. \tag{26}$$

Assuming the same signs for up- and downcoming waves in the pressure and horizontal component and opposite signs in the vertical, we can use the combination of up and down arrivals from (21)-(23) for the subscripted downcoming waves in (24)-(26). Therefore, the upcoming waves constructively build up in those relations. Then, with the downcoming removed, normalization factors for each component, C_P, C_x and C_z, should be evaluated to re-normalize their upcoming amplitudes back to their original scales. Equations (24)-(26) become

$$\mathscr{P}_{Up}(z_1^+) = C_P\{\mathscr{P}(z_1^+) + \kappa(1+r_{pp})[\varepsilon p V_x(z_1^+) + \frac{q}{(1-r_{pp})} V_z(z_1^+)]\}, \tag{27}$$

$$V_{xUp}(z_1^+) = C_x\{V_x(z_1^+) + \frac{1}{\varepsilon p}[\frac{\mathscr{P}(z_1^+)}{\kappa(1+r_{pp})} + \frac{q}{1-r_{pp}} V_z(z_1^+)]\}, \tag{28}$$

$$V_{zUp}(z_1^+) = -C_z\{V_z(z_1^+) + \frac{1-r_{pp}}{q}[\frac{\mathscr{P}(z_1^+)}{\kappa(1+r_{pp})} + \varepsilon p V_x(z_1^+)]\}. \tag{29}$$

DISCUSSIONS

This proposed method, of multicomponent combination to eliminate the downcoming waves, may sound even more complicated and awkward to compute in practice than the *P-Z* procedure. Given two extra components in the summation, the requirement that all of them have to be well calibrated, especially instrumental scaling, can be very challenging. We also realize that the horizontal, radial and transverse components tend to be noisy and have different bandwidths than the vertical component and pressure.

Nevertheless, we would like to try an alternative approach and hope to learn more about the full potential of the multicomponent data. It might lead us to some other useful applications. For example, the possibility of estimating this combination in the time domain which would be computationally more efficient and could be used in the first processing stage for a rough estimate. A hint of this notion lies in the comparison between the combination results in the *x-t* and in the *tau-p* domains from Bale (1998). The errors of downcoming removal in the *x-t* domain increase with offset and their phases at negative and positive offsets are opposite. It is known that the data in the horizontal components behave similarly. The possibility to be able to match them is appealing, with both vector and scalar relationships. We hope, in addition, that the seafloor elastic properties needed might be estimated using cross scaling among all components.

With an acceptable estimation in time domain, then we might want to include the transverse (*Y*) component into the combination for the real 4C OBC data. Some adjustment for the 3D relationship might be added. An ocean bottom 4-C physical modeling from Gallant et al (1996) should be very useful for this study. By comparing the modeling data, numerical to physical, and the numerical modeling to a real data set, we should be able to investigate the OBC experiment in some detail.

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