Conversion coefficients at a liquid/solid interface

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ABSTRACT

When upward-propagating rays transporting seismic energy are recorded at the earth's surface, the vertical and horizontal components of displacement are not the vector decomposition of the associated amplitude vector. Rather, a more involved interaction of the incident ray, either P or S_v, and two related reflected arrivals at this interface, the PP and PS_v in the first case and the S_vP and S_vS_v in the second, are incorporated. Although the earth's atmosphere is a fluid, it is usually assumed to be a vacuum so that no disturbance exists above the interface.

In the analogous situation at a liquid/solid interface, where the receivers are located on, and are assumed to be coupled to, the solid, the effect of the fluid becomes significant and must be included in the problem, as a compressional *(P)* ray of seismic interest propagates in the liquid. This results in a marginally more complex problem than the vacuum/solid interface. Also, this is a highly idealized statement of the problem, as in most realistic situations, a transition zone between the liquid and the solid exists which is quite different from the sharp discontinuity referred to above. However, it is thought to be instructive to first consider the simpler case before proceeding to the more difficult transition layer problem.

As reflection coefficients at a liquid/solid boundary are required in the derivation of these conversion coefficients, the formulae for the case of the solid being a transversely isotropic medium are presented which allows additional flexibility. For completeness, all reflection and transmission coefficients for a transversely isotropic medium in contact with a fluid are presented.

INTRODUCTION

When studying the vertical and horizontal components of displacement of a seismic disturbance at the earth's surface, a more complicated delineation process than vector decomposition of an arrival should be used. The results of this procedure have come to be known as *surface conversion coefficients* (Cerveny and Ravindra, (1970)).

A slightly more complex problem arises when geophones are located in a marine environment at the water-bottom boundary. The earth's atmosphere is usually assumed to be a vacuum in seismic problems, but in our first case it is a fluid. This approach must be modified in the liquid/solid interface case. Only the problem of waves arriving at the receivers at the liquid/solid interface from below will be considered here.

The computation of reflection and transmission coefficients at a liquid/solid interface has received fairly extensive treatment in the literature (for example, Graebner, 1992). The coefficients required here are taken from that work, where the case of a liquid halfspace overlying a transversely-isotropic elastic halfspace is developed, and the relevant formulae and definitions are presented in the Appendix. The motivation for this is, that in the above work and papers cited therein, typographical errors are present: the corrected formulae are given here.

This initial problem is somewhat simpler than the solid/solid interface case in that there are only three equations in three unknowns. Assuming either plane *P*–wave or S_v–wave incidence from the solid (lower) layer of unit amplitude, the unknowns are the *P*–wave transmission coefficient into the liquid and the *P* and S_v reflection coefficients at the liquid/solid interface.

Analogues of these may be derived for receivers located at a liquid/solid interface, in a similar manner to that employed in obtaining the free surface conversion coefficients.

THEORETICAL DEVELOPMENT

It is assumed that a receiver lies in the solid at some incremental depth $z = \varepsilon$ below the liquid/solid interface. The arrivals at this point due to plane wave incidence from below would be (a) for *P*-wave incidence: the direct *P* wave, the reflected *PP* wave and the reflected PS_v wave, and (b) for S_v-wave incidence: the direct S_v wave, the reflected S_vS_v wave and the reflected S_vP wave. In both cases, these arrivals are in addition to the *P*–wave transmitted into the liquid. (Figures (1) and (2)).

As plane wave incidence is assumed, the amplitude of either of the two incident wave types is set to unity. The time harmonic vector quantities describing the particle displacement at the point ε are

$$\mathbf{u_P}(x,\varepsilon,t) = \exp(-i\omega t)\left[\exp(iw\tau_P)\,\mathbf{e_P} + \exp(i\omega\tau_{PP})\,R_{P2P2}\,\mathbf{e_{PP}} + \exp(i\omega\tau_{PS})\,R_{P2S2}\,\mathbf{e_{PS}}\right] \tag{1}$$

for *P*-wave incidence, and

$$\mathbf{u_S}(x,\varepsilon,t) = \exp(-i\omega t)\left[\exp(iw\tau_S)\,\mathbf{e_S} + \exp(i\omega\tau_{SP})\,R_{S2P2}\,\mathbf{e_{SP}} + \exp(i\omega\tau_{SS})\,R_{S2S2}\,\mathbf{e_{SS}}\right] \tag{2}$$

for S_v incidence, with $z = 0$ corresponding to the liquid/solid boundary, and z chosen positive downwards.

The quantities involving **e** are the unit particle displacement (polarization) vectors for the given subscripted wave type (defined in the Appendix), as are the reflection coefficients R_{XIYJ} for a transversely isotropic solid medium. *X* and *Y* refer to the incident and reflected/transmitted wave types, respectively, and *I* and *J* to the incident and reflected/transmitted media with *1* denoting the liquid and *2* the solid. Those terms in τ are the travel times of the arrival specified by the subscript. These are identified in Figure 2.

If the limit $\varepsilon \to 0$ is taken in both equations (1) and (2), the travel-times of the three arrivals in each of the two cases are identical, so that

$$\mathbf{u_P}(x,\varepsilon=0,t)=\exp\left\{-i\omega\left(t-\tau_P(0)\right)\right\}\left[\mathbf{e_P}+R_{P2P2}\,\mathbf{e_{PP}}+R_{P2S2}\,\mathbf{e_{PS}}\right] \tag{3}$$

and

$$\mathbf{u_S}(x,\varepsilon=0,t)=\exp\left\{-i\omega\left(t-\tau_S(0)\right)\right\}\left[\mathbf{e_S}+R_{S2P2}\,\mathbf{e_{SP}}+R_{S2S2}\,\mathbf{e_{SS}}\right] \tag{4}$$

The square bracket terms in equations (3) and (4) are the analogues of the free-surface conversion vectors for the case of a wave arriving at a liquid/solid interface from below. These are the 2-component displacement values that are recorded by a receiver at the water bottom, assuming perfect coupling between the receiver and the underlying solid.

For the transversely isotropic case described in the Appendix, the expressions for the unit displacement (polarization) vectors, **e** are quite complex. However, for the isotropic case, they degenerate to combinations of sines and cosines as in the liquid layer. Introducing the expressions for the **e**'s, defined by equations (A.3) and (A.4) into (3) and (4) yields the horizontal and vertical, *(x , z),* components of the conversion vectors

$$C_x^{(P)}=\ell_{\alpha2}+\ell_{\alpha2}R_{P2P2}+m_{\beta2}R_{P2S2} \tag{5.a}$$

$$C_z^{(P)}=-m_{\alpha2}+m_{\alpha2}R_{P2P2}-\ell_{\beta2}R_{P2S2} \tag{5.b}$$

and

$$C_x^{(Sv)}=-m_{\beta2}+\ell_{\alpha2}R_{S2P2}+m_{\beta2}R_{S2S2} \tag{6.a}$$

$$C_z^{(Sv)}=-\ell_{\beta2}-m_{\alpha2}R_{S2P2}+l_{\beta2}R_{S2S2} \tag{6.b}$$

where the subscripts x or z indicate the component of the conversion vector, and the superscripts, *(P)* or *(S_v)*,, the type of the incident wave. In the isotropic case, the unit **e** vectors have the simpler forms

$$\mathbf{e_P}=\left(\sin\theta_P,-\cos\theta_P\right)\quad,\quad\mathbf{e_{PP}}=\left(\sin\theta_P,\cos\theta_P\right)\quad,\quad\mathbf{e_{PS}}=\left(\cos\theta_{Sv},-\sin\theta_{Sv}\right) \tag{7.a}$$

and

$$\mathbf{e_S}=\left(-\cos\theta_{Sv},-\sin\theta_{Sv}\right)\quad,\quad\mathbf{e_{SS}}=\left(\cos\theta_{Sv},-\sin\theta_{Sv}\right)\quad,\quad\mathbf{e_{SP}}=\left(\sin\theta_P,\cos\theta_P\right) \tag{7.b}$$

The acute angles, θ_P and θ_{Sv}, are those which the corresponding wavefront normal-types make with the vertical, (z), axis which has been chosen positive downwards. These angles, together with the related polarization vectors, are shown in Figure 2.

CONCLUSIONS

A derivation of conversion coefficients, for use in decomposition of an incident wavefield at a liquid/solid boundary into vertical and horizontal components, has been given. In general, this method of determining the displacement components that should be recorded by receivers located in this type of acquisition situation, differs from a straight vector decomposition. Assuming that good coupling between the receivers and the solid below the water layer exists, this is the proper way in which the incident wavefields should be decomposed into their constituent components. The problem has been highly idealized in that a sharp discontinuity is assumed between the liquid and solid layers. Further investigations of this topic should have provisions for replacing this sharp boundary with a transition layer in which the rigidity, μ, is allowed to increase from a zero value in the liquid to its value in the underlying solid.

APPENDIX: LIQUID/SOLID INTERFACE REFLECTION COEFFICIENTS

At a plane interface between a liquid (upper) medium and solid, elastic (lower) medium (assumed to be transversely isotropic with its meridional axes aligned with the plane interface), the vertical component of displacement, u_z, the normal stress, τ_{zz}, and the shear stress, τ_{xz}, must be continuous across the boundary separating them (Brekhovskikh, 1980).

In this configuration, there is the possibility of three modes of wave propagation; a quasi-compressional, qP, wave in the solid, a quasi-shear, qS_v, in the solid, and a compressional, P, wave in the liquid.

The compressional velocity in the liquid is specified as α_1, and the volume density ρ_1. If p is the horizontal component of the slowness vector, the vertical component of the slowness vector, q_1 is defined as

$$q_1 = \left(\alpha_1^{-2} - p^2\right)^{1/2} \tag{A.1}$$

and the components of the unit particle displacement (polarization) vector, $\mathbf{e}_1 = \left(\ell_1, m_1\right)$, are given by

$$\ell_1 = \sin\theta_1 \tag{A.2a}$$

$$m_1 = \cos\theta_1 \tag{A.2b}$$

In the transversely isotropic medium the elastic parameters required to describe the medium are C_{11}, C_{33}, C_{13} and C_{44} (Musgrave, 1970), together with the volume density ρ_s. The unit particle displacement (polarization) vectors for the qP and qS_v are $\mathbf{e}_{\alpha 2} = (l_{\alpha 2}, m_{\alpha 2})$ and $\mathbf{e}_{\beta 2} = (l_{\beta 2}, m_{\beta 2})$, respectively, and may be written as

$$\ell_{\alpha 2} = \left[\frac{\left(C_{33} q_{\alpha 2}^2 + C_{44} p^2 - 1\right)}{\left(C_{11} p^2 + C_{44} q_{\alpha 2}^2 - 1\right) + \left(C_{33} q_{\alpha 2}^2 + C_{44} p^2 - 1\right)} \right]^{1/2} \tag{A.3a}$$

$$m_{\alpha 2} = \left[\frac{\left(C_{11} q_{\alpha 2}^2 + C_{44} p^2 - 1\right)}{\left(C_{11} p^2 + C_{44} q_{\alpha 2}^2 - 1\right) + \left(C_{33} q_{\alpha 2}^2 + C_{44} p^2 - 1\right)} \right]^{1/2} \tag{A.3b}$$

for qP wave propagation and

$$\ell_{\beta 2} = \left[\frac{\left(C_{11} p^2 + C_{44} q_{\beta 2}^2 - 1\right)}{\left(C_{11} p^2 + C_{44} q_{\beta 2}^2 - 1\right) + \left(C_{33} q_{\beta 2}^2 + C_{44} p^2 - 1\right)} \right]^{1/2} \tag{A.4a}$$

$$m_{\beta 2} = \left[\frac{\left(C_{33} q_{\beta 2}^2 + C_{44} p^2 - 1\right)}{\left(C_{11} p^2 + C_{44} q_{\beta 2}^2 - 1\right) + \left(C_{33} q_{\beta 2}^2 + C_{44} p^2 - 1\right)} \right]^{1/2} \tag{A.4b}$$

for qS_v propagation. The prefix "q" in the notation is used to indicate "quasi" as these vectors do not, in general, lie parallel or horizontal to either the wavefront normal (phase) vector, or the ray (group) vector in an anisotropic medium. The quantities in the above which require definition are the horizontal component of slowness, p, and the vertical components of slowness, q, which is obtained in terms of p from the eikonal equation as

$$q_{\alpha 2} = \frac{1}{\sqrt{2}} \left[-K_1 - \left(K_1^2 - 4K_2\right)^{1/2} \right]^{1/2} \tag{A.5a}$$

and

$$q_{\beta 2} = \frac{1}{\sqrt{2}} \left[-K_1 + \left(K_1^2 - 4K_2\right)^{1/2} \right]^{1/2} \tag{A5.b}$$

with

$$K_1 = \frac{\left\{ \left[C_{44}\left(C_{11} + C_{33}\right) - C_D^2 \right] p^2 - \left(C_{33} + C_{44}\right) \right\}}{C_{33} C_{44}} \tag{A.6a}$$

$$C_D^2 = \left(C_{13} + C_{44}\right)^2 - \left(C_{11} - C_{44}\right)\left(C_{33} - C_{44}\right) \tag{A.6b}$$

$$K_3 = \frac{1 + \left[C_{11} C_{44} p^2 - \left(C_{11} + C_{44}\right) \right] p^2}{C_{33} C_{44}} \tag{A.6c}$$

The reflection coefficients required in the text at the liquid/solid interface for both qP and qS_v are R_{P2P2}, R_{P2S2}, R_{S2S2} *and* R_{S2P2}, where the first two subscripted characters indicate the incident-wave type and the incident medium, and the last two, the reflected wave type and medium of propagation. Medium *2* is taken to be the solid.

These coefficients, together with the other 5 (marked with the superscript "*"), which may exist at an interface of this type, are given explicitly as

$$R_{P2P2} = \frac{-m_{\alpha 2}\left[Hb + p\left(q_{\beta 2} + q_{\alpha 2}\right)\right] + dc}{D} \tag{A.7}$$

$$R_{P2S2} = \frac{2m_{\alpha 1}Kf}{D} \tag{A.8}$$

$$R_{S2S2} = \frac{m_{\ell}\left[Hb + p\left(q_{\beta 2} + q_{\alpha 2}\right)J\right] + dc}{D} \tag{A.9}$$

$$R_{S2P2} = \frac{-2m_{\alpha 2}Le}{D} \tag{A.10}$$

$$R^{*}_{P1P1} = \frac{m_{\alpha 1}\left[Ga + p\left(q_{\beta 2} - q_{\alpha 2}\right)I\right]}{D} \tag{A.11}$$

$$R^{*}_{P1P2} = \frac{2m_{\alpha 1}de}{D} \tag{A.12}$$

$$R^{*}_{P1S2} = -\frac{2m_{\alpha 1}df}{D} \tag{A.13}$$

$$R^{*}_{P2P1} = \frac{2Kc}{D} \tag{A.14}$$

$$R^{*}_{S2P1} = \frac{2Lc}{D} \tag{A.15}$$

where

$$D = m_{\alpha}\left[Ga + p\left(q_{\beta} - q_{\alpha}\right)I\right] + dc \tag{A.16}$$

and the expressions within the above equations are defined as

$$a = m_{\alpha 2} m_{\beta 2} + \ell_{\alpha 2} \ell_{\beta 2} \tag{A.17}$$

$$b = m_{\alpha 2} m_{\beta 2} - \ell_{\alpha 2} \ell_{\beta 2} \tag{A.18}$$

$$c = q_{\alpha 2} \ell_{\alpha 2} \ell_{\beta 2} + q_{\beta 2} m_{\alpha 2} m_{\beta 2} \tag{A.19}$$

$$d = \rho_1 \alpha_1^2 \left(p \ell_{\alpha 1} + q_{\alpha 1} m_{\alpha 1} \right) \tag{A.20}$$

$$e = q_{\beta 2} m_{\beta 2} - p \ell_{\beta 2} \tag{A.21}$$

$$f = q_{\alpha 2} \ell_{\alpha 2} + p m_{\alpha 2} \tag{A.22}$$

$$G = q_{\alpha 2} q_{\beta 2} C_{33} - p^2 C_{13} \tag{A.23}$$

$$H = q_{\alpha 2} q_{\beta 2} C_{33} + p^2 C_{13} \tag{A.24}$$

$$I = m_{\alpha 2} \ell_{\beta 2} C_{33} + \ell_{\alpha 2} m_{\beta 2} C_{13} \tag{A.25}$$

$$J = m_{\beta 2} \ell_{\alpha 2} C_{13} - m_{\alpha 2} \ell_{\beta 2} C_{33} \tag{A.26}$$

$$K = p \ell_{\alpha 2} C_{13} + q_{\alpha 2} m_{\alpha 2} C_{33} \tag{A.27}$$

$$L = p m_{\alpha 2} C_{13} - q_{\beta 2} \ell_{\beta 2} C_{33} . \tag{A28}$$

REFERENCES

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FIGURES

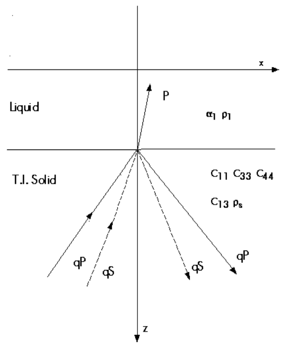

Figure 1. The possible waves types which exist at a liquid/solid interface for *P* and S_v incidence from the solid (lower) transversely isotropic medium.

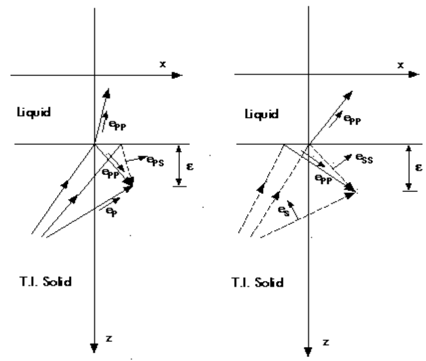

Figure 2. Schematics of *P*- and S_v -wave incidence at a liquid/solid interface. The directions of the polarization vectors are defined as there is a degree of freedom in choosing the orientation of the shear vector. The polarization vectors are shown as being either perpendicular or parallel to the wavefront normal vectors. This only occurs in an isotropic medium. In a transversely isotropic medium they may have similar orientations, but not, in general, perpendicular and parallel to the wavefront normals, thus deviating somewhat from what is shown in the figure, depending upon the degree of anisotropy.