Radial trace computational algorithms at CREWES

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ABSTRACT

In another chapter in this report we introduced the use of the radial trace domain for coherent noise attenuation and demonstrated it on both real and synthetic data (Henley, 1999, 1, 2). This chapter describes the ProMAX modules we have written to perform the radial trace transform and its inverse, and to perform other useful radial trace operations. We consider some of the underlying issues involved in implementing the radial trace transform as a discrete mapping algorithm and give heuristic arguments for our programming choices. For each module, key parameters are discussed and criteria given for their selection. Some future research opportunities are outlined as well, based on these modules and their capabilities, which extend somewhat beyond the standard radial trace regime.

INTRODUCTION

The radial trace transform is a simple mapping from the usual X-T domain of seismic trace gathers to a domain described by coordinates of apparent velocity and travel time, the R-T domain. An algorithm for achieving this mapping and its inverse is easy to construct, but there are several considerations that must be addressed in order to ensure a completely *invertible* transform that is also fast enough to use routinely in processing flows. In the modules we describe here, we have attempted to deal adequately with most of these considerations and to set default values for parameters that will protect the user from common problems in using the R-T domain. We will first describe the numerical considerations to be addressed in the R-T transform and its inverse and how we deal with them in actual software. Then we will describe each of our four ProMAX modules in terms of its known applications and potential uses, and provide some guidelines for selection of key parameters.

PRACTICAL CONSIDERATIONS

The transformation of coordinate axes embodied in the radial trace transform R and its inverse R^{-1} applied to map a seismic wavefield S(x,t) to the R-T domain wavefield S'(v,t') and back to X-T can be described by:

$$R\{S(x,t)\} = S'(v,t') \tag{1}$$

$$R^{-1}\{S'(v,t')\} = S(x,t) \tag{2}$$

where

$$t' = t;\ v = x/t. \tag{3}$$

This can be generalized to place the radial transform origin at an arbitrary point (x_0, t_0) in the X-T plane:

$$t' = t - t_0;\ v = (x - x_0)/(t - t_0). \tag{4}$$

These equations completely describe the mapping of seismic amplitude values, S, from the X-T domain to the R-T domain. In the continuum world, the only difficulty is encountered at $t - t_0 = 0$, where we assume no data in order to avoid the singularity.

Discretization of the seismic wavefield, however, leads to complications due to the geometric distortion of sample intervals by the mapping formula. A wavefield that is uniformly sampled without spatial aliasing in one domain will transform to the other domain in such a way that the sampling is no longer uniform, and aliasing of data in portions of the input plane may be caused by the transform process. Thus, transform software must adequately address the creation of an R-T wavefield uniformly sampled in both velocity and travel time from the corresponding X-T wavefield sampled uniformly in source-receiver offset and travel time; as well as similar restrictions for the inverse. The two mathematical concerns raised are therefore anti-alias protection and interpolation. A third concern is mainly an accounting issue raised by the nature of the R-T transform: *partial mapping*. It is entirely possible, and sometimes desirable, to map only a portion of the X-T domain to R-T, or vice-versa. To allow this in a seamless way, any two-way transform module needs to retain the original input panel so that the portion extracted for R-T domain operations can be replaced. We will now describe how each of these considerations is dealt with in the ProMAX modules.

Anti-aliasing

Because radial trace trajectories have a common origin in the X-T plane, and because the trajectories diverge away from this origin, radial traces tend to over-sample the X-T domain at small travel times and offsets near this origin and to under-sample it at larger times and offsets. Since the travel time axis is taken to be the same in both the X-T and R-T domains, sampling in that dimension is not a concern. The sampling of the velocity dimension in the R-T domain, however, is what most directly affects the degree of over- or under-sampling of the X-T panel. By choosing a smaller velocity sample interval (more radial traces), we reduce under-sampling of the X-T domain. As we indicate in (Henley, 1999, 1), the safest way to ensure no aliasing in the R-T mapping process is to select a large enough number of radial traces that the widest spacing of the radial trace trajectories is less than or equal to the sample spacing along the edges of the X-T panel that those trajectories intersect. This ensures that every sample in the X-T domain is represented by at least one sample in the R-T domain. An estimate of this number can be easily obtained by adding the number of time samples per trace to the number of traces in the X-T domain for a leading or trailing spread geometry, and adding *two times* the number of time samples to the number of traces for a split-spread geometry. This estimate in either case is a rough estimate of the number of discrete samples along the outside edges of the X-T panel. In actual practice, for an X-T panel whose maximum frequency is less than the half-Nyquist frequency, we can reduce the above estimate by a factor of at least two without risking alias in the R-T transform. Since X-T data are more likely to be aliased in spatial wavenumber, it is always worthwhile to check that the number of radial traces selected for the R-T transform ensures at least one radial trace per X-T trace along the bottom edge (maximum travel time) of the panel. The easiest way to do this is to use the radial trace display module to compute and plot a trial R-T transform and check that the live data at the bottom of the R-T panel spans at least as many R-T traces as there are X-T traces.

Interpolation

Interpolation in radial trace computations was discussed by (Claerbout, 1983) mostly in the context of removing aliasing of wave modes by interpolating extra traces into an X-T gather to decrease trace spacing in the X-T domain. In that setting, interpolation *direction* was used to favor the interpolation of one mode into the new traces in preference to others. Our main interpolation concerns here are to ensure reflection image fidelity in the radial trace *transform* and its inverse, and to maximize speed and convenience in computation; hence our explicit *transform* interpolation is horizontal *only.* Interpolation in directions *other* than horizontal, in order to preferentially enhance specific wavefield modes is basically what happens during high-pass or low-pass filtering (a form of interpolation) in the R-T domain and is dealt with elsewhere (Henley, 1999, 1).

The digitally sampled seismic wavefield can be viewed as a matrix in either R-T space, with columns of constant v and rows of constant t', or in X-T space, with columns of constant x and rows of constant t. Because the two matrices have exactly corresponding rows, the fastest and most convenient way of finding elements in R-T from those in X-T is by using the formula (3) to compute the x coordinate for each value of v at a fixed time t and then interpolating the amplitudes from the x coordinate grid of the X-T panel to the new x coordinate grid of the R-T panel. Doing this interpolation for every time sample t will result in a complete R-T transform. The inverse is done in the same way. Each row in R-T is interpolated from the computed x-grid values of the R-T panel to the x-grid values of the corresponding row in the X-T domain. The radial trace transform and its inverse thus reduce to simple one-dimensional interpolation of each row of one matrix with one set of grid points to the corresponding row of a second matrix with a new set of grid points computed from the transform formula (3).

Since proper interpolation is crucial to the R-T transform, we must pay close attention to the choice of algorithm to ensure both data fidelity and algorithm speed. After considerable experimentation, we constructed an algorithm which seems reasonably fast and efficient while allowing the user some control over the fidelity of data passed through the transform. Currently, we offer the choice of three related interpolation methods for computing the amplitude of a sample at a new grid point lying between two old grid points:

- Linearly interpolate from the sample amplitudes at the two nearest old grid points
- Choose the sample amplitude of the old grid point lying closest to the new grid point (nearest neighbor interpolation)
- Compute the new sample from the two nearest sample amplitudes at the old grid points weighted by an exponential power of their complementary distance from the new grid point (“soft neighbor” interpolation).

We find that the linear option causes not only the most lateral smearing when comparing an R-T transform-filter-inverse R-T transform sequence with the input, but also the most noise attenuation. The nearest-neighbor interpolation causes no smearing, but the action of a filter in the R-T domain causes sharp discontinuities to appear on traces in some parts of the X-T domain after inverse R-T transformation using the nearest-neighbor option. The "soft neighbor" interpolation, however, seems to provide an intermediate method which smears less than the linear method, yet suppresses the artifacts of the nearest-neighbor option. Choice of unity as the exponent reduces the method to linear interpolation, while choice of a large exponent forces it closer to nearest neighbor interpolation. While there are a variety of multi-point interpolation methods available, such as polynomials, splines, filter response functions, many of which offer a high degree of data fidelity protection, we chose to implement only the simple two-point methods described above because of their computation speed, and because they require neither the old grid nor the new one to be uniformly spaced.

To illustrate the relative effects of the three types of interpolation, we show in figure 1 a close-up of a shot gather containing a *single* 60 Hz noise trace. Because this trace is totally different in character from its neighboring traces, it is easy to tell whether or not it is smeared laterally by radial trace operations. While most of the potential smear will be due to the interpolation methods used in the R-T transform and its inverse, some smear and/or other disturbance can be attributed to the effect of the filter response function applied in the R-T domain. The gather in figure 1 has been subjected to two passes of R-T domain filtering, both using the linear interpolation option. It is obvious that the 60 Hz noise trace has been smeared laterally so that both neighboring traces are now contaminated with this noise as well, but we see no overt evidence of disturbance due to the filter response. Figure 2 is a close-up of the same gather after two passes of R-T filtering using the nearest-neighbor interpolation option. Indeed, this figure shows *no* lateral smearing, but the abrupt discontinuities on the traces show the effect of applying a filter in the R-T domain. The effectiveness of the "soft neighbor" interpolation is shown in figure 3. No evidence of lateral smearing can be seen, and there are no visible filter artifacts either; this interpolation is our preferred method for routine use.

Partial mapping

In contrast to integral transforms like the F-K transform, in which the entire X-T panel must be transformed to F-K in order to yield any single value in the new domain, some mapping transforms require no more than a single point in the old domain to obtain a point in the new one. Furthermore, a fragmentarily mapped data set can be modified in the new domain and then replaced seamlessly in the old domain without disturbing unmapped points. One instance where this might be desirable in R-T domain filtering is in applying a filter to a low velocity "noise cone" as an alternative to late muting, without transforming and filtering the entire shot gather. The noise can be extracted with an appropriately designed fragmentary R-T transform (low velocity wedge), filtered, and the result replaced in the original X-T panel. Our ProMAX R-T filter module automatically provides for this by retaining the entire X-T input panel through to the final X-T output stage, so that samples from the filtered R-T traces can be remapped into the old X-T domain. Any samples not originally gathered by the R-T transform, however, are untouched by the entire process.

While fragmentary R-T transforms and their inverses are meaningful, there is no meaningful way in which to extrapolate the inverse R-T transform to regions of the X-T plane not originally occupied by the input X-T panel. Our ProMAX transform module allows the user to transform to R-T and then to transform back to a different set of X coordinates, which may be linearly or even quadratically spaced, but these coordinates must always lie within the range of the original X-T seismic data panel coordinates.

PROMAX MODULES

Radial filter

This ProMAX module is the workhorse for R-T domain coherent noise filtering and wavefield separation. It consists of both forward and inverse R-T transforms with a number of filtering and amplitude scaling options available in the R-T domain. The radial filter module is designed to operate on any ensemble of traces having legitimate values in the ‘signed offset’ trace headers, so any arbitrary trace ensemble can be used as long as these trace headers are present and non-decreasing across the gather. This is naturally true for shot and/or receiver gathers sorted in ascending order of offset. The only pre-processing that might be desirable for input ensembles is a pass through a spike and noise burst editor, since R-T domain filtering, like all 2-D filter algorithms will place filter response artifacts on an ensemble at the positions of any spikes or bursts on the input panel. To use the radial filter module, the processor should first display an example of an input trace ensemble, and for all coherent noises locate their apparent origins and velocities, even if an origin apparently lies outside the boundaries of the ensemble itself. For many gathers, much of the coherent noise will share a common origin, indicating that an R-T fan filter is the most appropriate choice. Other gathers may show systems of parallel coherent noises that have no common origin but a common velocity, indicating an R-T dip filter as the most appropriate application. Once the user has chosen whether to apply a fan filter or dip filter and has measured apparent velocities and/or found apparent origins of noise systems, he may choose parameters within the menu of the radial filter module as follows:

spread flag -- This seldom-used parameter allows the user to specify that the data gathers are a mix of leading and trailing spread gathers, and that the filter parameters are based on a leading spread gather only. This parameter is best defaulted and the data simply properly sorted prior to filtering.

number of radial traces – This is one of the most important parameters, in that it controls the fidelity of the wavefield transformed to the R-T domain. A safe choice for this parameter is the number of samples per trace in the X-T domain plus the number of X-T traces…the default value should be considered a minimum number only. The correct choice of this parameter provides anti-alias protection for the forward transform.

switch for dip filter – The choice of filter type (dip or fan) is made at this point, and later parameters are displayed or hidden based on this choice.

nominal filter velocity – If a dip filter is chosen above, this parameter is active and should be set to the apparent velocity of the noise to be filtered.

velocity range for dip filter – If a dip filter is chosen, this parameter is used to determine the width of the narrow fan of radial traces which span the input gather for the dip filter application. There is no minimum range, but values smaller than 10 m/sec are discouraged.

minimum radial trace velocity – If a fan filter is chosen above, this parameter must be supplied to indicate the velocity of the first radial trace in the R-T domain. For a conventional radial fan with origin at the shot origin, we typically transform the whole X-T gather, and this minimum velocity would be that of the first arrivals for the negative offsets for a split spread or trailing spread (negative velocity), or zero for a leading spread.

maximum radial trace velocity – Choice of a fan filter forces choice of this parameter to specify the velocity of the last radial trace to be generated in the R-T domain. For a conventional radial fan with origin at the shot origin, this maximum velocity would be zero for a trailing spread, or the apparent velocity of the positive offset direct arrivals for a split spread or leading spread geometry (positive velocity).

time coordinate for radial trace origin – This parameter is chosen only for a fan filter; in the case of a dip filter it is computed. It is the point in time at which to place the origin of the radial trace fan for the R-T transform, often at or near the time of the shot, but sometimes at the apparent time of some “virtual” source.

offset coordinate for radial trace origin – This parameter may be chosen only for a fan filter; choice of a dip filter forces computation of the value. It is the offset at which to place the origin of the R-T transform, often at or near the position of the shot, but sometimes selected to correspond to the position of some “virtual” source. Both time and offset coordinates must be specified to locate the origin of the R-T fan filter.

nominal offset increment – This is just the station spacing for a shot gather or the shot spacing for a receiver gather…it is not a crucial parameter and can be defaulted.

time-reverse switch for X-T traces – This switch allows a gather of traces to be time reversed before and after the application of an R-T domain filter to emulate the effect of a filter applied from a “virtual” source beneath the gather…useful for “backscatter”

interpolation method for radial transform – This parameter allows selection of one of the three methods described earlier for interpolation. The default “soft neighbor” is recommended.

exponent for ‘soft neighbor’ interpolation – If “soft neighbor” interpolation is chosen, this parameter selects the actual exponent used in the interpolation. The default value works well.

domain in which to apply filter – With this parameter the user can choose to filter the R-T traces either in the time domain by convolution or in the frequency domain by multiplication…in this application the time-domain filters seem to work best and are the default.

type of filter to apply – At this point, the user selects either low-cut, bandpass, low-pass, or input minus low-pass, depending upon the desired results. Low-cut or bandpass work best for most noise attenuation applications, low-pass for wavefield separation.

low-stop frequency for filter – This is the designated frequency below which all response is zero (or unity, for low-pass).

low-cut frequency for filter – This is the frequency above which all response is unity (or zero, for low-pass).

high-cut frequency for filter – The frequency below which all response is unity. This parameter is only used for a bandpass filter.

high-stop frequency for filter – The frequency above which all response is zero. this parameter is only used for a bandpass filter.

length of filter – This parameter is only used for time-domain filters and helps determine the overall attenuation of the filter. The default value works well.

type of normalization in the radial domain – If it is desirable to equalize the amplitudes of the R-T traces because of the strength of the noise, this switch chooses the type of equalization to be used. The current choices are none (the default), normalize traces by their RMS amplitude, normalize traces by average absolute amplitude, or apply AGC to the traces. Unless you know what you're doing, this option is best left alone. Never apply normalization if filter type is chosen as input minus low-pass.

gate length for normalization – This parameter is needed only if some type of trace normalization in the R-T domain is desired. It is the length of the derive gate for trace amplitude normalization or the length of the running window for AGC.

begin time for normalization – This is only used in conjunction with the gate length above to determine the portion of the R-T traces to be used for normalization factor derivation.

normalization level – For all the normalization methods, this is the desired output level for the R-T traces.

refractive index computation method – The refractive index is a vector of values, one for each time sample, that can be used to cause convergence or divergence of the radial traces from their normal linear trajectories. More about this later…this parameter should be defaulted, which selects unity for all vector values and keeps the radial traces linear.

Radial trace display

This ProMAX module was originally written as a forward R-T transform intended to produce R-T traces for diagnostic display. It has since been superceded by the newer module 'radial trace transform' which provides both forward and inverse transforms, as well as a choice of options for output coordinate grids. The parameters for the radial trace display module mirror those for the radial filter module, except that there are no filter or amplitude normalization parameters.

Radial trace transform

This is the most recently completed radial trace module; it provides either the forward R-T transform or its inverse, so that X-T panels can be transformed to the R-T domain, operations performed on the R-T traces, and the R-T traces transformed back to the X-T domain. Several options have been added to the inverse transform mode, so that in addition to regularizing offsets on the X-T panel by giving them a uniform increment, the offsets can be chosen to have a quadratic distribution (X**2-T domain), or as a final step, the time axis can be made quadratic as well (X**2-T**2 domain), which provides a useful mapping for velocity analysis and dip filtering. The latter options are mainly intended for research purposes and remain to be thoroughly tested. Most of the parameters mirror those of the radial filter module, so we will emphasize those that differ:

transform switch – the forward or inverse R-T transform is chosen by this parameter.

number of traces – the number of traces in the output panel, whether R-T traces from the forward transform or X-T traces from the inverse. This parameter provides the opportunity in the inverse transform to interpolate traces into the original X-T panel.

The following parameters are identical to those for the radial filter module and won't be repeated here:

switch for dip transform

nominal transform velocity

velocity range for dip transform

minimum radial trace velocity

maximum radial trace velocity

time coordinate for radial trace transform

offset coordinate for radial trace transform

nominal offset increment

time-reverse switch for X-T traces

interpolation method for radial transform *exponent for 'soft neighbor' interpolation*

refractive index computation method

The following, however, are important for the inverse R-T transform and must be specified:

minimum source-receiver offset – This value need not be the same as the minimum offset on the original input X-T panel, but if it is outside the range of the original offsets, it will simply lead to blank traces at the illegitimate offset values thereby created.

maximum source-receiver offset – This value need not be the same as the maximum offset on the original X-T panel, but if it is outside the range of the original offsets it will lead to creation of blank traces at the illegitimate offset values.

offset increment method – This is a key parameter, since it specifies how the R-T traces are to be interpolated back to the X-T domain. The default is linear offset distribution (X-T), but quadratic offset distribution (X**2-T) and quadratic offset and travel time (X**2-T**2) are also possible for diagnostic purposes. A final option is to recover linear travel time from quadratic (the X**2-T**2 panels must first go back to the R-T domain via a forward R-T transform).

Make ensemble

This is not an actual ProMAX module, but instead is a ProMAX macro, whose purpose is to convert a group of traces (for example, a stacked section) to a legitimate ensemble with useable values in the 'signed offset' trace headers, so that the traces may be filtered or otherwise diagnosed by the other radial trace modules. It has only three parameters, which are described as follows:

number of traces in stack – This helps set up the proper geometry, so the number should be close to the actual number of traces in the panel, but never smaller.

stack trace spacing in metres – This determines the scale used to measure velocity of dipping events on the trace panel, so it should be accurate.

number of traces in output ensemble – This number is normally the same as the first parameter, but is an integer rather than part of a string variable, as is the first parameter.

Hardware considerations

The current radial trace software modules are reasonably *fast,* so that CPU time is not normally an important consideration, but the algorithms *are* memory-intensive. This means that if enough free RAM is not available during execution of an R-T transform routine, the execution wall-clock time may become excessive, even though CPU time is minimal, as the machine simulates RAM with its disk space. Having exclusive access to 64 mbytes of RAM during an R-T operation is highly recommended.

DISCUSSION

The intent of this phase of ProMAX software development has been to provide tools for the processor needing easy access to radial trace technology for noise attenuation and/or wavefield diagnostics, and to provide tools intended for research efforts exploring further expansions and uses of the radial trace transform and other mapping transforms. To that end, the radial filter, radial trace display, and make ensemble modules have been created with the general user in mind, and the radial trace transform module with the experimenter in mind. The radial trace transform module should be useful in amplitude-vs-angle (AVA) studies, and particularly in its X**2-T**2 form, useful for the study and separation of various wave propagation modes. All of the modules should be considered experimental software, and used with appropriate caution, but the radial filter module in particular has been found to be relatively robust and should provide useful results under most circumstances.

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FIGURES

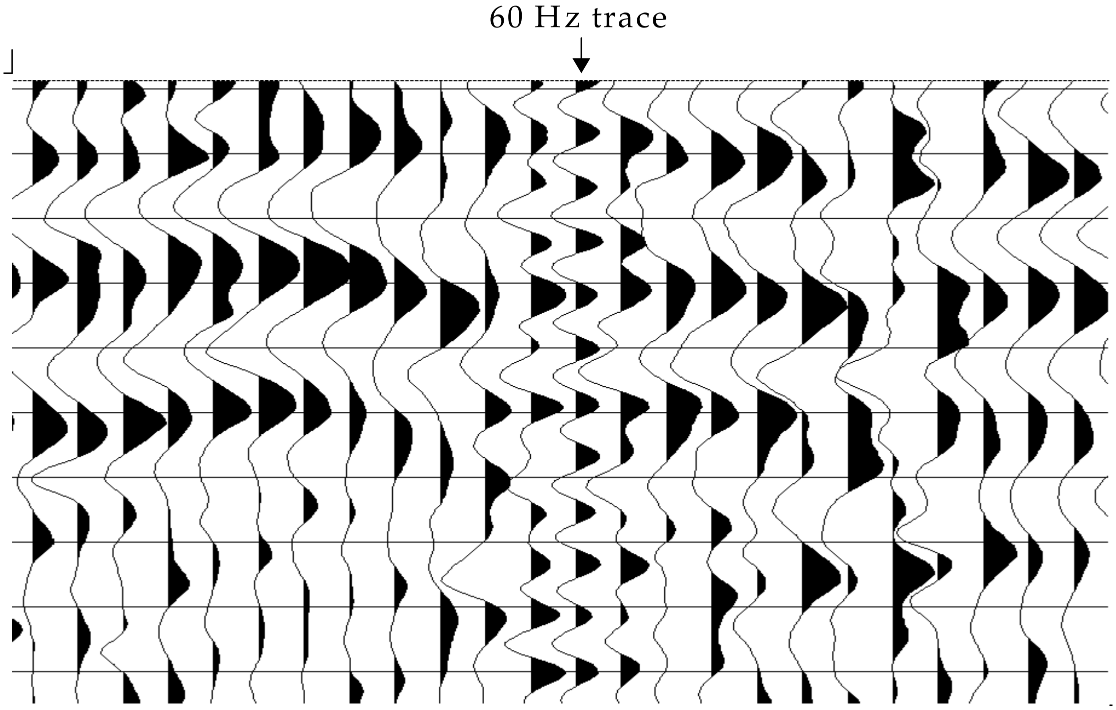

Figure 1 – Lateral smearing of noise trace due to use of linear interpolation in two passes of R-T filtering.

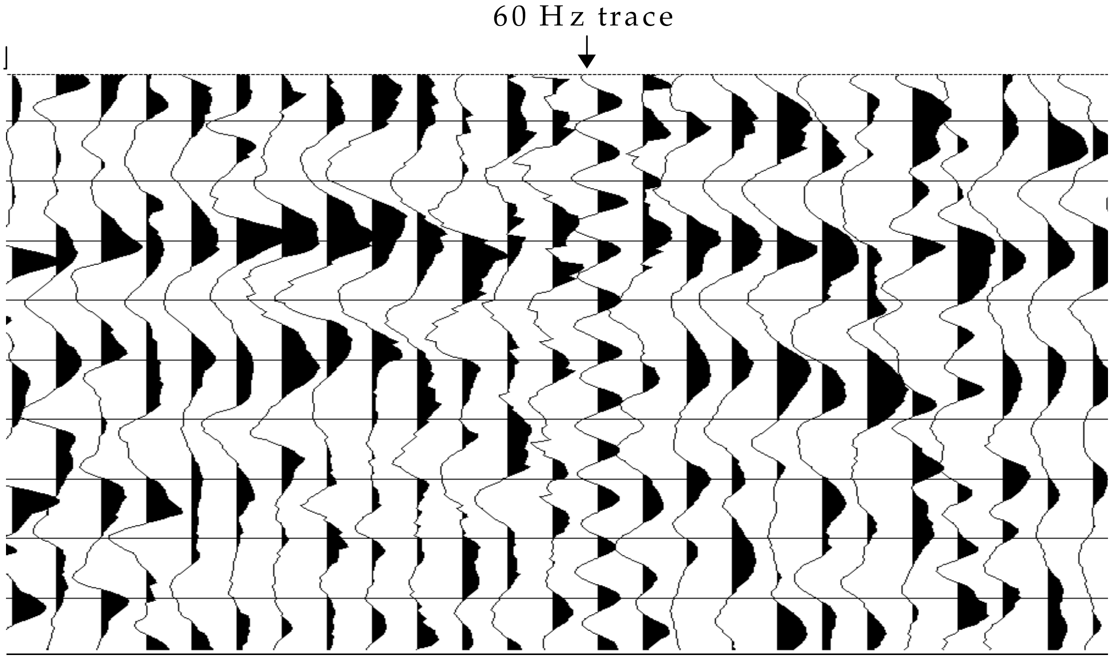

Figure 2 – Nearest neighbor interpolation creates high-frequency artifacts in two passes of R-T filtering

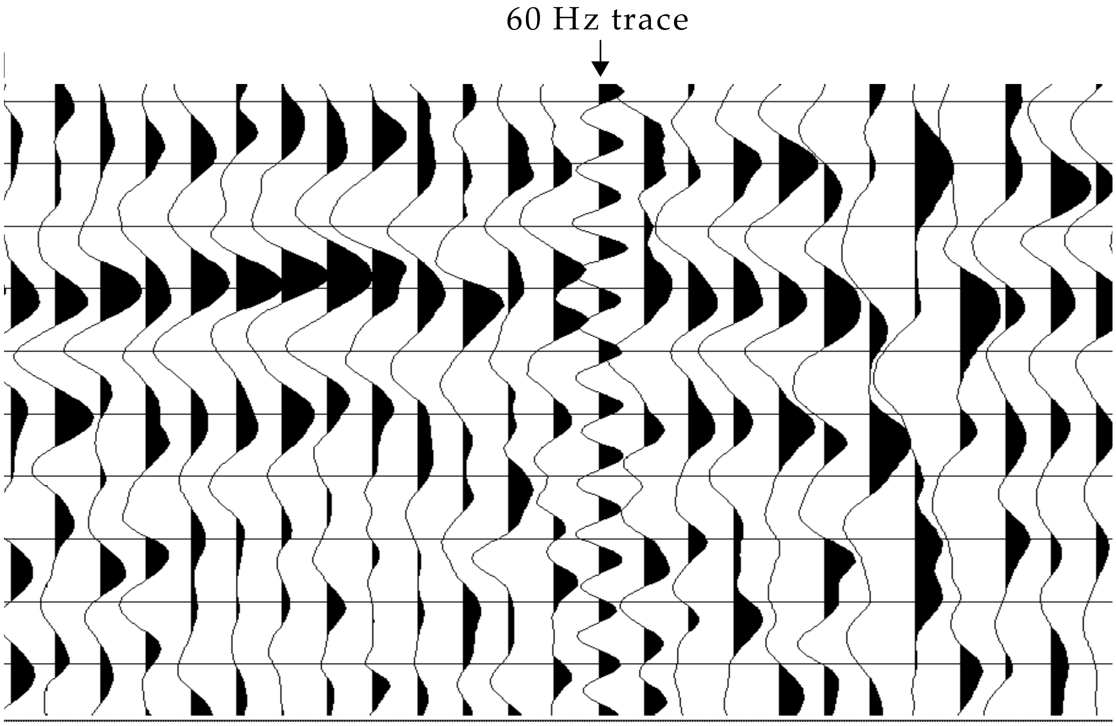

Figure 3 – "Soft neighbor" interpolation used in two passes of R-T filtering to preserve lateral discontinuities.