Direct traveltime inversion of VSP data for elliptical anisotropy in layered media

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ABSTRACT

This paper presents an exact method for inversion of VSP data for anisotropic parameters that uses traveltimes directly, rather than phase slownesses computed from traveltime differences. There is a tradeoff in the present traveltime-based method compared with the slowness-based method. In our method we have to assume some function representing the vertical velocity variation, something that is not necessary in the phase-slowness method. There is, however, a higher numerical accuracy in the direct traveltime method. This is due partly to our using observed traveltimes directly in the computations rather than taking differences between values of comparable size, which greatly magnifies the relative errors. Error analysis shows that there also are other intrinsic reasons why our technique has less error. A numerical example representing a typical VSP yielded errors from the phase-slowness method (requiring two sources and two receivers) that were about 18 times larger than for a single determination by the direct traveltime method. Furthermore, in cases where the VSP has been acquired with a single source offset, e.g., for some offshore wells, the traveltime method will yield results, whereas the slowness method will not.

INTRODUCTION

In the last decade or two there has been a greatly increased recognition that significant seismic anisotropy is rather common in sedimentary rocks. It has also been shown that neglecting even a seemingly modest degree of anisotropy can lead to significant degradation in processed seismic images (e.g., Winterstein, 1986; Wright, 1987; Larner, 1993; Tsvankin, 1996; Chen and Castagna, 2000) or to significant errors in vertical or lateral positioning of subsurface features (e.g. Banik, 1984, 1987; Isaac and Lawton, 1999; Vestrum et al., 1999).

Gaiser (1990) determined how to estimate the anisotropic parameters of a transversely isotropic medium from VSP data using vertical and horizontal phase-slowness measurements. Miller and Spencer (1994) and Miller et al. (1994) presented methods for inverting phase slowness estimates from walkaway VSPs for anisotropy parameters. Using a similar method, Leaney et al. (1999) have inverted multiazimuthal multioffset VSP P-wave data over a marine carbonate reservoir to determine azimuthal anisotropy in the carbonate and transverse isotropy in the overlying shale. Leslie and Lawton (1999) present a method for inverting refraction seismic data for the anisotropic parameters of steeply dipping strata. Tsvankin and Thomsen (1994) and Alkhalifah and Tsvankin (1995) deal with the problem of inverting for longer spreads, beyond the near-vertical elliptic-anisotropic approximation, in order to recover velocity fields for processing.

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This paper investigates the problem of determining the degree of anisotropy in a buried layer under certain assumptions. Principally, we assume some knowledge of the section above the anisotropic layer of interest, i.e., its velocity as a function of depth and its anisotropic properties. Further we assume the target medium to be transversely isotropic with a vertical symmetry axis (TIV) and we consider the case of elliptical anisotropy, which is valid for SH-wave propagation at all angles of incidence and for P-wave propagation in the short-spread approximation. This method will use traveltimes directly, rather than phase slownesses, thereby avoiding the loss in numerical accuracy associated with taking differences between values of comparable size, and allowing recovery of at least some results when a VSP has been acquired at only a single source offset.

MATHEMATICAL BACKGROUND

Group and phase velocities

In a medium composed of n horizontal layers, the traveltime, t, between a point source and a point receiver can be written as the sum of traveltimes in all layers,

$$t = \sum_{i=1}^{i=n} \frac{l_i}{V_i} \, , \tag{1}$$

where l_i is the distance traveled in the ith layer and V_i is the corresponding group velocity. If the medium is anisotropic, the magnitude of the group velocity depends on the direction of propagation.

In a plane of symmetry, group velocity, V, can be expressed in terms of phase velocity, v, as

$$V[\phi(\theta)] = \sqrt{v^2(\theta) + \left(\frac{dv}{d\theta}\right)^2} \, , \tag{2}$$

where ϕ and θ are group and phase angles, respectively. The relation connecting group and phase angles is given by the equation for the slope of the normal to the phase-slowness curve, $1/v$ versus θ, expressed in polar coordinates, as

$$\tan[\phi(\theta)] = \frac{\tan\theta + \frac{1}{v(\theta)}\frac{dv}{d\theta}}{1 - \frac{\tan\theta}{v(\theta)}\frac{dv}{d\theta}} \, , \tag{3}$$

(e.g., Berryman, 1979; Brown et al., 1991).

Elliptical anisotropy

The expression for phase velocity of an SH wave in a transversely isotropic medium (and following his notation) is given by Thomsen (1986) as:

$$v_{SH}(\theta) = \beta_0 \sqrt{1 + 2\gamma \sin^2 \theta}\,, \tag{4}$$

where

$$\gamma = \frac{C_{66} - C_{44}}{2C_{44}} = \frac{v^2{}_{SH}(\pi/2) - \beta^2{}_0}{2\beta^2{}_0} \tag{5}$$

and $\beta_0 \equiv v_{SH}$ (0) is the SH-wave speed along the symmetry axis, which we take to be the vertical direction. The elastic stiffnesses, C_{ij}, are as defined by Thomsen (1986).

Substituting equation (4) into equation (2) gives an equation for the magnitude of the group velocity as a function of the phase angle and Thomsen parameter, namely,

$$V_{SH}[\phi(\theta)] = \beta_0 \sqrt{\frac{1 + 4\gamma(1+\gamma)\sin^2\theta}{1 + 2\gamma \sin^2\theta}}\,. \tag{6}$$

Similarly, inserting equation (4) into equation (3) gives an expression for the group angle as a function of the phase angle:

$$\tan[\phi(\theta)] = (1 + 2\gamma)\tan\theta\,. \tag{7}$$

Solving equation (7) explicitly for θ and substituting into equation (6) yields an expression for group velocity as a function of group angle (Aggarwala et al., 1997),

$$V_{SH}(\phi) = \beta_0 \sqrt{\frac{1}{1 - \frac{2\gamma}{2\gamma+1}\sin^2\phi}} \tag{8}$$

which may be written as:

$$\frac{1}{V_{SH}^2(\phi)} = \frac{\cos^2\phi}{\beta_0^2} + \frac{\sin^2\phi}{\beta_0^2(1+2\gamma)}\,, \tag{9}$$

showing clearly the elliptical nature of the group-velocity or wavefront surface. This is what is meant by the term elliptical anisotropy (e.g. Daley and Hron, 1979).

The short-spread approximation, i.e., at near-vertical incidence, for phase velocity of a P wave in a transversely isotropic medium, which can be derived from equations (10a), (10d) and (17) of Thomsen (1986) by neglecting terms in $\sin^4\theta$ and higher, is:

$$v_P(\theta) = \alpha_0 \sqrt{1 + 2\delta \sin^2\theta}\,, \tag{10}$$

which is wholly analogous to equation (4), the phase-velocity expression for v_{SH}. In equation (10):

$$\delta = \frac{(C_{13} + C_{44})^2 - (C_{33} - C_{44})^2}{2C_{33}(C_{33} - C_{44})} \tag{11}$$

and $\alpha_0 \equiv v_P(0)$ is the P-wave speed along the vertical axis (Thomsen, 1986).

Since equations (4) and (10) are of identical form, representing elliptical anisotropy, the ensuing discussion could equally well apply to P waves, in the near-vertical approximation, or to SH waves exactly; and P-wave equations exist that are analogous to equations (6) to (9) for SH. In the rest of this paper we shall generally drop the subscripts SH and P, though we shall continue using γ and α_0 (since the equations are exact for the SH case), realizing that the discussion also applies to the near-vertical P case, with $\gamma \Rightarrow \delta$ and $\beta_0 \Rightarrow \alpha_0$.

TRAVELTIME INVERSION

Isotropic homogeneous upper layer

Consider transmission through a two-layer medium as illustrated in Figure 1. It is assumed that the media belong to the symmetry class of transverse isotropy with a vertical symmetry axis (TIV). Assume further that the anisotropic parameter in the upper (surface) layer is known and denoted by γ_1, and that the linear-vertical-inhomogeneity parameters, a and b in the group-velocity expression $V(z) = a + bz$ also are known. For clarity of presentation, the upper layer is assumed to be isotropic and homogeneous, i.e., $\gamma_1 = 0$, and $b = 0$, respectively. However, upon minor modifications, the method is applicable in the anisotropic or the inhomogeneous cases, $\gamma_1 \neq 0$ or $b \neq 0$, provided γ_1 and b are known.

We wish, based on traveltime measurements, to determine the anisotropic parameter, γ, in the lower (buried) layer. Using the symbols shown in Figure 1, one can rewrite equation (1), as:

$$t = \frac{\sqrt{(X-r)^2 + H^2}}{V} + \frac{\sqrt{\dfrac{r^2}{1+2\gamma} + Z^2}}{\beta_0}, \tag{12}$$

Since this is a problem with two unknowns, γ and r, we need a second equation for its solution. The second equation comes from the calculus of variations, or Fermat's principle of stationary time, namely,

$$\frac{dt(r)}{dr} = 0\,. \tag{13}$$

As it stands, equation (13) is difficult to use in traveltime inversion: inverting for γ requires extremizing $t(r)$ subject to the constraint of the measured traveltime, $t = t_0$.

The key to a practical inversion scheme is the condition:

$$\frac{\partial \gamma}{\partial r} = 0\,, \tag{14}$$

which follows from equation (13) as shown in the following lemma.

Lemma: With the traveltime, *t*, being a function of the refraction point, *r*, and anisotropic parameter, γ:

$$t = f(r, \gamma)\,, \tag{15}$$

Fermat's principle of stationary time [equation (13)] implies $\frac{\partial \gamma}{\partial r} = 0$ [equation (14)]. For *t* a minimum, the critical point of $\gamma(r)$ is also a minimum.

Proof: Note that γ, although unknown, is constant, i.e., $\gamma = \gamma_0$. Thus the total derivative with respect to *r* in equation (13) is equivalent to a partial derivative,

$$\left.\frac{dt}{dr}\right|_{\substack{r=r_0\\ \gamma=\gamma_0}} = \frac{\partial f}{\partial r}(r_0, \gamma_0) = 0. \tag{16}$$

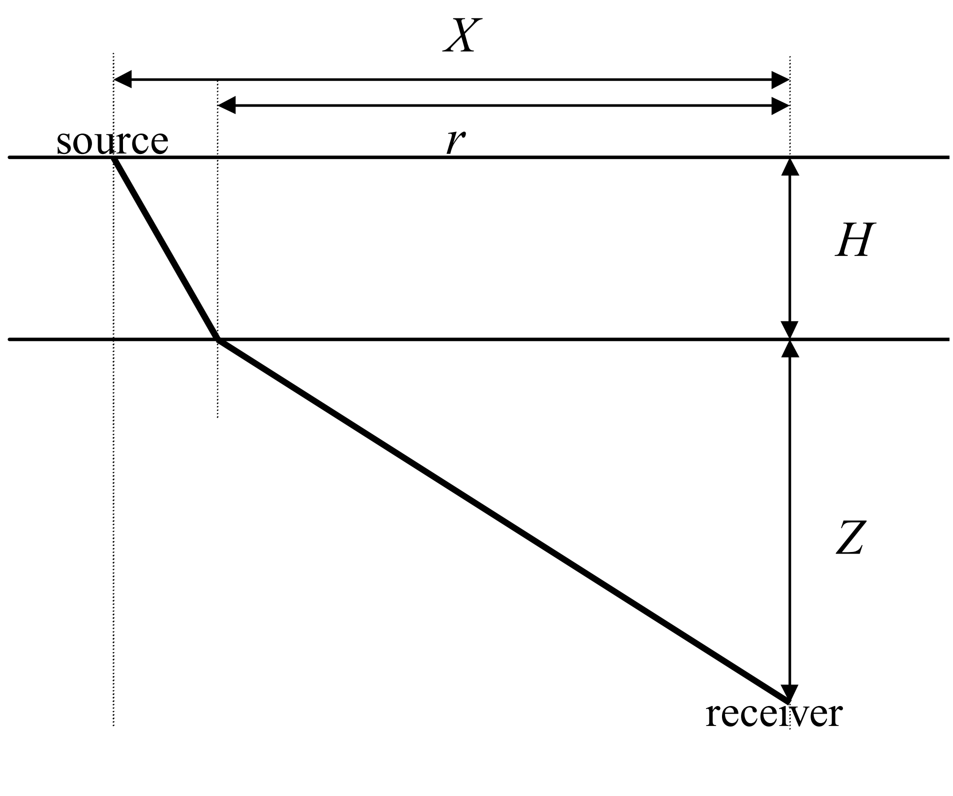

Figure.1. The two-layer model. Top layer thickness is denoted by *H*. The depth of the receiver in the buried layer is given by *Z*. The horizontal source-receiver offset is denoted as *X*. The symbol *r* corresponds to the lateral distance between the receiver and the refraction point. For simplicity, in this illustration the upper layer is homogeneous. The exact expressions derived in this paper, however, accommodate media exhibiting linear velocity function yielding circular-arc raypaths.

Fixing the measured traveltime, $t = t_0$, equation (15) may be solved for γ. Substituting back into equation (15) evaluated at $t = t_0$ yields an implicit equation for r,

$$f\left[r, \gamma(r, t_0)\right] = t_0 . \tag{17}$$

Differentiating equation (17) with respect to r, since t_0 is a constant, it follows that

$$\frac{\partial f}{\partial r}(r_0, \gamma_0) + \frac{\partial f}{\partial \gamma}(r_0, \gamma_0)\frac{\partial \gamma}{\partial r}(r_0, t_0) = 0 . \tag{18}$$

By Fermat's principle, as stated in equation (16), the first term vanishes; while by physical reasoning, $\partial f/\partial \gamma$ is never zero [see equation (12)]. Therefore, $\frac{\partial \gamma}{\partial r} = 0$ [equation (14)] showing that the physical value of $\gamma = \gamma_0$ is a critical point.

Differentiating equation (17) again, one can show that if t_0 is a minimum at r_0, then the critical point of $\gamma(r, t_0)$ is also a minimum. Thus,

$$\frac{\partial^2 f}{\partial r^2} + \frac{\partial^2 f}{\partial r \partial \gamma}\frac{\partial \gamma}{\partial r} + \frac{\partial f}{\partial \gamma}\frac{\partial^2 \gamma}{\partial r^2} = 0 . \tag{19}$$

The first term is positive since it is the second derivative of the traveltime function evaluated at a minimum; the second term vanishes because $\partial \gamma/\partial r = 0$; while in the third term, $\partial f/\partial \gamma$ is negative since the traveltime decreases with increasing γ [see equation (12)]. Therefore,

$$\frac{\partial^2 \gamma}{\partial r^2} > 0 , \tag{20}$$

indicating a minimum of γ; Q.E.D.

Having measured the traveltime $t(r, \gamma) = t_0$, we can uniquely determine the actual value of the anisotropic parameter, γ, by first solving equation (14) for r. Specifically, in the present case we may invert equation (12) to obtain an explicit formula for γ,

$$\gamma = \frac{1}{2}\left\{ \frac{V^2 r^2}{\beta_0{}^2\left[Vt - \sqrt{(X-r)^2 + H^2}\right]^2 - V^2 Z^2} - 1 \right\} . \tag{21}$$

The derivative is then:

$$\frac{\partial \gamma}{\partial r} = \frac{V^2 r}{\beta_0^2 \left[Vt - \sqrt{(X-r)^2 + H^2} \right]^2 - V^2 Z^2}$$

$$- \frac{V^2 \beta_0^2 r^2 (X-r) \left[Vt - \sqrt{(X-r)^2 + H^2} \right]}{\sqrt{(X-r)^2 + H^2} \left\{ \beta_0^2 \left[Vt - \sqrt{(X-r)^2 + H^2} \right]^2 - V^2 Z^2 \right\}^2} . \tag{22}$$

If the phase-slowness surface has no concavities (Helbig, 1994), the appropriate real value of r must lie between 0 and X (Figure 1) and correspond to a minimum of γ. This follows from the fact that (with the exception of the singular point at $x = -0.5$) the factor $1/(1+2\gamma)$ in the traveltime equation (12) decreases monotonically with increasing γ. That is,

$$\frac{d}{d\gamma} \sqrt{\frac{r^2}{1+2\gamma} + Z^2} < 0 \tag{23}$$

for physically meaningful values of r and Z.

Once the value of r is found from equations (14) and (22), it is inserted into equation (21) and the corresponding value of γ calculated. Note that, rather than numerically solving equation (15) for $d\gamma/dr = 0$, it is preferable to minimize the function $\gamma(r)$ directly, as we do numerically using a Newton-Raphson algorithm. This preference is suggested by a number of standard results in numerical analysis, which indicate direct minimization techniques are typically more robust than root-finding applied to a derivative. (Press et al., 1992, pp. 382-395).

Ansotropic or inhomogeneous upper layer

If the medium above the layer in question is assumed to be homogeneous and anisotropic, the first term on the right-hand side of equation (12), i.e., the traveltime in the upper medium, becomes,

$$\frac{\sqrt{\frac{(X-r)^2}{1+2\gamma_1} + H^2}}{V} , \tag{24}$$

where the speed, V, along the vertical axis, and the anisotropic parameter, γ_1, are known.

If the medium above the layer in question is assumed to be isotropic and linearly inhomogeneous, i.e., $v(z) = a + bz$, the first term on the right-hand side of equation (12), i.e., the traveltime in the upper medium, becomes,

$$\frac{1}{b}\left|\ln\left[\left(\frac{a+bH}{a}\right)\left(\frac{1-\sqrt{1-(sa)^2}}{1-\sqrt{1-\left(s(a+bH)\right)^2}}\right)\right]\right|, \tag{25}$$

where,

$$s=\frac{2b(X-r)}{\sqrt{\left[\left(b(X-r)\right)^2+a^2+(a+bH)^2\right]^2-\left[2a(a+bH)\right]^2}}, \tag{26}$$

(Slawinski and Slawinski, 1998).

NUMERICAL EXAMPLE

Our direct traveltime method

Subsurface model

Consider the following scenario. An isotropic, vertically inhomogeneous medium overlies an anisotropic, homogeneous medium. The interface between the two media is horizontal. The upper layer is 1000 m thick and has a linearly increasing velocity given by:

$$V(z)=2000+0.8z \quad \text{(SI units)}. \tag{27}$$

The lower medium exhibits elliptical anisotropy with a vertical symmetry axis described by the parameter $\gamma = 0.25$. The speed along the vertical symmetry axis is $\beta_0 = 2500$ m/s.

Such a scenario can be viewed as a good model for basins where, for instance, an anisotropic shale is situated below a thick clastic sequence. Slotnick (1959), for example, stated that in his experience, P-wave velocity in many basins is closely approximated by a linear function of depth down to about 6000 m. Acheson (1981) found a power-law relationship to represent P-wave velocity-depth variation most precisely, but that the power law approached a linear variation as his parameter, *n*, approached unity. He found *n* to lie between 0.83 and 1.0 at virtually all wells studied in western and northern Canada. Finally, Jain (1987) stated that most logs in the western Canadian basin justify a linear increase in velocity with depth down to the Paleozoic unconformity.

Recording geometry

To illustrate our method based on the type of data which one could obtain by a standard field acquisition method, let the VSP geometry be represented by a source S_1 located at a horizontal distance of 800 m from the receiver and a receiver, R_1, located at 100 m below the interface separating the isotropic and anisotropic media.

Forward traveltime

Following the forward model based on the exact traveltime expressions for linear-velocity media (e.g., Slawinski and Slawinski, 1998), and using our Mathematica® code (available from the authors), the exact traveltime is $t = 0.5635204778989572$ s.

Inversion using direct traveltimes

Following the inverse method presented herein and using our Mathematica® code (available from the authors), the result is $\gamma = 0.25$, as expected.

Traveltime-error sensitivity

For the sensitivity analysis consider the range of values $t \pm 0.001$ s, namely, $t_{\min} = 0.5625204779$ s and $t_{\max} = 0.5645204779$ s. This range of traveltime errors results in the anisotropy parameters given by $\gamma_{\min} = 0.302223$ and $\gamma_{\max} = 0.197557$, respectively.

The phase-slowness method

The phase-slowness method (e.g. Gaiser, 1990; Miller and Spencer, 1994) is based on estimates of the horizontal slowness, p, and the vertical slowness, q, wherein for the TIV case, one can use the equation:

$$q^2 + p^2(1+2\gamma) = \frac{1}{\beta_0^2}, \tag{28}$$

to solve for the anisotropy parameter, namely,

$$\gamma = \frac{1}{2}\left[\frac{1}{(p\beta_0)^2} - \left(\frac{q}{p}\right)^2 - 1\right], \tag{29}$$

where $p = \frac{dt}{dx} \approx \frac{\Delta t}{\Delta x}$ and $q = \frac{dt}{dz} \approx \frac{\Delta t}{\Delta z}$.

Considering two sources and two receivers, one can write

$$p_1 \approx \frac{t_{21} - t_{11}}{\Delta x} \quad \text{and} \quad p_2 \approx \frac{t_{22} - t_{12}}{\Delta x} \tag{30}$$

and

$$q_1 \approx \frac{t_{12} - t_{11}}{\Delta z} \quad \text{and} \quad q_2 \approx \frac{t_{22} - t_{21}}{\Delta z}, \tag{31}$$

where t_{ij} is the traveltime between the ith source and jth receiver. Hence, taking the average, namely

$$p = \frac{p_1 + p_2}{2}, \tag{32}$$

and

$$q = \frac{q_1 + q_2}{2}, \tag{33}$$

one obtains the values to be used in equation (29).

Recording geometry

Due to distance increments, as shown in expressions (30) and (31), the slowness method requires traveltime measurements at two surface locations and two depth locations. To provide additional insight to the application of this approach, let the VSP acquisition geometry be represented by three sources, S_1, S_2, and S_3, located at 750 m, 800 m and 1250 m from the vertical wellbore, respectively, and by two receivers, R_1 and R_2, located at 85 m and 100 m, respectively, below the interface separating the isotropic and anisotropic media.

Inversion using horizontal and vertical slowness

Considering S_1 and S_2, and using the exact traveltimes, the anisotropy-parameter value, resulting from the slowness method, is γ= 0.2502393252714912.

Distance-increment sensitivity

The slowness method is derived based on the concept of local slowness. Consequently, the change of distance increments used in expressions (30) and (31) affects the results. Considering the exact traveltimes from the sources S_1 and S_3, the value resulting from the slowness method is γ= 0.2601712990007443.

Traveltime-error sensitivity

In view of equations (30) to (33), the systematic error does not affect the slowness method. As an alternative, rounding off to the three decimal points was used to give an indication of the sensitivity of this method to the traveltime errors. This yields γ= 0.162.

Discussion of numerical results

The direct traveltime method:

- requires single source and single receiver;
- assumes linear vertical velocity function in the medium overlying the anisotropic medium;
- assumes no lateral velocity variation.

The slowness method:

- requires two sources and two receivers to yield a single result;
- assumes no lateral velocity variation.

Comparison of the two methods

Velocity-model considerations

In principle, the slowness method is more general than the traveltime method since it does not require any knowledge of the vertical-velocity function. On the other hand, the direct traveltime method is based on the assumption of the linear vertical-velocity function.

Acquisition considerations

The direct traveltime method requires a single source and a single receiver while the slowness method requires a couple of sources and a couple of receivers. The direct traveltime method uses the source and receiver locations as two points, while the slowness method uses horizontal and vertical increments and, hence, requires the same elevation of sources and the same horizontal location of receivers. In other words, unlike for the direct traveltime method, the application of the slowness method depends on surface topography and on the shape of the wellbore.

Sensitivity

Offset increments

The slowness method depends on distance increments, namely Δx and Δz – a nonexistent consideration for the direct traveltime method. As the distance increments grow, the error increases. On the other hand, the independence of each source-receiver couple, in the direct traveltime method, gives certain statistical indication of results.

Traveltime

Using exact data, both the traveltime and the slowness methods give good results. Slowness method would also yield an exact value – as exhibited by the traveltime method – if $\Delta x \to 0$ and $\Delta z \to 0$, in expressions (30) and (31). The direct traveltime method appears to be significantly less sensitive to the traveltime errors than the slowness method.

ANALYTIC ERROR ANALYSIS

One valuable feature of the above inversion technique is that it has fairly good performance in the presence of measurement error. In particular, errors in measured traveltime results in an (absolute) error in γ which is roughly an order of magnitude larger. This compares very favourably with other techniques, such as the slowness method, where the resulting error in γ can be two orders of magnitude larger. One reason for this good performance is the fact our technique is derived from exact equations. Another reason, intuitively, is that the value of γ is computed at a local minimum, and hence changes very slowly near this minimum.

More precisely, suppose $t = t_0$ is the exact traveltime, $r = r_0$ is the exact minimizer to the function $\gamma = \gamma(r, t_0)$ (thus an exact root to $\frac{\partial \gamma}{\partial r}(r, t_0) = 0$), and $\gamma_0 = \gamma(r_0, t_0)$ is the exact value for the Thomsen parameter. If there is an error of Δt in the measured traveltime, the inversion method begins with the exact time t_0 replaced by $t_0 + \Delta t$. Minimizing the function $\gamma(r) = \gamma(r, t_0 + \Delta t)$ yields a new root to the perturbed equation:

$$0 = \frac{\partial \gamma}{\partial r}(r, t_0 + \Delta t) \tag{34}$$

To first order, this amounts to solving

$$\begin{aligned} 0 &= \frac{\partial \gamma}{\partial r}(r_0 + \Delta r, t_0 + \Delta t) \\ &\approx \frac{\partial \gamma}{\partial r}(r_0, t_0) + \frac{\partial^2 \gamma}{\partial r^2}(r_0, t_0)\Delta r + \frac{\partial^2 \gamma}{\partial t \partial r}(r_0, t_0)\Delta t \\ &= 0 + \frac{\partial^2 \gamma}{\partial r^2}(r_0, t_0)\Delta r + \frac{\partial^2 \gamma}{\partial t \partial r}(r_0, t_0)\Delta t, \end{aligned} \tag{35}$$

and thus solving for Δr in terms of Δt, we find

$$\Delta r = \frac{\dfrac{\partial^2 \gamma}{\partial t \partial r}}{\dfrac{\partial^2 \gamma}{\partial r^2}}(r_0, t_0). \tag{36}$$

Now, the error in γ computed to first order, is given by:

$$\gamma(r_0 + \Delta r, t_0 + \Delta t) \approx \gamma(r_0, t_0) + \frac{\partial \gamma}{\partial r}(r_0, t_0)\Delta r + \frac{\partial \gamma}{\partial t}(r_0, t_0)\Delta t \tag{37}$$

but of course the middle term is zero, since $\frac{\partial \gamma}{\partial r}(r_0, t_0) = 0$. Thus the error in γ is proportional to the partial derivative $\frac{\partial \gamma}{\partial t}(r_0, t_0)$.

With $\gamma(r, t)$ given by equation (21) we easily compute:

$$\frac{\partial\gamma}{\partial t} = -\frac{1+2\gamma}{\dfrac{Vt-\sqrt{(X-r)^2+H^2}}{V} - \dfrac{Z^2V}{\beta_0^2\left(Vt-\sqrt{(X-r)^2+H^2}\right)}}. \tag{38}$$

For a numerical example with homogeneous upper layer and values:

$$\begin{aligned} V &= 2000 \text{ m/s} \\ \beta_0 &= 2500 \text{ m/s} \\ X &= 800 \text{ m} \\ H &= 1000 \text{ m} \\ Z &= 100 \text{ m} \\ \gamma_0 &= 0.25 \\ t_0 &= 0.65938 \text{ s} \\ r_0 &= 171.84 \text{ m} \end{aligned} \tag{39}$$

one finds

$$\frac{\partial\gamma}{\partial t} = -32.8 \text{ s}^{-1}.$$

In particular, an error in measured *t* of one millisecond ($\delta t = 0.001$ s) leads to an error in γ of about 0.033 ($\Delta\gamma = -0.033$). These derivative results are consistent with the results found numerically in this paper.

By comparison, in the slowness method, we have

$$\gamma = \frac{1}{2p^2}\left(\frac{1}{\beta_0^2} - q^2\right) - \frac{1}{2} \tag{40}$$

where for horizontal and vertical slownesses, *p* and *q*, one uses average measures of the inverse velocities $\Delta t/\Delta x$ and $\Delta t/\Delta z$, respectively. An error of ε in the Δt values leads to errors in *p* and *q* of $\varepsilon/\Delta x$ and of $\varepsilon/\Delta x$, respectively. To first order, the error in γ is found by:

$$\begin{aligned} \gamma(p+\Delta p, q+\Delta q) - \gamma(p,q) &\approx \frac{\partial\gamma}{\partial p}\Delta p \pm \frac{\partial\gamma}{\partial q}\Delta q = \delta\gamma \\ &= \frac{\partial\gamma}{\partial p}\frac{\varepsilon}{\Delta x} \pm \frac{\partial\gamma}{\partial q}\frac{\varepsilon}{\Delta z} \\ &= -\frac{1+2\gamma}{p}\frac{\varepsilon}{\Delta x} \pm \frac{q}{p^2}\frac{\varepsilon}{\Delta z} \end{aligned} \tag{41}$$

where we have written ± since the time measurement errors in p and q may be in different directions. In fact, for the uncertainty in γ we can write:

$$\delta\gamma = \left|\frac{1+2\gamma}{p}\frac{\varepsilon}{\Delta x}\right| + \left|\frac{q}{p^2}\frac{\varepsilon}{\Delta z}\right| \tag{42}$$

For the numerical examples comparable to the above case, we take Δx = 50 m and Δz = 15 m, and compute p = 0.0002436 s/m and q = 0.0002661 s/m. We then obtain

$$\gamma(p+\Delta p, q+\Delta q) \approx 421\varepsilon \quad \text{(SI units).} \tag{43}$$

That is, for an error of 0.001 s in the traveltime measurements, we would have ε = 0.0014 s, i.e., for Δt. Thus we get errors in γ of 0.0328 (13.1%) and 0.595 (238%) for the traveltime and slowness methods, respectively, a factor of about 18 difference.

DISCUSSION AND CONCLUSIONS

The proposed inversion scheme allows one to calculate the value of the Thomsen anisotropic parameter, γ, for horizontally layered TIV media. The information required for inversion consists of oblique traveltimes, layer thickness and vertical wave speeds. The acquisition context of vertical seismic profiles, with known source and receiver locations, is particularly suitable for this inversion scheme. Layer thickness can be obtained directly from wellbore information, while the vertical wave speed can be reliably established from the zero-offset survey.

The inversion is expressly formulated for the case of elliptical anisotropy, which applies to both SH waves exactly and P waves in the short-spread approximation, and is characterized by a single Thomsen parameter, γ. Although in this paper the method is illustrated for a two-layer case, it can be modified for a multilayer setting. In such a scenario, the inversion is progressively iterated for deeper layers, once the parameters of all the shallower layers are known (Slawinski, 1996).

The results of the traveltime inversion are quite sensitive to errors in the traveltime measurement (e.g., Monagan and Slawinski, 1998). However, greater reliability of the inversion results will be provided by a multiplicity of measurements. Under the assumption of TIV media, the same value of γ is expected for all source-receiver configurations where the receiver is located within the same layer. This property provides a measure of consistency.

The anisotropic parameter, γ, is defined directly in terms of both vertical, β_0, and horizontal, $v(\pi/2)$, velocities [equation (5)]. In TIV media, the vertical velocity, β_0, is measured directly and, in this inversion method, its value is assumed to be known. Thus, instead of inverting for the anisotropic parameter, γ, which contains both horizontal and vertical velocities, one could invert explicitly for the unknown horizontal phase velocity, $v(\pi/2)$. Hence, from equations (4) and (21):

$$v(\pi/2) = \frac{\beta_0 Vr}{\sqrt{\beta_0^2 \left[Vt - \sqrt{(X-r)^2 + H^2} \right]^2 - V^2 Z^2}}, \tag{44}$$

and the present inversion method entails finding the unique local minimum of $v(\pi/2)$, as a function of $r \in (0, X)$. Then, the value of the parameter γ can be obtained from its definition. Note that the propagation in TIV media implies that the magnitudes of phase and group velocities are the same along the symmetry axes. Therefore, for horizontal propagation in TIV media, group and phase angles coincide and equation (6) or (8) can be used to yield the magnitude of the horizontal group velocity, $V(\pi/2)$, by setting the angle to $\pi/2$ in the above-mentioned equations, namely, $V(\pi/2) = \beta_0 \sqrt{1+2\gamma}$.

Error analysis has shown that our direct traveltime technique has less error than the phase-slowness method. A numerical example representing a typical VSP yielded errors from the phase-slowness method that were about 25 times greater than for a single determination by the direct traveltime method. Furthermore, in cases where the VSP has been acquired with a single source offset, e.g., for some offshore wells, the direct traveltime method will at least yield some results, whereas the phase-slowness method will not.

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