Reflections on phase

Ayon K. Dey and Laurence R. Lines

ABSTRACT

In seismic exploration, statistical wavelet estimation and deconvolution are standard tools. Both of these processes make a minimum phase assumption about the actual wavelet embedded in the trace. The validity of this assumption is examined by using well-log reflectivity sequences, synthetic seismic traces, and by using a procedure for evaluating the resulting deconvolutions. Because of these investigations, this work presents a simple group of tests to be used in evaluating the validity of the minimum phase assumption.

INTRODUCTION

The processing of a digital seismic section is one of the most widely practiced activities in the field of exploration seismology. Signal deconvolution is a processing step that is usually carried out after exponential gain recovery and before or after velocity analysis, depending on the situation. The purpose of deconvolution is to improve data resolution by increasing the sharpness of the seismic reflections. In practice, this process attempts to shorten the seismic wavelet, broaden the wavelet's spectrum, remove wavelet phase delay, and to stabilize the wavelet from trace to trace.

Discussed below is the problem of estimating the wavelet's phase spectrum. In order to completely define the wavelet, its phase spectrum is required. Using a minimum phase wavelet (for dynamite sources) or a zero phase wavelet (for vibroseis sources) often determines the phase spectrum. The critical role of phase estimation in deconvolution can be established by making comparisons between wavelet deconvolutions where this assumption of minimum phase is valid and where the assumption is violated. Comparisons will allow for an evaluation of this minimum phase assumption. This comparison and evaluation will occur with a model data set because, with model data, the correct answers are known and therefore the assumption can be objectively analyzed. Speaking in a strictly theoretical sense, deconvolution eludes a rigorous mathematical justification because of the non-unique nature of the solution to the deconvolution problem. These investigations hope to evaluate how well the phase assumptions work in practice. Establishing the validity of the minimum phase assumption is critical because this is the assumption used for estimating the phase for dynamite and tuned air gun sources.

METHODOLOGY

The issue of concern is that of phase. As stated in the introduction, a fundamental assumption that is made about the seismic source signature (or seismic wavelet) is that it is minimum phase for impulsive sources. While this assumption is essential to the process of statistical signature deconvolution, it is generally accepted as being invalid. That being the case, it is essential to determine how poor this minimum phase assumption for the seismic wavelet is. In practical terms, it would be worthwhile to know how this assumption affects the deconvolution. To that end, a three pronged approach is taken.

Firstly, the effect of assuming that the wavelet is minimum phase is investigated with respect to resolving kernels. That is, wavelets of various phase are given as input to an optimum spiking deconvolution program. This program determines the optimum spiking position based on the input wavelet and then generates a resolving kernel that is the convolution of the input wavelet and the deconvolution filter estimated from it. A model scenario is considered. For this model, four wavelets are investigated: a model minimum phase wavelet, a mixed phase wavelet, another mixed phase wavelet, and a maximum phase wavelet.

After analyzing the resolving kernels, the investigation focuses on how the minimum phase assumption affects trace deconvolution. Here, a well-log test for phase effects is used. Again, the model scenario discussed above will serve as the arena of investigation. For this investigation, convolving the wavelets and a primaries only reflectivity sequence generates synthetic seismograms. These synthetics are then used to estimate Hilbert transform and Wiener-Levinson minimum phase wavelets. After this, the estimated wavelets are used to generate deconvolution filters. Convolving these filters with the synthetic trace will give two estimates of the reflectivity sequence used to create the trace. By comparing the three (the actual reflectivity sequence and the two estimates), the effect of the minimum phase assumption is evaluated. The use of noise-free sequences is to show how phase affects trace deconvolution in the simplest case. As complexity in the trace increases, the effect of phase compounds.

Finally, there is an investigation of another proposed test for phase. Here, the various synthetic sections are used to statistically compute deconvolution filters. Then the actual wavelet used to make the section in question is convolved with the estimated filter. Comparisons of these results to the ideal result of a spike allow for further evaluation of the minimum phase assumption.

Proposed tests for the phase assumption

An analysis of the phase assumption made in statistical deconvolution has led to the development of two tests for phase. The first test involves a well-log derived reflectivity sequence. Statistically estimated minimum phase wavelets are created from the input trace. These wavelets are then used to estimate minimum phase deconvolution filters. The filters are convolved with the input trace to generate reflectivity estimates. Comparisons of these estimates to the well-log derived reflectivity will allow one to ascertain the goodness of the phase assumption being made. The pseudocode outlining this test is as follows.

BEGIN: *A well-log validity test for the minimum phase assumption.*

for each *seismic trace* **do**

compute desired statistically estimated wavelet **;**

compute deconvolution filter based on said wavelet **;**

compute the trace deconvolution **;**

compare to the well-log derived reflectivity sequence **end do**

END

The 2nd test to check the validity of the phase assumption is fairly simple and easy to apply. It assumes that a trace and that some estimate of the wavelet exists. From this trace, a deconvolution filter is estimated based on the seismic trace. The design of the deconvolution filter is based on the seismic trace since this is what is generally available in the case of real data. Then, the actual wavelet is convolved with the estimated filter to produce a resolving kernel which is compared to the ideal spiking response. The resolving kernel's similarity to a spike will dictate the goodness of the phase assumption being made. A pseudocode describing this test is outlined below.

BEGIN: *A validity test for the minimum phase assumption.*

for each *seismic trace* **do**

compute Wiener-Levinson deconvolution filter **;**

compute the resolving kernel **;**

compare to the desired output delta function **end do**

END

Both of these tests are used on the model environment discussed above. The results section presents the model environment and a comprehensive set of results regarding the investigations on the phase assumption with this model.

RESULTS

Presented in the following subsection is the model environment where the minimum phase assumption is investigated. This scenario incorporates a minimum phase wavelet, a mixed phase wavelet, another mixed phase wavelet, and a maximum phase wavelet. This model also contains the synthetic traces associated with each of the wavelets and the reflectivity sequence used to create the synthetics. In addition, the minimum phase synthetic is used to generate two statistical minimum phase wavelet estimates and their deconvolution filters. The trace deconvolution investigations use these statistically estimated filters.

Model environment

Figures 1-4 show the various wavelets used in this model, the common reflectivity sequence for the model, and the associated synthetic traces. Note that the reflectivity is a primaries only reflectivity sequence (i.e. no multiples or noise) and is convolved with each of the wavelets to give the traces shown to the right. The final two images show the statistically estimated minimum phase wavelets. Figure 5 shows the model minimum phase wavelet, a Hilbert transform wavelet estimate, and the deconvolution filter associated with it. This figure shows that the model wavelet and the Hilbert transform estimate are similar waveforms but have different time delays and the statistical estimate has some minor noise in its tail. Figure 6 also shows the model minimum phase wavelet but the statistical wavelet and its filter are generated by the Wiener-Levinson method. Here, the waveforms are very dissimilar. In particular, the Wiener-Levinson double inverse wavelet is quite front loaded with minimal amounts of tail energy and is extremely characteristic of a spike response.

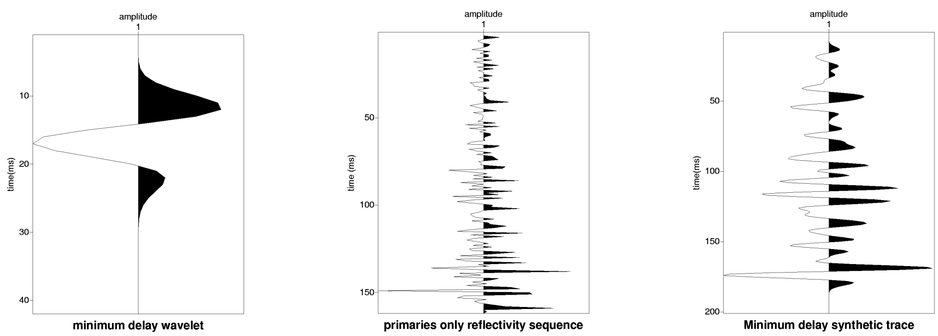

Figure 1: The minimum phase wavelet, $(-1.1+z)^2(1.75+z)^{38}$, and the primaries only reflectivity sequence are convolved to give the synthetic trace to the right.

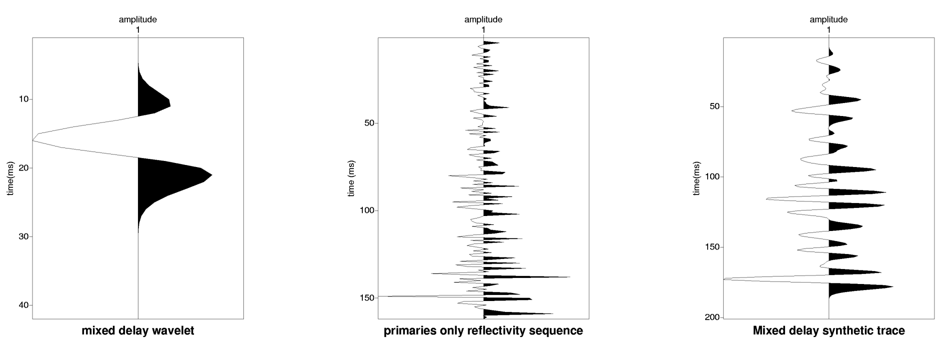

Figure 2: A mixed phase wavelet, $(1-1.1z)^2(1.75+z)^{38}$, and the primaries only reflectivity sequence are convolved to give the synthetic trace to the right.

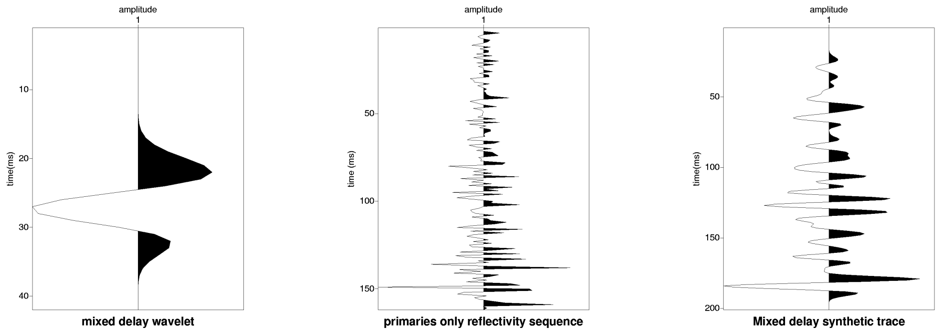

Figure 3: Another mixed phase wavelet, $(-1.1+z)^2(1+1.75z)^{38}$, and the primaries only reflectivity sequence are convolved to give the synthetic trace to the right.

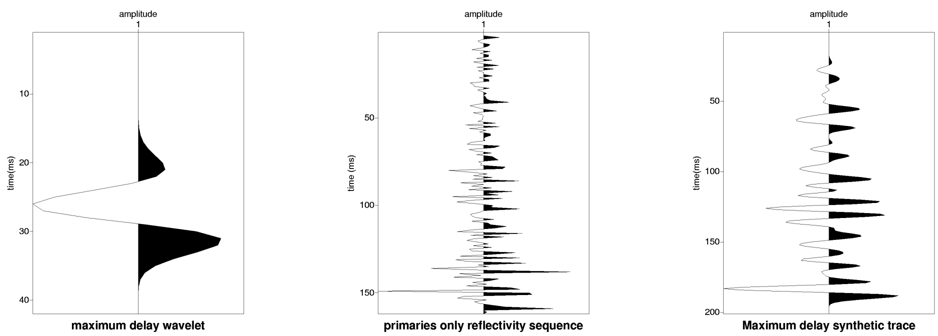

Figure 4: The maximum phase wavelet, $(1-1.1z)^2(1+1.75z)^{38}$, and the primaries only reflectivity sequence are convolved to give the synthetic trace to the right.

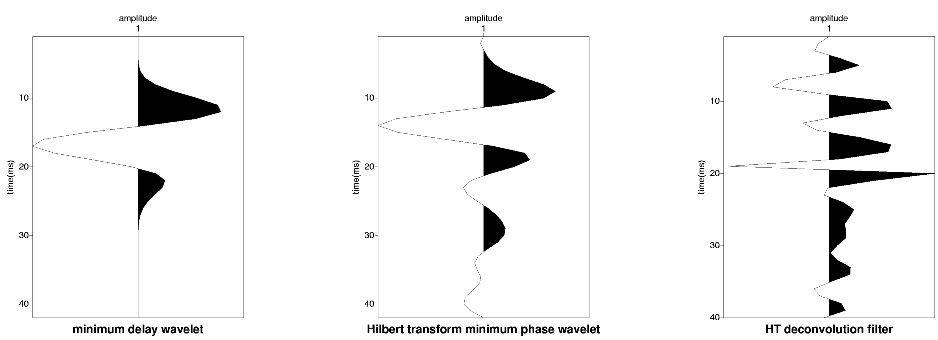

Figure 5: The actual minimum phase wavelet (left) for the model, the Hilbert transform minimum phase wavelet estimate (center) for this model and the deconvolution filter (right) estimated from it.

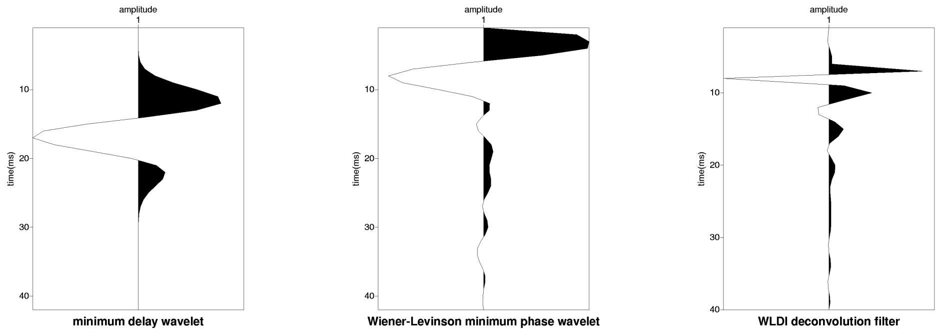

Figure 6: The actual minimum phase wavelet (left) for the model, the Wiener-Levinson minimum phase wavelet estimate (center) for this model and the deconvolution filter (right) estimated from it.

Phase and resolving kernels

Figures 7 through 10 show the resolving kernel tests for the model minimum phase wavelet and its selected permutations. The minimum phase wavelet has the z-dipole form $(-1.1+z)^2(1.75+z)^{38}$ and the permutations considered are the mixed phase wavelet $(1-1.1z)^2(1.75+z)^{38}$, another mixed phase wavelet $(-1.1+z)^2(1+1.75z)^{38}$, and the maximum phase wavelet $(1-1.1z)^2(1+1.75)^{38}$.

In Figure 7, a spiking deconvolution filter for the minimum phase wavelet is shown along with the wavelet itself and the resulting resolving kernel. The convolution of the Wiener deconvolution filter with the wavelet produces the resolving kernel shown in this figure. This is a sharp resolving kernel with a narrow bandwidth. The optimal spiking position is at 25ms and there is insignificant energy in the tail of the kernel. If the dipoles for the 1^{st} term are interchanged, the mixed phase wavelet shown in Figure 8 results. The deconvolution, in this case, produces a resolving kernel that is almost identical to the minimum phase situation with the exception being that now the optimal spiking position is at 44ms. Similarly, in Figure 9, the second mixed phase wavelet has a sharp resolving kernel with narrow bandwidth and an optimum spiking position at 36ms. Figure 10 shows the resolving kernel result for the maximum phase wavelet. In this case, the kernel is sharp with a spiking position at 55ms but the kernel has a broader bandwidth than the previous three images. The general observed trend is that as the phase of the wavelet changes, so does the optimum spiking position. Comparing the optimum spiking position of the various resolving kernels shows this. In other words, if the wavelet is known, the Wiener deconvolution filter can produce an equally good resolving kernel. The key question now becomes the following: *if the wavelet is not known and certain wavelet phase assumptions must be made in order to perform seismic deconvolution, how well will the deconvolved traces estimate the earth's reflectivity?*

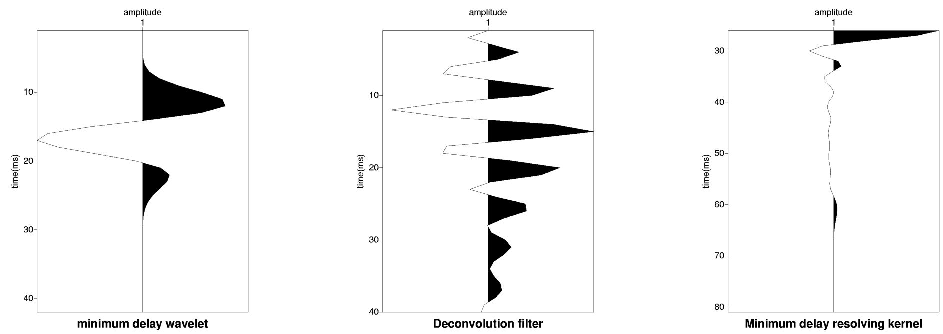

Figure 7: The model minimum phase wavelet, $(-1.1+z)^2(1.75+z)^{38}$, its deconvolution filter, and its resolving kernel. The optimal spiking position is at 25ms.

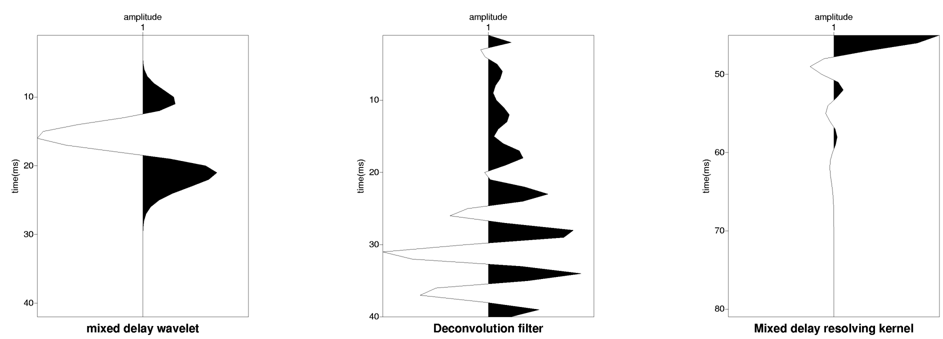

Figure 8: A mixed phase version of the model wavelet, $(1-1.1z)^2(1.75+z)^{38}$, its deconvolution filter, and its resolving kernel. The optimal spiking position is at 44ms.

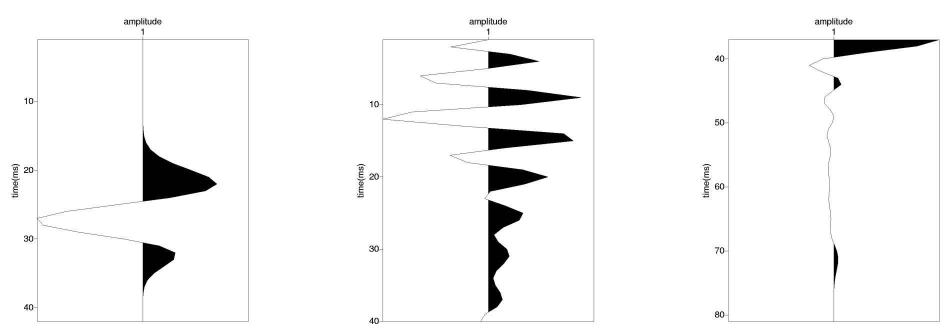

Figure 9: Another mixed phase version of the model wavelet, $(-1.1+z)^2(1+1.75z)^{38}$, its deconvolution filter, and its resolving kernel. The optimal spiking position is at 36ms.

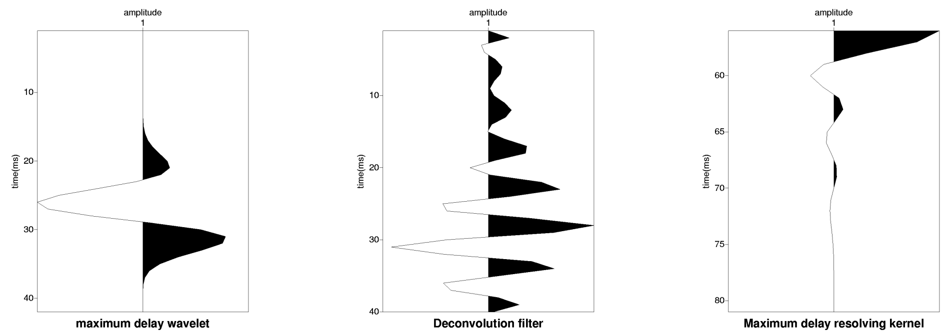

Figure 10: The maximum delay version of the model wavelet, $(1-1.1z)^2(1+1.75z)^{38}$, its deconvolution filter, and its resolving kernel. The optimal spiking position is at 55ms.

Well-log phase test

The next 4 images display the effect of the minimum phase assumption on the deconvolution of the entire trace. Each synthetic is created by the convolution of a wavelet (minimum, mixed, or maximum phase) with the model reflectivity sequence. The minimum delay trace is used to estimate minimum phase wavelets by the Hilbert transform and Wiener-Levinson double inverse methods.

This section introduces the problems that often arise when using the minimum phase assumption. Figures 11 through 14 illustrate just how problematic and damaging the minimum phase assumption can be. Shown in Figure 11 is the actual reflectivity sequence used in this analysis, the Hilbert transform deconvolution of the minimum phase trace, and the Wiener-Levinson deconvolution of the minimum phase trace. Both of these reflectivity estimates have a band-passed nature to them and suffer from significant amplitude attenuation. Both also seem to have strong impulses for minor reflections but the major reflectivities are absent. It also appears as if the Wiener-Levinson estimate suffers more from these shortcomings than the Hilbert transform estimate. Similar results are seen in Figure 12 where the results of a mixed phase trace deconvolution are shown. Presented here are the same shortcomings listed previously except that there seems to be a significant phase mismatch. Next (Figure 13) are the deconvolution results for the second mixed phase trace. These reflectivity estimates cannot, not even in the broadest sense, be considered representative of the model reflectivity. The time delay is particularly noticeable. This could very well be due to the fact that the other major assumptions of wavelet stationarity and reflectivity randomness are coming into play. Finally, in Figure 14, there are the results of the deconvolutions for the maximum phase trace. Here, the statistical reflectivity estimates are very time delayed, have a significant bandpassed nature, suffer from severe phase mismatches, and there is major amplitude attenuation.

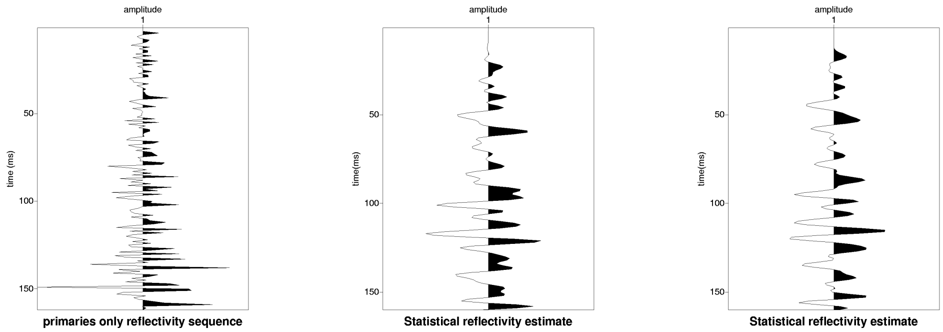

Figure 11: The actual reflectivity sequence (left), spiking deconvolution of the minimum phase synthetic trace based on the Hilbert (center), and spiking deconvolution of the minimum phase synthetic trace based on the Wiener-Levinson wavelet (right).

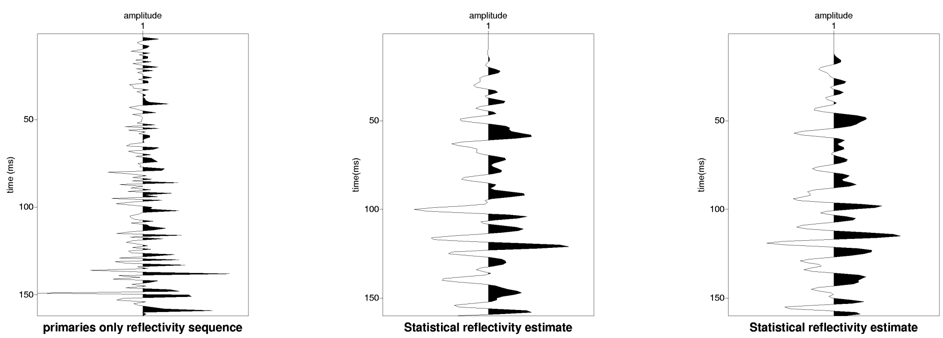

Figure 12: The actual reflectivity sequence (left), spiking deconvolution of the mixed phase synthetic trace based on the Hilbert (center), and spiking deconvolution of the mixed phase synthetic trace based on the Wiener-Levinson wavelet (right).

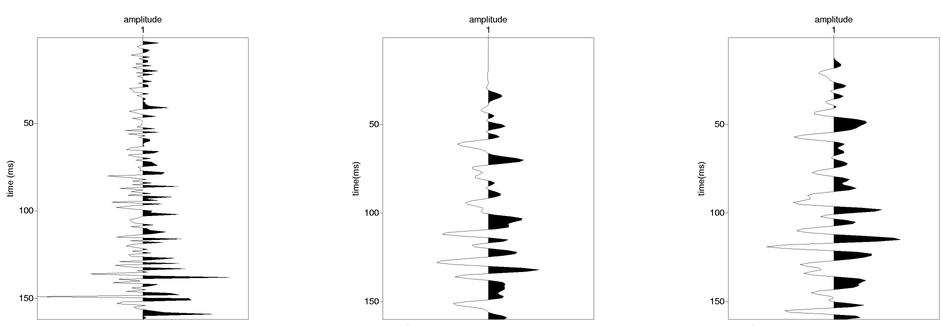

Figure 13: The actual reflectivity sequence (left), spiking deconvolution of the second mixed phase synthetic trace based on the Hilbert (center), and spiking deconvolution of the second mixed phase synthetic trace based on the Wiener-Levinson wavelet (right).

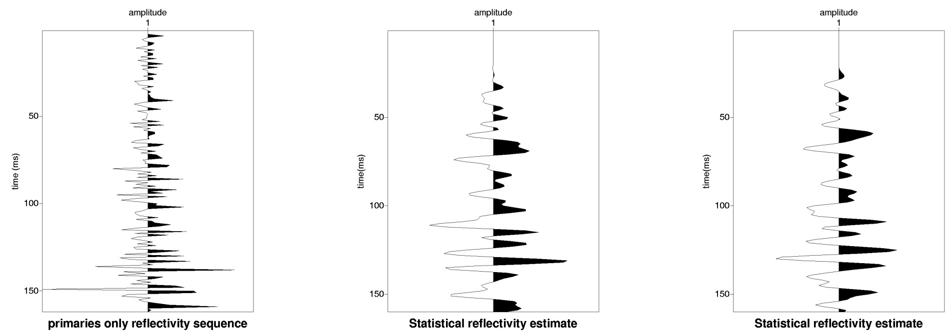

Figure 14: The actual reflectivity sequence (left), spiking deconvolution of the maximum phase synthetic trace based on the Hilbert (center), and spiking deconvolution of the maximum phase synthetic trace based on the Wiener-Levinson wavelet (right).

Wavelet phase test

This final subsection illustrates a set of results using the remaining proposed test for phase discussed earlier. The model uses the synthetic traces to statistically estimate Wiener-Levinson deconvolution filters. These filters, computed using the minimum phase assumption, are then convolved with the actual wavelets to generate resolving kernels. The similarity to an ideal spike will give a measure of how well or how poor the minimum phase assumption is working.

Given the rather poor performance of the statistical methods in the trace deconvolution section, another approach is taken. As described above, the model synthetic traces are used with the minimum phase assumption to spike the wavelets used. That is, the synthetic traces generate the statistically estimated minimum phase filter that convolves with the actual wavelets to give a good resolving kernel where the major peak greatly dwarfs the residual peaks in the kernel. The intrigue in Figure 16 and Figure 17 comes from the mixed phase nature of the actual wavelets. Figure 16 shows a model mixed phase wavelet that is similar to the maximum phase wavelet (see Figure 18), except that the mixed phase wavelet is not as time delayed as the maximum phase wavelet. The mixed phase wavelet spikes at 39ms, while the maximum delay wavelet spikes at 49ms. Its resolving kernel is a version of the kernel for the maximum phase wavelet except at an earlier time. The same holds when considering the mixed phase wavelet in Figure 17 and the minimum phase wavelet (see Figure 15). This mixed phase wavelet has a close resemblance to the minimum phase wavelet and its resolving kernel is that of the minimum delay wavelet but it spikes at 22ms instead of 11ms.

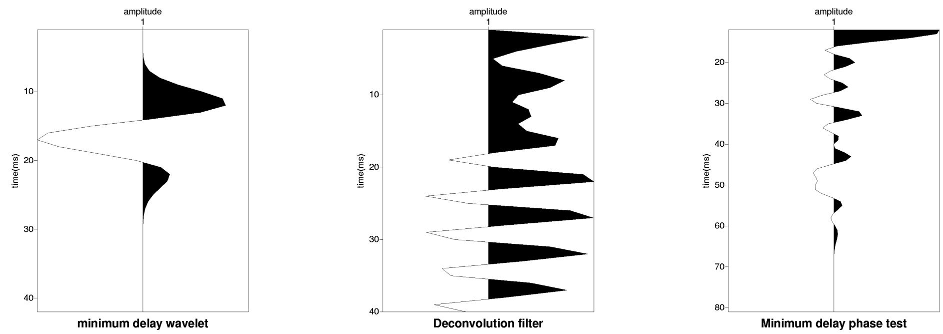

Figure 15: The actual minimum phase wavelet used to create the previously shown synthetic is convolved with the deconvolution filter, estimated from the synthetic trace, to give the resolving kernel to the right.

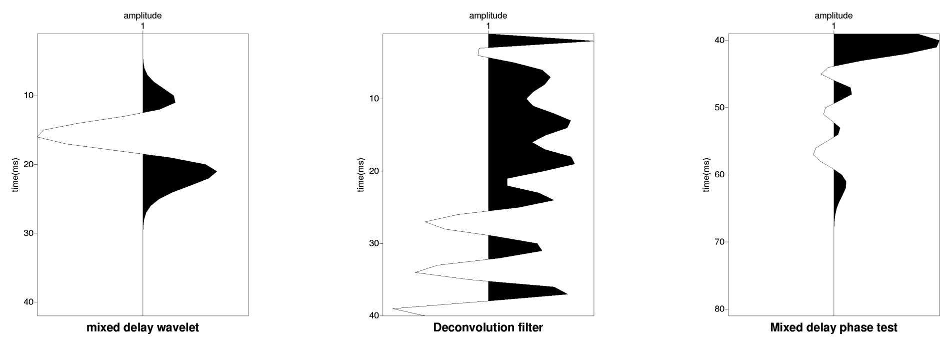

Figure 16: The actual mixed phase wavelet used to create the previously shown synthetic is convolved with the deconvolution filter, estimated from the synthetic trace, to give the resolving kernel to the right.

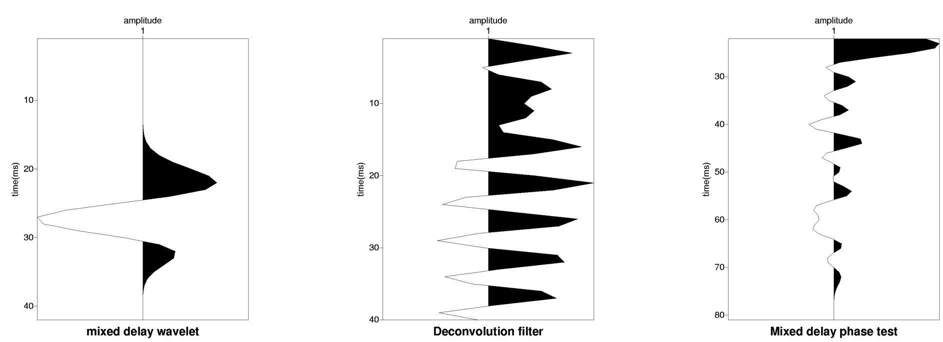

Figure 17: The second mixed phase wavelet used to create the previously shown synthetic is convolved with the deconvolution filter, estimated from the synthetic trace, to give the resolving kernel to the right.

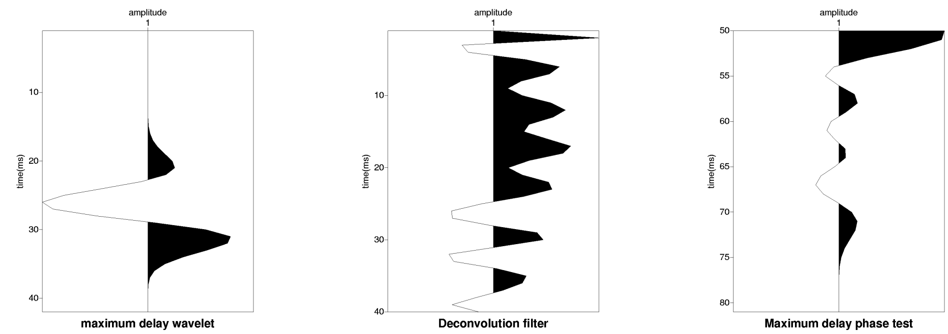

Figure 18: The actual maximum phase wavelet used to create the previously shown synthetic is convolved with the deconvolution filter, estimated from the synthetic trace, to give the resolving kernel to the right.

CONCLUSIONS

From the investigations of this research, it is clear that the minimum phase assumption is critical in determining the success or failure of statistical deconvolution methods. Proper investigation of the minimum phase assumption must be done in a synthetic framework because a knowledge of how the wavelet phase differs from minimum phase is required to know how these phase differences affect statistical deconvolution. The complete picture involves investigating resolving kernels and trace deconvolutions. It is evident that if there is *a priori* knowledge of the wavelet, then the statistical methods used in this work will consistently produce a well-defined, sharp, resolving kernel. Also seen from the study is that statistically estimating minimum phase wavelets from input traces and then using these wavelets to deconvolve the trace is a disastrous path to follow. Far better results are seen when the statistical deconvolution filters are estimated from the trace themselves. Convolving the filter estimated from the trace with the actual wavelet used to create the trace shows this. For the situations considered in this paper, the previous process creates good resolving kernels and, therefore, it can be concluded that the filters will effectively spike the embedded wavelet. As a whole, it is seen that the problems of wavelet phase greatly beset the statistical deconvolution problem. In addition, using traces to estimate wavelets and then estimating filters from these wavelets is a flawed process. To assess the usefulness of statistical deconvolution as a processing tool, use the proposed tests along with the best interpretive judgement available.

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REFERENCES

[1] Claerbout, J., 1976, *Fundamentals of seismic data processing*: McGraw-Hill, New York, New York.

[2] Danielsen, V., and T. V. Karlsson, 1984, Extraction of signatures from seismic and well data: First Break, 2, 15-22.

[3] Lindsey, J. P., 1987, Measuring wavelet phase from seismic data: SEG 1987 fall distinguished lecture.

[4] Lindsey, J. P., 1996, A note on phase unwrapping: Geophysics reprint series, #18, 301-307.

[6] Lines, L. R., and S. Treitel, 1991, Wavelets, well-logs, and Wiener filters: First Break, 3, 9-14.

[7] Lines, L. R., and T. J. Ulrych, 1977, The old and the new in seismic deconvolution and wavelet estimation: Geophysical Prospecting, 25, 512-540.

[8] Robinson, E. A., and S. Treitel, 1980, *Geophysical signal analysis*: Prentice-Hall, Englewood Cliffs, New Jersey.

[9] White, R. E., and P. N. S. O'Brien, 1974, Estimation of the primary seismic pulse: Geophysical Prospecting, 22, 627-651.