

Dual extrapolation algorithms towards optimum efficiency and accuracy

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ABSTRACT

In heterogenous media with velocity field $v(x,z)$, Fourier domain wavefield extrapolators are typically applied recursively over depth to perform migration or modelling. Due to the approximation in the theory, errors accumulate through depth steps. In this paper a dual algorithm is formulated that employs high-accuracy, integral, wavefield extrapolation to compute quality control wavefields at a grid coarser than the imaging grid and employs algorithms of lower accuracy to compute the intermediate wavefields.

PSPI AND NSPS IN ACCURATE FORM AND PRACTICAL IMPLEMENTATION

In accurate form, PSPI becomes an integral over horizontal wave number k_x , which performs wavefield extrapolation simultaneously with an inverse Fourier transform. It can be written as (Margrave and Ferguson 1999)

$$\psi(x, z, \omega) = \frac{1}{(2\pi)^2} \int \alpha(x, k_x, z, \omega) \varphi(k_x, 0, \omega) \exp(-ik_x x) dk_x, \quad (1)$$

where $\alpha(x, k_x, z, \omega)$ is

$$\alpha(x, k_x, z, \omega) = e^{-i\Delta z \sqrt{\frac{\omega^2}{v^2(x)} - k_x^2}}. \quad (2)$$

$\psi(x, z, \omega)$ is the FX domain expression of the wavefield at depth z and $\varphi(k_x, 0, \omega)$ is the FK domain expression of the wavefield at depth $z=0$.

Equation (1) has a complementary form, NSPS, which performs wavefield extrapolation simultaneously with a forward Fourier transform,

$$\varphi(k_x, z, \omega) = \int \alpha(x, k_x, z, \omega) \psi(x, 0, \omega) \exp(ik_x x) dx, \quad (3)$$

where $\alpha(x, k_x, z, \omega)$ is expressed by equation (2).

Margrave and Ferguson (1999b) showed that the NSPS and PSPI can be naturally combined into a symmetric wavefield extrapolator, SNPS. A Taylor series derivation of PSPI, NSPS and SNPS show that SNPS has a smaller error and is more stable than either PSPI or NSPS alone. In practice, PSPI and NSPS are computed using a set of spatial windows and results in the windows are collected. PSPI can be written as

$$\Psi(x, z, \omega) = \sum_j \Omega_j \frac{1}{2\pi} \int_{-\infty}^{\infty} \alpha_j(k_x, z, \omega) \phi(k_x, 0, \omega) \exp(-ik_x x) dx, \quad (4)$$

and NSPS can be written as

$$\phi(k_x, z, \omega) = \sum_j \alpha_j(k_x, z, \omega) \int_{-\infty}^{\infty} \Psi(x, 0, \omega) \Omega_j \exp(ik_x x) dx, \quad (5)$$

where the window Ω_j is

$$\Omega_j(x) = \begin{cases} 1, & v(x) = v_j \\ 0, & \text{otherwise} \end{cases}. \quad (6)$$

$\alpha_j(k_x, z, \omega)$ becomes a local split-step phase-shift operator,

$$\alpha_j(k_x, z, \omega) = \exp \left(iz \left(\sqrt{\left(\frac{\omega}{v_j} \right)^2 - k_x^2} - \omega(s_j(x) - s_{j,0}) \right) \right), \quad (7)$$

where $s_j(x)$ is the slowness in the j^{th} window and $s_{j,0}$ is reference slowness in the j^{th} window.

A more precise approach is to compute the integrals (1) and (3) directly at each imaging step. Equation (1) can be written in matrix form

$$\Psi(x, z + \Delta z, \omega) = \begin{pmatrix} e^{i \left(\Delta z \sqrt{\frac{\omega^2}{v_1^2} - k_{x1}^2} \right)} & \cdot & \cdot & \cdot & e^{i \left(\Delta z \sqrt{\frac{\omega^2}{v_1^2} - k_{x1}^2} - x_1 k_{x1} \right)} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ e^{i \left(\Delta z \sqrt{\frac{\omega^2}{v_n^2} - k_{xn}^2} \right)} & \cdot & \cdot & \cdot & e^{i \left(\Delta z \sqrt{\frac{\omega^2}{v_n^2} - k_{xn}^2} - x_n k_{xn} \right)} \end{pmatrix} \begin{pmatrix} \phi(k_{x1}, z) \\ \cdot \\ \cdot \\ \cdot \\ \phi(k_{xn}, z) \end{pmatrix}, \quad (8)$$

and equation (3) can be written as

$$\varphi(kx, z + \Delta z, \omega) = \begin{pmatrix} e^{i\left(\Delta z \sqrt{\frac{\omega^2}{v_1^2} - k_{x1}^2} - x_1 k_{x1}\right)} & & & e^{i\left(\Delta z \sqrt{\frac{\omega^2}{v_n^2} - k_{xn}^2} - x_n k_{xn}\right)} \\ & \ddots & & \\ & & \ddots & \\ & & & \ddots \\ e^{i\left(\Delta z \sqrt{\frac{\omega^2}{v_1^2} - k_{x1}^2} - x_1 k_{x1}\right)} & & & e^{i\left(\Delta z \sqrt{\frac{\omega^2}{v_n^2} - k_{xn}^2} - x_n k_{xn}\right)} \end{pmatrix} \begin{pmatrix} \Psi(x_1, z) \\ \vdots \\ \Psi(x_n, z) \end{pmatrix} \quad (9)$$

Computation of either equation (8) or (9) is required at every imaging step and it provides higher accuracy than equations (4) and (5). However the computation effort is generally not acceptable in real industrial imaging projects.

LARGE-STEP FORMULAE FOR PSPI AND NSPS

In homogeneous media, the phase shift can be split into a static phase-shift term and a focusing term

$$\begin{aligned} \Phi &= \Phi_{static} + \Phi_{focus} \\ \Phi_{static} &= \frac{\omega \Delta z}{v} \\ \Phi_{focus} &= \frac{\omega \Delta z}{v} \left(\sqrt{1 - \frac{k_x^2 v^2}{\omega^2}} - 1 \right), \end{aligned} \quad (10)$$

where Φ_{static} can be applied in FX domain, and the focusing term has to be applied in the FK domain.

For $v(x, z)$ media, the static phase shift through multiple steps can be accumulated and the average velocity can be used for one-step extrapolation

$$\Phi_{static} = \int_0^z \frac{\omega}{v(x, z')} dz' = \frac{\omega z}{v_{ave}(x, z)}. \quad (11)$$

The focusing term can also be accumulated,

$$\Phi_{focus} = \int_0^z \frac{\omega}{v(x, z')} \left(\sqrt{1 - \frac{k_x^2 v(x, z')^2}{\omega^2}} - 1 \right) dz'. \quad (12)$$

Expanding the square-root term with power series leads to,

$$\begin{aligned} \Phi_{focus} &= -\frac{k_x^2 z}{2\omega} \frac{1}{z} \int_0^z v(x, z') dz' - \frac{k_x^4 z}{8\omega^3} \frac{1}{z} \int_0^z v(x, z')^3 dz' + \frac{k_x^6}{16\omega^5} \frac{1}{z} \int_0^z v(x, z')^5 dz' - \dots \\ &= -\frac{k_x^2 z}{2\omega} v_{mean}(x, z) - \frac{k_x^4 z}{8\omega^3} \frac{1}{z} \int_0^z v(x, z')^3 dz' + \frac{k_x^6}{16\omega^5} \frac{1}{z} \int_0^z v(x, z')^5 dz' - \dots \end{aligned} \quad (13a)$$

The v_{mean} is the mean velocity over depth, which is defined as

$$v_{mean} = \frac{1}{z} \int_0^z v(z') dz' \quad (13b)$$

The infinite series of equation (13) can then be approximated by

$$\Phi'_{focus} = \frac{\omega z}{v_{mean}(x, z)} \left(\sqrt{1 - \frac{k_x^2 v_{mean}^2(x, z)}{\omega^2}} - 1 \right) \quad (14)$$

Expanding the equation (14) with power series and then subtracting equation (12) leads to the higher-order error terms

$$\Phi_{error} = \frac{k_x^4 z}{8\omega^3} \left(\frac{1}{z} \int_0^z v(x, z')^3 dz' - v_{mean}^3(x, z) \right) + \frac{k_x^6 z}{16\omega^5} \left(\frac{1}{z} \int_0^z v(x, z')^5 dz' - v_{mean}^5(x, z) \right) + \dots \quad (15)$$

which generally can be ignored.

Note the one-step focusing term, equation (14), is dependent on x . The nonstationary wavefield extrapolation theory can be applied solely to the focusing term after a FX domain static phase-shift with the average velocity. Equations (1) and (3) now become

$$\psi(x, z, \omega) = \frac{1}{(2\pi)^2} \int \alpha'(x, k_x, z, \omega) \left[F_x \left(\psi(x, 0, \omega) \exp \left(\frac{i\omega \Delta z}{v_{ave}} \right) \right) \right] \exp(-ik_x x) dk_x \quad (16)$$

and

$$\varphi(k_x, z, \omega) = \int \alpha'(x, k_x, z, \omega) \left[F_x^{-1} (\varphi(k_x, 0, \omega)) \exp \left(\frac{i\omega \Delta z}{v_{ave}} \right) \right] \exp(ik_x x) dx, \quad (17)$$

where $\alpha'(x, k_x, z, \omega)$ is the nonstationary focusing phase-shift term

$$\alpha'(x, k_x, z, \omega) = e^{i \frac{\omega z}{v_{mean}} \left(\sqrt{1 - \frac{k_x^2 v_{mean}^2(x, z)}{\omega^2}} - 1 \right)}, \quad (18)$$

and F_x and F_x^{-1} denote forward and inverse Fourier transforms.

Similar to the natural combination of the recursive NSPS and PSPI (Ferguson and Margrave, 1999b), (16) and (17) can also be combined naturally to form a one-step symmetric wavefield extrapolator.

Equations (16) and (17) require the same computation efforts as the recursive PSPI and NSPS integral as described before. However they allow a much larger depth step. There is a trade-off between the vertical extrapolation step size and the horizontal velocity variation. If the velocity variation is strong, the extrapolation step has to be reduced to keep the higher order error term ignorable. We shall demonstrate the accuracy of one-step PSPI integral and this trade-off relationship with two zero-offset extrapolation examples using velocities from depths 0-200 m and 1200-1400 m of the Marmousi model. Figure 1 highlights the two velocity zones. Forward modelling is done by recursive application of the PSPI integral (equation (1)), which should produce very accurate synthetic data. For each velocity field, 11 impulses were forward extrapolated recursively through 20, 30 and 40 layers at a 4m step size. The quality of the inverse extrapolation with the one-step PSPI integral of the three synthetic datasets indicates how robust the algorithm is when different horizontal velocity variations are present.

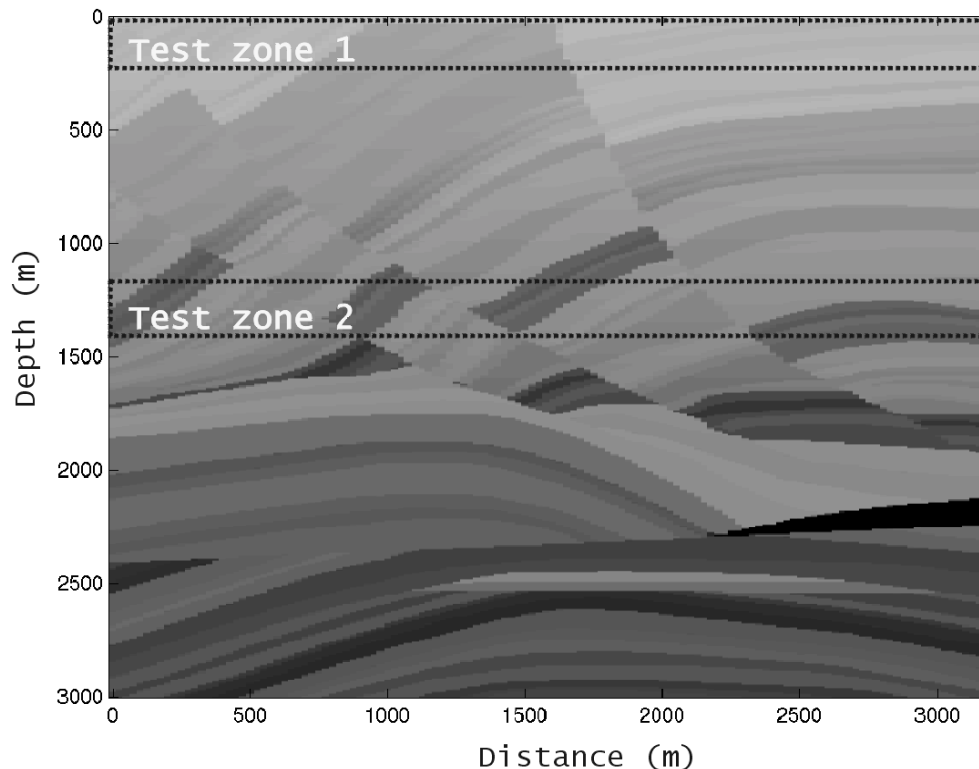


Figure 1. Two velocity zones with different horizontal velocity variation are chosen to test the one-step PSPI integral wavefield extrapolator.

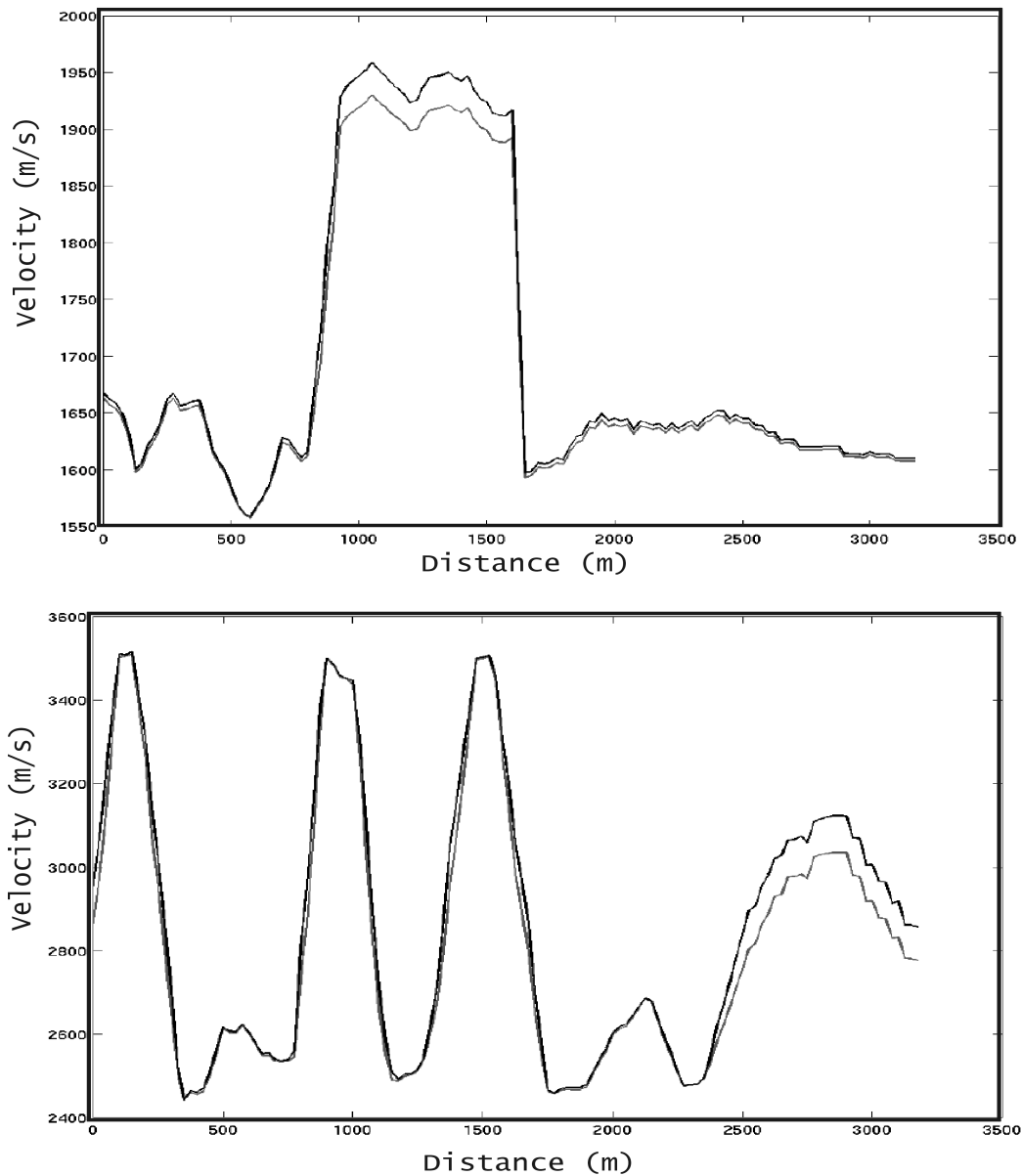


Figure 2. 40-step mean velocity and average velocity variation for velocity zone 1 (top) and 2 (top). The solid black line is average velocity and the gray line is the mean velocity.

Figure 2 shows the 40-step average and mean velocities for the two velocity zones. Note the maximum percentage average velocity variation is about 20% for velocity zone 1 while about 50% for velocity zone 2. We should expect the maximum allowable depth step in velocity zone 2 to be less than that in velocity zone 1.

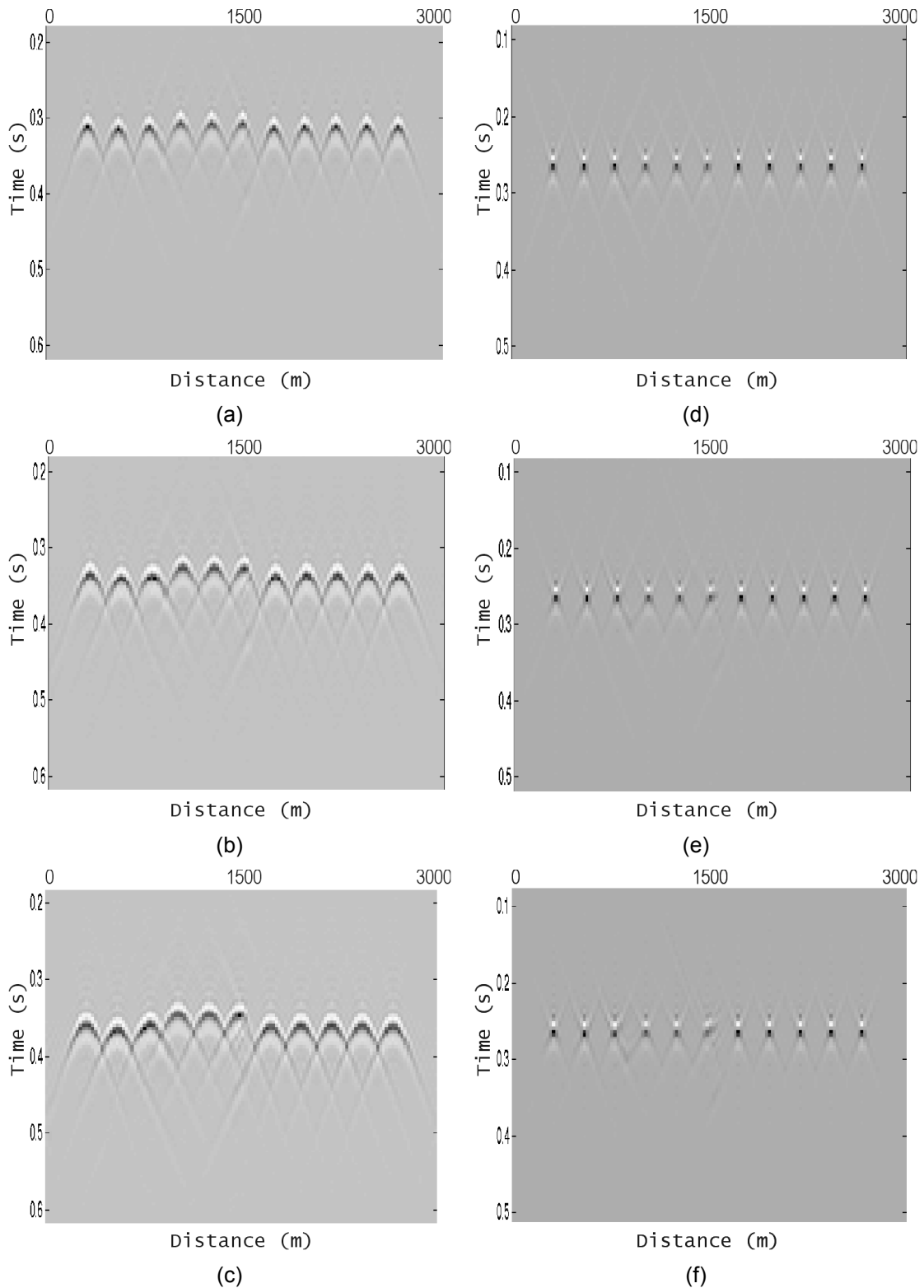


Figure 3. Extrapolation test in velocity zone 1. (a) (b) and (c) are forward modeling by recursive PSPI integral through 20, 30 and 40 layers. The step size is 4 m. (d), (e) and (f) are inverse extrapolation of (a), (b) and (c) using the one-step algorithm.

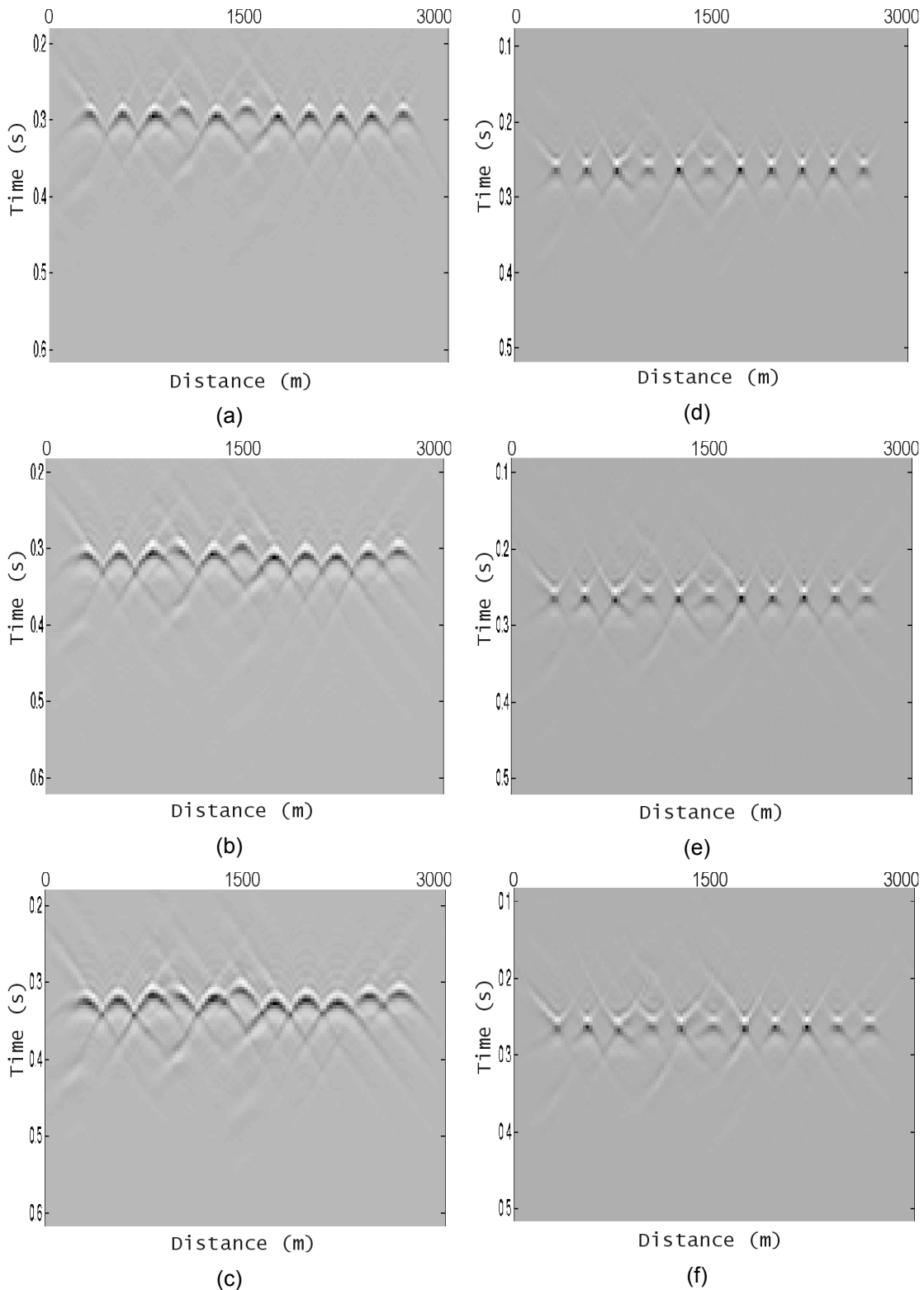


Figure 4. Extrapolation test in velocity zone 2. (a) (b) and (c) are forward modelling by recursive PSPI integral through 20, 30 and 40 layers. The step size is 4 m. (d), (e) and (f) are inverse extrapolations of (a), (b) and (c) using the one-step algorithm with a step size of 40 m.

Figure 3 shows the extrapolation test in the velocity zone 1. In all three cases, the impulses are very well recovered and doubling the step size of inverse extrapolation from 80 m to 160 m does not have much negative impact on the image quality even at sharp velocity boundaries. The quality of the image of the 40-step case suggests that even larger extrapolation step can be used without losing accuracy when the horizontal velocity variation is not quite severe.

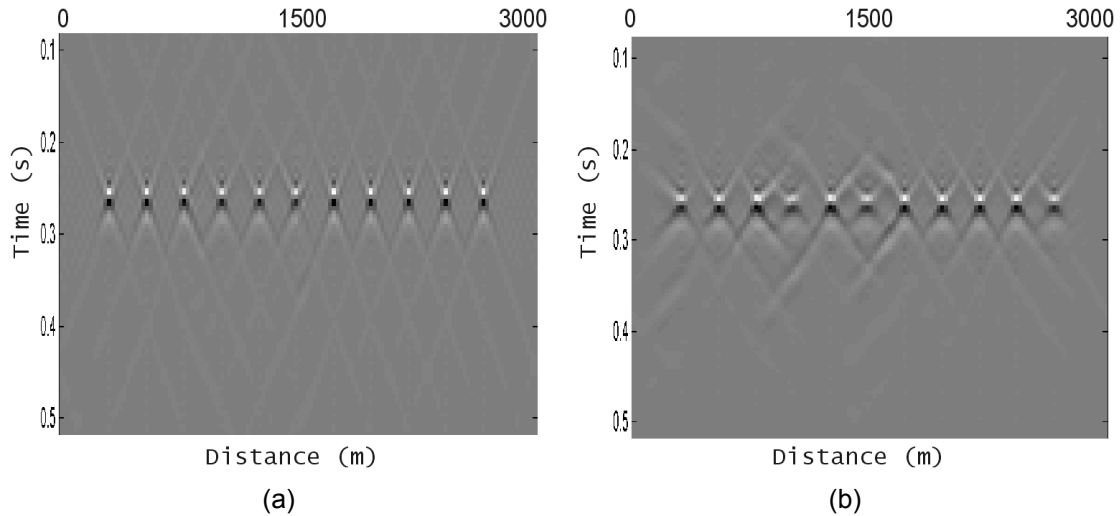


Figure 5. Inverse extrapolation of Figure 3d and Figure 4d with recursive PSPI integral. The step size is 4 m.

Figure 4 shows the extrapolation test in the velocity zone 2. Figure 5 shows the results of inverse extrapolation of Figure 3d and Figure 4d with recursive PSPI integral extrapolator. Note the inverse extrapolation with large-step PSPI integral extrapolator in both the shallow and deep part of the model produce very similar results to the 4-m PSPI integral extrapolator.

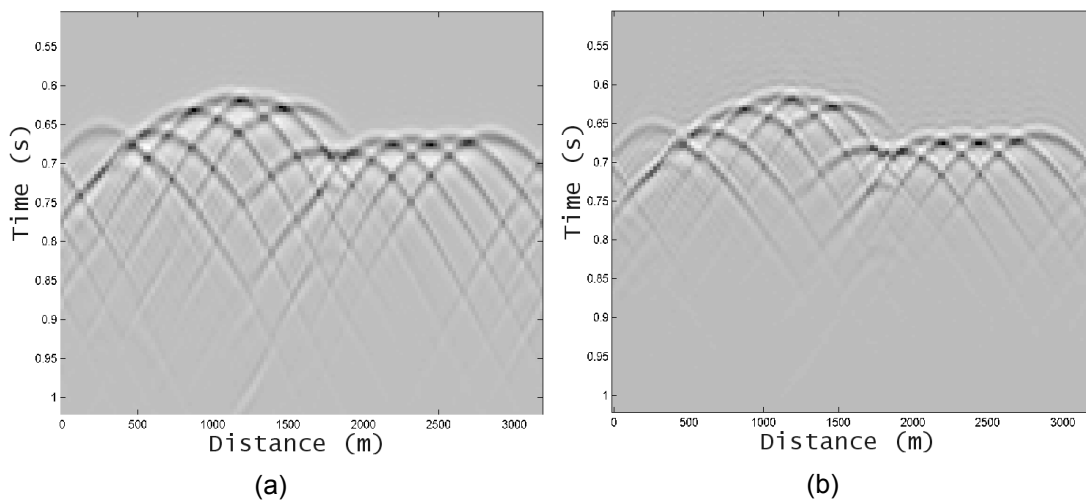


Figure 6. Zero-offset wavefields at 760 m of Marmousi model. (a) was extrapolated by large-step PSPI integral extrapolator of 40 m step size and (b) was extrapolated by recursive PSPI integral of 4 m step size.

Figure 6 shows a comparison of 11 impulse responses after extrapolation through the upper 760 m of the Marmousi model. The computation of Figure 6a is ten times faster than that of Figure 6b.

DUAL EXTRAPOLATION ALGORITHM

For traditional, recursive, Fourier-domain imaging algorithms, the error caused by approximation of the extrapolator accumulates in every imaging step. The idea of the dual algorithm is to provide relatively accurate reference wavefields at a coarser depth interval and compute the intermediate images from these reference wavefields with a faster algorithm such as Gazdag's PSPI (Gazdag and Sguazzero, 1984) or the split-step Fourier algorithm (Stoffa et. al., 1990). When the distance between reference wavefields is not large, the intermediate image can be computed by interpolation of the reference wavefields after a static phase-shift. There are many interpolation possibilities. The example shown in this paper is the simplest combination of one-step PSPI integral extrapolator for computing reference wavefields and linear interpolation of the upper and lower reference wavefields after static phase-shift.

Consider two monofrequency reference wavefields at z_1 and z_2 , $\psi(x, z_1, \omega)$, $\psi(x, z_2, \omega)$, which are computed by the large-step PSPI integral extrapolator described above, an intermediate wavefield between z_1 and z_2 , $\psi(x, z, \omega)$, can be computed by either forward extrapolation from the wavefield at z_1 or by inverse extrapolation from the wavefield at z_2 . However, computation of the focusing term is time consuming. If only the static phase-shift is applied instead of full extrapolation, the two resulting wavefields

$$\psi'_1(x, z, \omega) = \psi(x, z_1, \omega) e^{\frac{i(z-z_1)\omega}{v_{ave1}(x)}} \quad (19)$$

and

$$\psi'_2(x, z, \omega) = \psi(x, z_2, \omega) e^{\frac{i(z-z_2)\omega}{v_{ave2}(x)}} \quad (20)$$

are under- and over-focused approximations to the correct wavefield $\psi(x, z, \omega)$, where v_{ave1} and v_{ave2} are average velocities from z_1 to z and z to z_2 . A linear interpolation between $\psi'_1(x, z, \omega)$ and $\psi'_2(x, z, \omega)$ roughly cancel the focusing error if v_{ave1} and v_{ave2} are close.

$$\psi(x, z, \omega) \approx \frac{z - z_2}{z_2 - z_1} \psi_1'(x, z, \omega) + \frac{z - z_1}{z_2 - z_1} \psi_2'(x, z, \omega) \quad (21)$$

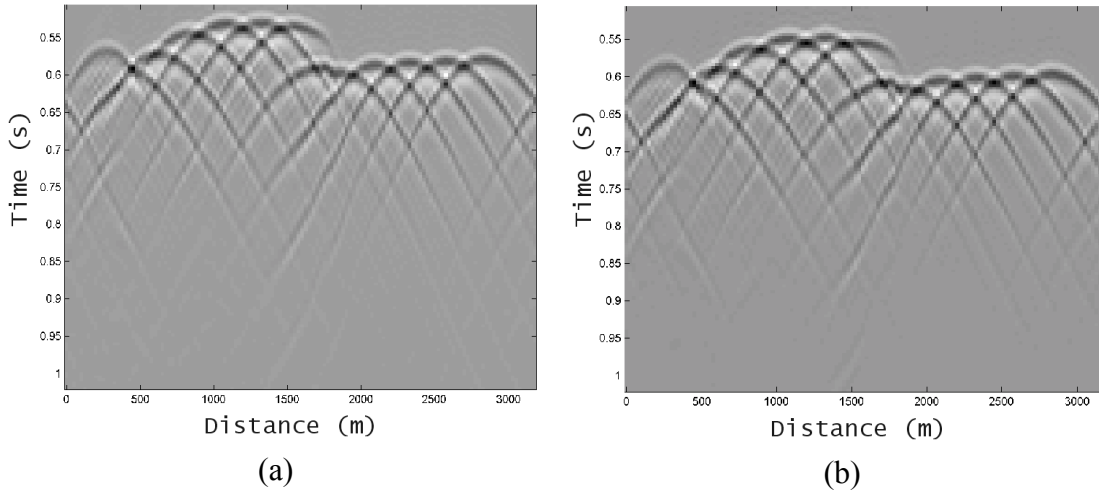


Figure. 7 Zero-offset wavefield extrapolation of 11 impulses to (a) 560 m and (b) 600 m of the Marmousi model.

Figure 7 and Figure 8 show the zero-offset forward extrapolation of 11 impulses starting at 0.25s through the shallow part of the Marmousi model with the dual algorithm. Two reference wavefields at 560 m and 600 m were computed with the large-step PSPI integral extrapolator with 40-m step size (Figure 7). The intermediate wavefields at 564 m, 572 m, 580 m, 588 m and 596 m were computed with equation (21).

The quality of intermediate image degrades slightly as the wavefield approaches the middle of the reference wavefields. Inverse extrapolation of Figure 8a-e with 4-m recursive PSPI integral extrapolator are shown in Figure 9. There is little difference between them, which indicates that a larger depth interval between the reference wavefields could be used without significant loss of accuracy.

FUTURE WORK

By using the average velocity for the static phase-shift and the mean velocity for the focusing phase-shift, a large depth step can be used to produce images with quality comparable to the recursive PSPI integral with small extrapolation steps, while computing speed is tremendously improved. Instead of PSPI integral, NSPS integral will be tested in the same way and it should produce similar results. From previous research on symmetric nonstationary wavefield extrapolation (Margrave and Ferguson, 1999), an even larger extrapolation step can be also used without significant loss of accuracy.

There is a relationship between the allowable focusing phase error and the largest step size that can be used in computing the reference wavefields. This question needs to be addressed in future research work.

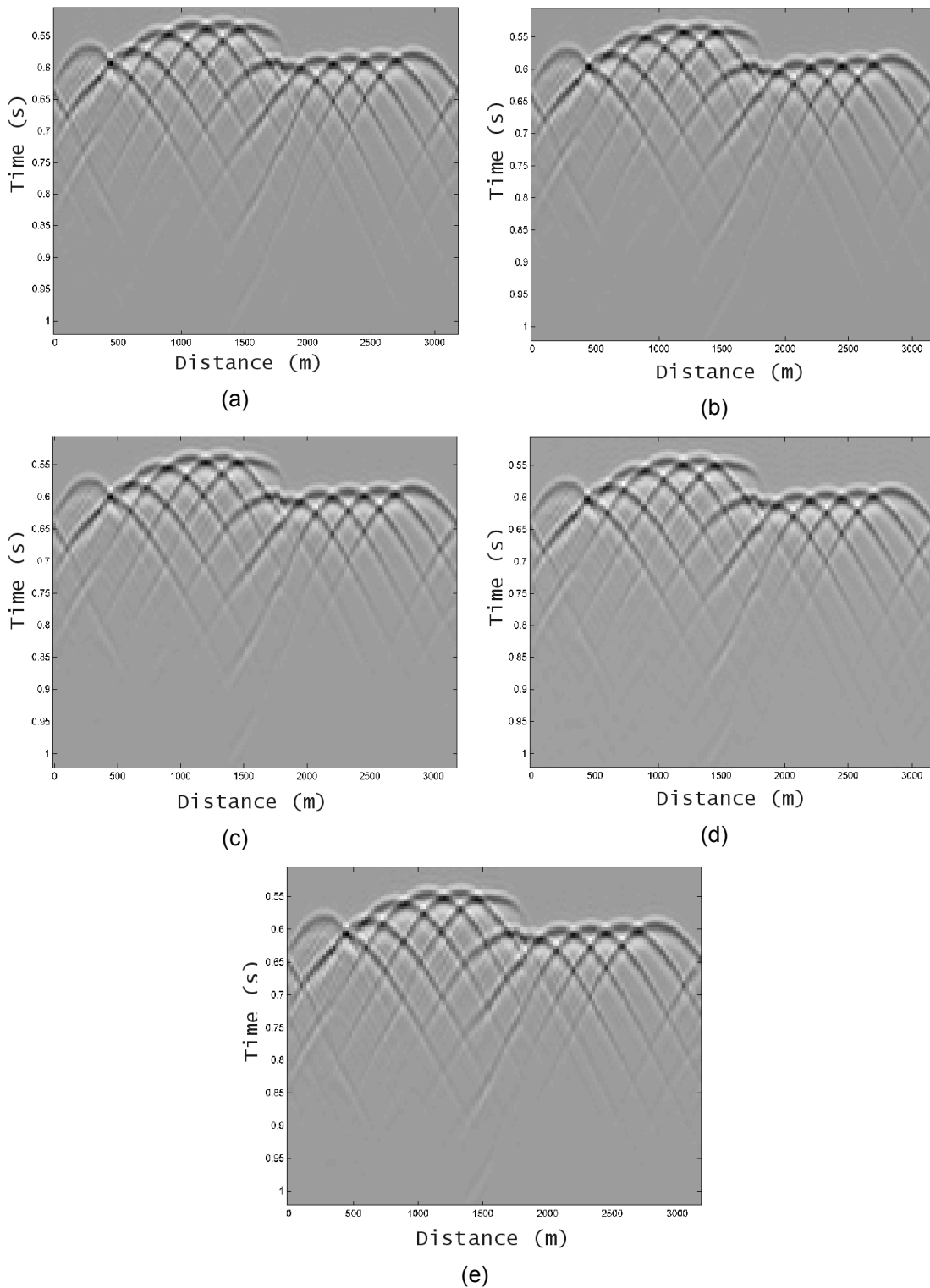


Figure 8. Intermediate wavefields at (a) 564 m, (b) 572 m, (c) 580 m, (d) 588 m and (e) 596 m were computed by linear interpolation between the reference wavefields at 560 m and 600 m.

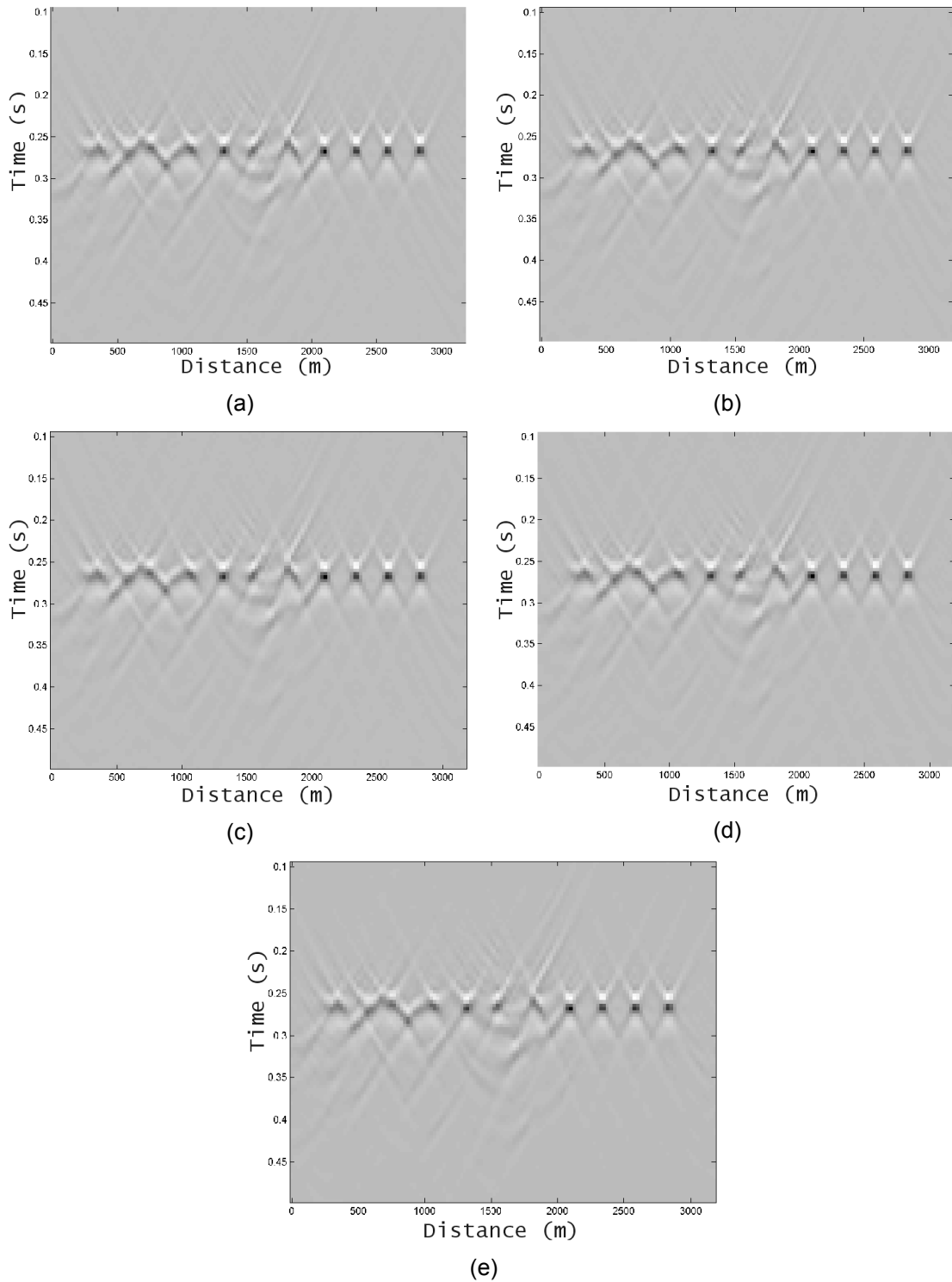


Figure 9. Inverse extrapolation of the intermediate wavefields at (a) 564 m, (b) 572 m, (c) 580 m, (d) 588 m and (e) 596 m with the 4-m step recursive PSPI integral operator.

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