Extending AVO inversion techniques

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ABSTRACT

A software tool is introduced for assessment of AVO inversion techniques. It employs synthetic offset gathers with spike wavelets and models a wide distribution of earth parameters for each lithology. It is used to demonstrate that a wide range of values that can be obtained for a given lithology, and to obtain the limits of accuracy available for two-parameter inversions based on the Aki-Richards approximation. The method of Fatti et al. (1994) is shown to be an improvement on that of Smith and Gidlow (1987). A Full Offset method is introduced and shown to be superior to that of Fatti et al. A generalized Gardner relation is also introduced and applied. It does not yield improved inversion results for P- and S-wave changes, but can be employed to develop new density inversion techniques. The results show promise for the development of a linear density inversion method.

INTRODUCTION

In principle, linear AVO inversion can yield three parameters, $\Delta\alpha/\alpha$, $\Delta\beta/\beta$, and $\Delta\rho/\rho$, where α is the P-wave velocity, β is the S-wave velocity, and ρ is the density. These parameters represent the difference in properties between adjoining earth layers, divided by the average across the two layers. The linear inversion is based on the Aki-Richards approximation (Aki and Richards, 1980) to the P-P reflection coefficient (R_{PP}), which can be written as

$$R_{PP} = A\frac{\Delta\alpha}{\alpha} + B\frac{\Delta\beta}{\beta} + C\frac{\Delta\rho}{\rho} \tag{1}$$

where $A = ½\ (\ 1 - 4\ (\beta/\alpha)^2\sin^2\theta\ )$, $B = ½\ (\ 1 + \tan^2\theta\ )$, $C = -\ 4\ (\beta/\alpha)^2\sin^2\theta$, and θ is the average of the angle of incidence and the angle of P-P transmission. In the case of joint inversion (Stewart, 1990) one also employs the converted wave reflection coefficient (R_{PS}), which in the Aki-Richards approximation is given as

$$R_{PS} = D\frac{\Delta\beta}{\beta} + E\frac{\Delta\rho}{\rho} \tag{2}$$

where

$$D = \frac{\alpha\tan\varphi}{2\beta}\left(\frac{4\beta^2}{\alpha^2}\sin^2\theta - \frac{4\beta}{\alpha}\cos\theta\cos\varphi\right) \tag{3}$$

$$E = -\frac{\alpha\tan\varphi}{8\beta}\left(1 - \frac{2\beta^2}{\alpha^2}\sin^2\theta + \frac{2\beta}{\alpha}\cos\theta\cos\varphi\right) \tag{4}$$

and φ is the average of P-S reflection and transmission angles. Given experimentally measured values of R_{PP} (and R_{PS} for joint inversion) at several offsets (or values of θ), one can use a least-squares approach to optimize values of $\Delta\alpha/\alpha$, $\Delta\beta/\beta$, and $\Delta\rho/\rho$, which may be called the α or P-wave contrast, the β or S-wave contrast, and the ρ or density contrast, respectively. In practice it is difficult to obtain accurate estimates of all three values, and a more realistic objective is to manipulate the equations so that one only solves for two parameters. One well-known approach is that of Smith and Gidlow (1987) who invoke Gardner's relation (Gardner et al., 1974) in differential form to replace $\Delta\rho/\rho$ with $\Delta\alpha/\alpha$:

$$\rho = F\alpha^{1/4} \quad \text{(Gardner's Relation)} \tag{5}$$

$$d\rho = \frac{1}{4}F\alpha^{-3/4}d\alpha \quad \text{(differential form)} \tag{6}$$

$$\frac{d\rho}{\rho} = \frac{1}{4}\frac{d\alpha}{\alpha} \quad \text{(ratio of Equations (6) and (5))} \tag{7}$$

Thus $\Delta\rho/\rho$ can be replaced by ¼ $\Delta\alpha/\alpha$, and one solves for just $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$.

$$R_{PP} = (A + C/4)\frac{\Delta\alpha}{\alpha} + B\frac{\Delta\beta}{\beta} \tag{8}$$

There are of course other density-velocity relations, such as those by Birch (1961) and Lindseth (1979), but they do not lead to a simple substitution such as Equation (7).

Another approach is that of Fatti et al. (1994), who group $\Delta\rho/\rho$ terms together with $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$ terms to generate an expression of the form

$$R_{PP} = AI_P + BI_S + (C - A - B)\frac{\Delta\rho}{\rho} \tag{9}$$

where $I_P = \Delta\rho/\rho + \Delta\alpha/\alpha$ and $I_S = \Delta\rho/\rho + \Delta\beta/\beta$, the zero-offset impedances for P- and S-waves. The coefficient of the remaining $\Delta\rho/\rho$ term is of second order in $\sin(\theta)$ (or of fourth order if one assumes $V_P/V_S = 2$) and is discarded. This leaves two variables for which to solve.

Smith-Gidlow and Fatti et al. are both approximations to the Aki-Richards approximation, and comparing them to the Aki-Richards R_{PP} shows them to be complementary in behaviour, as illustrated in Figure 1 below. Fatti et al. give an approximation that is exact at zero-offset, but poor at large angles. Smith-Gidlow on the other hand is not exact even at zero-offset, but can be of reasonable accuracy at large angles.

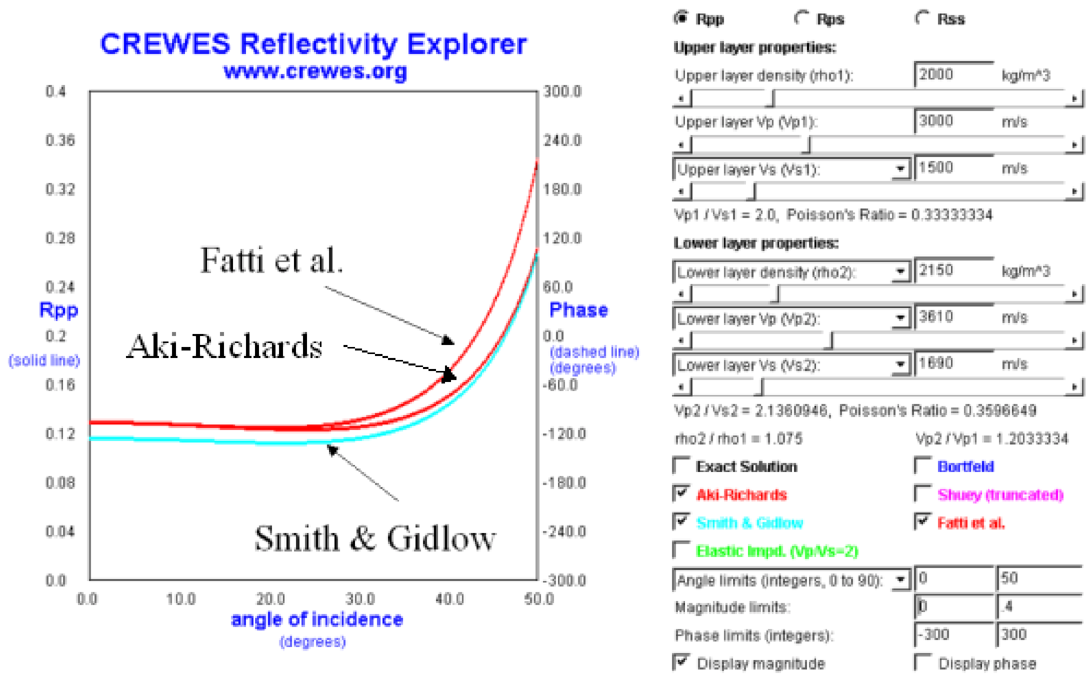

FIG. 1. A screen capture from the CREWES Reflectivity Explorer illustrating the contrasting behavior of the Smith-Gidlow and Fatti et al. approximations to the Aki-Richards theory for small and large angles of incidence.

For this report we have developed a simple tool for testing the viability of various approximations in the context of AVO inversion at reservoir tops. This has been implemented in an interactive Java applet. It will be used to explore several AVO inversion methods. To begin we will consider the fundamental Aki-Richards approximation, and then the Smith-Gidlow and Fatti et al. approaches. Then we will consider some new ideas. First we will consider a method to combine the strengths of both the Smith-Gidlow and Fatti et al. approaches, which we will refer to as the Full Offset method. Second we will consider a more general Gardner's relation involving V_S, which may be employed in the Smith-Gidlow procedure in place of the standard Gardner's relation to yield an approximation we will refer to as Smith-Gidlow(V_S). The Full Offset method also employs Gardner's relation, so we can also consider a Full Offset (V_S) approximation. In addition, a Gardner's relation involving V_S will allow us to replace $\Delta\beta/\beta$ instead of $\Delta\rho/\rho$ in Equation (1), and to invert for $\Delta\alpha/\alpha$ and $\Delta\rho/\rho$ instead of for $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$. We will refer to this as the α-ρ approximation. We also consider an analogous β-ρ approximation. Together these studies will permit us to present a number of conclusions regarding AVO inversion methods.

THEORY

In this section we outline the new approximations we wish to test.

The Full Offset Approximation

The first step in this method is to carry out the Fatti et al. approximation of Equation (9), but without discarding the $\Delta\rho/\rho$ term. Instead we write

$$\frac{\Delta\rho}{\rho}=\frac{1}{5}\left(4\frac{\Delta\rho}{\rho}+\frac{\Delta\rho}{\rho}\right)\approx\frac{1}{5}\left(\frac{\Delta\alpha}{\alpha}+\frac{\Delta\rho}{\rho}\right)=\frac{1}{5}I_P \tag{10}$$

where we have invoked Equation (7) (Gardner's relation) in the second step. This allows us to replace $\Delta\rho/\rho$ in Equation (9) with I_P. We refer to the resulting approximation as Full Offset.

This method is exact at the zero-offset (as is Fatti et al.), but is expected to be reasonably accurate at large offsets (as is Smith-Gidlow) since no terms have been discarded.

Generalized Gardner's Relation

Previous workers have demonstrated the feasibility of a Gardner-like approximation involving β instead of α (Dey and Stewart, 1997; Potter and Stewart, 1998; Potter, 1999). Ursenbach (2001) develops a generalized version of Gardner's relation of the form

$$\rho=G\alpha^{H}\beta^{J} \tag{11}$$

In differential form this is

$$d\rho=G(H\alpha^{H-1}\beta^{J}d\alpha+J\alpha^{H}\beta^{J-1}d\beta) \tag{12}$$

which, combined with Equation (11), yields the relation

$$\frac{d\rho}{\rho}=H\frac{d\alpha}{\alpha}+J\frac{d\beta}{\beta} \tag{13}$$

This can be used in the Smith-Gidlow approximation instead of the regular Gardner's relation to replace $\Delta\rho/\rho$ with $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$:

$$R_{PP}=(A+HC)\frac{\Delta\alpha}{\alpha}+(B+JC)\frac{\Delta\beta}{\beta}. \tag{14}$$

We refer to this approximation as Smith-Gidlow(V_S).

We could similarly use the generalized Gardner relation to obtain

$$\frac{\Delta\rho}{\rho}\approx\frac{H}{1+H+J}I_P+\frac{J}{1+H+J}I_S \tag{15}$$

This yields the approximation

$$R_{PP}=\left(A+\frac{H(C-A-B)}{1+H+J}\right)I_P+\left(B+\frac{J(C-A-B)}{1+H+J}\right)I_S \tag{16}$$

Inverting for $\Delta\rho/\rho$

From Equation (1) and Equation (13) we obtain the expression

$$R_{PP} = \left(A - \frac{H}{J} B \right) \frac{\Delta \alpha}{\alpha} + \left(C + \frac{1}{J} B \right) \frac{\Delta \rho}{\rho}. \tag{17}$$

This relation cannot be obtained using Gardner's relation alone, as it does not have any β dependence. Similarly, from Equations (1), (13), and (15) we can obtain

$$R_{PP} = \left(B - \frac{J}{H} A \right) \frac{\Delta \beta}{\beta} + \left(C + \frac{1}{H} A \right) \frac{\Delta \rho}{\rho}, \tag{18}$$

$$R_{PP} = \left(A - \frac{H}{J} B \right) I_P + \left(C - A + \frac{1+J}{H} B \right) \frac{\Delta \rho}{\rho}, \tag{19}$$

$$R_{PP} = \left(B - \frac{J}{H} A \right) I_S + \left(C - B + \frac{1+H}{J} A \right) \frac{\Delta \rho}{\rho}. \tag{20}$$

It is not expected that these will all lead to practical inversion methods, but their analysis can provide useful information if one possesses a convenient evaluation technique. This is the subject of the following section.

EVALUATION METHOD

One of the objectives of this project was to develop a software tool that would allow a convenient and rapid assessment of AVO inversion methods, particularly in the context of reservoir tops. A Java applet was created based on the following procedure:

1) Choose a cap rock and a reservoir rock. A set of checkboxes is available for this.

2) For each chosen lithology, create a statistical sampling of typical earth parameters (ρ, α, β). The control panel permits the user to specify the number of samples desired. Within the program each lithology is assigned a typical density range. These are given in Table I below. A set of densities is randomly generated from the density range. The compressional velocity is then selected using Equation (6) from Wang (2000), which is a lithology-specific equation of the form $V = a\,\rho^b$, a rearrangement of the Gardner form. A random component of ± 500 m/s is then added in order to mimic the actual scatter of experimental values (see Wang, 2000, Figure 3). The shear velocity is then generated using V_S / V_P relations (Mavko, 1998, Section 7.8). This time a random component of ± 100 m/s is added to mimic the scatter (see Mavko, 1998, Figures 7.8.1-8). The one exception to this procedure is for anhydrite, for which the pure mineral value is used for velocities. This completes the selection of the earth parameter values.

3) For each earth parameter sample, evaluate the exact Zoeppritz R_{PP} coefficient for a range of incident angles (0 to 30 degrees). A very fine grid is employed (360 points). This constitutes a synthetic offset gather with a spike wavelet. Fit the offset data to each approximation to be tested. For each approximation this yields two of the values $\Delta\alpha/\alpha$, $\Delta\beta/\beta$, $\Delta\rho/\rho$, I_P, and I_S. Compare these with the exact values and calculate percent errors.

4) After performing step 3) for each earth parameter sample, average the percent errors over the ensemble to obtain a mean result for the given lithology.

5) Display all results for the given lithology. The results are displayed in two ways. First they are displayed as simple averages, one for the percent error of each quantity obtained through inversion. Second, they are displayed as a scatter plot of the percent error of the individual earth parameter results.

It is convenient for the program to display individual %-errors logarithmically. For instance, the average results for the Smith-Gidlow approximation when the cap rock is shale and the reservoir is sandstone are %-error ($\Delta\alpha/\alpha$) = 39% and %-error ($\Delta\beta/\beta$) = 35%. These appear to be rather large errors, but alone they do not tell the whole story, as is evident from the scatter plot:

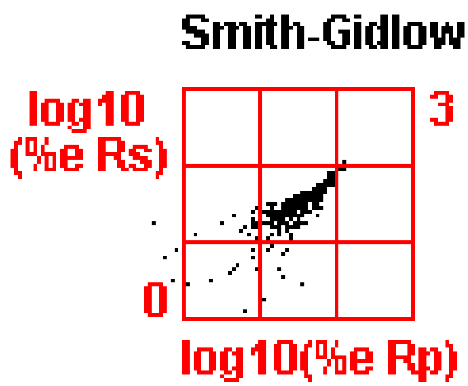

FIG. 2. A logarithmic scatter plot of %-errors for $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$ for a distribution of earth parameters typical of shale over sandstone. Inversion is by the Smith-Gidlow method using Castagna's mudrock relation for V_P/V_S. The grid outlines a scale extending from 1% error to 1000% error in each dimension.

From this we see that both $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$ errors vary over a few orders of magnitude, from ~0.3% to ~100%. This demonstrates both the necessity of using log plots, and more significantly the danger in trying to validate a method by reference to only one or a few case studies. Another advantage of using scatter plots is that, for such a wide distribution, the average itself may vary somewhat from run to run, even for very large samples, because of the behaviour of a few very large numbers in the average. The general shape of the scatter plot however will always be consistent, so that it can give, not only a more detailed, but also a more dependable view of a method's inversion behaviour. (In our results below we will normally report only typical averages, but scatter plots may easily be reproduced with the Java utility, and one example of scatter plots is also given later in Figure 3.)

Table 1. Ranges of densities values employed for generating input data for each lithology.

Lithology	Sandstone	Anhydrite	Shale	Limestone	Dolomite
Density Range (g/cm^3)	2.10-2.60	2.96	2.30-2.70	1.85-2.75	2.45-2.85

RESULTS

We first consider the fundamental Aki-Richards approximation, then two established methods, Smith-Gidlow and Fatti et al., and then the new approximations introduced above.

The Aki-Richards approximation

The Aki-Richards expression, Equation (1), may be used as the basis for a three-parameter inversion, but must be approximated further for a two-parameter inversion. However, in order to see the maximum amount of accuracy that can be obtained from a two-parameter inversion, we simply insert the exact value for one of the parameters and solve for the other two, e.g.,

$$R_{PP} - C\left(\frac{\Delta\rho}{\rho}\right)_{exact} = A\frac{\Delta\alpha}{\alpha} + B\frac{\Delta\beta}{\beta} \tag{21}$$

Table 2. Typical average %-errors of parameters predicted by the AVO Inversion Explorer using Equation (21) and its two analogues at five different reservoir top lithologies.

Quantity (method)	$\Delta\alpha/\alpha$ (α-β, α-ρ)	I_P (α-ρ)	$\Delta\beta/\beta$ (α-β, β-ρ)	I_S (β-ρ)	$\Delta\rho/\rho$ (α-ρ, β-ρ)
shale/sandstone	1.18, 8.83	2.28	6.24, 6.87	10.25	155, 18.4
shale/limestone	0.95, 14.4	2.33	22.9, 23.8	41.1	258, 11.2
shale/dolostone	0.41, 22.1	0.41	14.7, 14.7	11.4	376, 13.0
anhydrite/limestone	0.81, 16.0	3.02	20.1, 21.1	141	14.6, 0.51
anhydrite/dolostone	1.86, 58.1	4.99	40.1, 41.4	49.8	272, 8.64

This of course is not intended as a practical method, as $(\Delta\rho/\rho)_{exact}$ is only available for synthetic data, but is designed only to assess the value of the Aki-Richards approximation as a starting point for other AVO inversion methods. Equation (21) we refer to as the α-β method, and analogous expressions are obtained using $(\Delta\alpha/\alpha)_{exact}$ and $(\Delta\beta/\beta)_{exact}$ instead, which we call the β-ρ and α-ρ methods. Average %-errors are contained in Table II below:

We see that the Aki-Richards approximation provides an excellent basis for the prediction of $\Delta\alpha/\alpha$ from the α-β method. It is predicted more poorly by the α-ρ method, but when combined with $\Delta\rho/\rho$, it yields a satisfactory IP. $\Delta\beta/\beta$ is not predicted as well, but the results are very consistent between α-β and β-ρ. The prediction of IS from β-ρ is generally worse. The prediction of $\Delta\rho/\rho$ by α-ρ is extremely poor, but is reasonable (probably usable) by the β-ρ method. For the anhydrite over dolomite it is excellent.

These results indicate the maximum quality that we can expect from approximations to Equation (1).

Smith-Gidlow and Fatti et al.

The methods of Smith and Gidlow (1987) and Fatti et al. (1994) are important contributions that have been influential in developing the AVO method. Their success however is based upon the results of a small number of case studies. The advantage of the present study using simplified synthetics is that we may readily consider hundreds of cases in order to get a clearer idea of a method's robustness. In Table 3 below we present the average %-errors for these two theories, and compare them to the relevant Aki-Richards limits taken from Table 2.

Table 3. Typical average %-errors of parameters predicted using the Smith-Gidlow and Fatti et al. methods, compared to Aki-Richards limits.

Quantity (method)	$\Delta\alpha/\alpha$ (α-β, Smith-Gidlow)	I_P (α-ρ, Fatti et al.)	$\Delta\beta/\beta$ (α-β, Smith-Gidlow)	I_S (β-ρ, Fatti et al.)
shale/sandstone	1.18, 38.8	2.28, 3.07	6.24, 35.1	10.25, 9.99
shale/limestone	0.95, 39.2	2.33, 2.46	22.9, 113	41.1, 29.8
shale/dolostone	0.41, 133	0.41, 0.31	14.7, 249	11.4, 14.5
anhydrite/limestone	0.81, 130	3.02, 10.5	20.1, 168	141, 88.7
anhydrite/limestone (0 to 50 degrees)	0.80, 137	14.5, 82.5	20.4, 239	53.6, 143
anhydrite/dolostone	1.86, 39.4	4.99, 1.60	40.1, 69.4	49.8, 48.5

Clearly, the Fatti et al. approach is much more successful than Smith-Gidlow in approaching the Aki-Richards limit. In some cases it even surpasses it, but remains of the same order of magnitude, probably indicative of a fortuitous cancellation of errors. The success of Fatti et al. should not be surprising as it is most accurate at small angles, and our chosen range is 0 to 30 degrees. For comparison we have included additional data in Table 3 for a 0 to 50 degree inversion of anhydrite/limestone. This lithology was chosen because it's first critical angles are all greater than 50 degrees, while the first critical angles for the other four reservoir tops all start between 30 and 40 degrees. In any case, we can see that Fatti et al. suffers substantially from the increased angle range, while Smith-Gidlow is only slightly affected. Thus the results of this paper cannot be applied incautiously to very large offset AVO. For typical pre-critical ranges though, the method of Fatti et al. definitely appears superior.

It may appear surprising that Smith-Gidlow gives such large %-errors given its empirical success, but recall Figure 2 which displays the distribution of individual values on a logarithmic scale for the shale/sandstone lithology. There certainly are cases when excellent results can be obtained by this method, but there is as yet no key to discerning when those will occur.

The Full Offset Approximation

The purpose of the Full Offset approximation was to combine the zero-offset accuracy of Fatti et al. with the large angle accuracy of Smith-Gidlow. We present results for this method below, based on the combination of Equations (9) and (10) and compare them with Fatti et al. and with the Aki-Richards limit.

Table 4. Typical average %-errors of parameters predicted using the Full Offset method, compared to the Aki-Richards limit and Fatti et al.

Quantity (method)	I_P (α-ρ, Fatti et al., Full Offset)	I_S (β-ρ, Fatti et al., Full Offset)
shale/sandstone	2.28, 3.07, 2.74	10.25, 9.99, 11.1
shale/limestone	2.33, 2.46, 2.30	41.1, 29.8, 32.8
shale/dolostone	0.41, 0.31, 0.23	11.4, 14.5, 15.3
anhydrite/limestone	3.02, 10.5, 10.43	141, 88.7, 83.2
anhydrite/dolostone	4.99, 1.60, 1.04	49.8, 48.5, 47.8

There appears to be generally a small decrease in %-error of I_P relative to Fatti et al., although for dolostone the improvement is more substantial. Since the Full Offset is always better for I_P and is apparently neutral for I_S (sometimes better and sometimes worse, but always similar) there appears to be some advantage in using the Full Offset method, particularly as it would be extremely simple to implement in any program that already has already implemented Fatti et al.

The Generalized Gardner Relation

The purpose of introducing the Generalized Gardner Relation was to replace the original Gardner relation with an expression that could provide more lithology specific information, without requiring separate expressions for each lithology. It can be used as a replacement in the Smith-Gidlow and Full-Offset approximations which both employ the Gardner relation. Below we display a screen capture of the applet output for the shale over sandstone lithology:

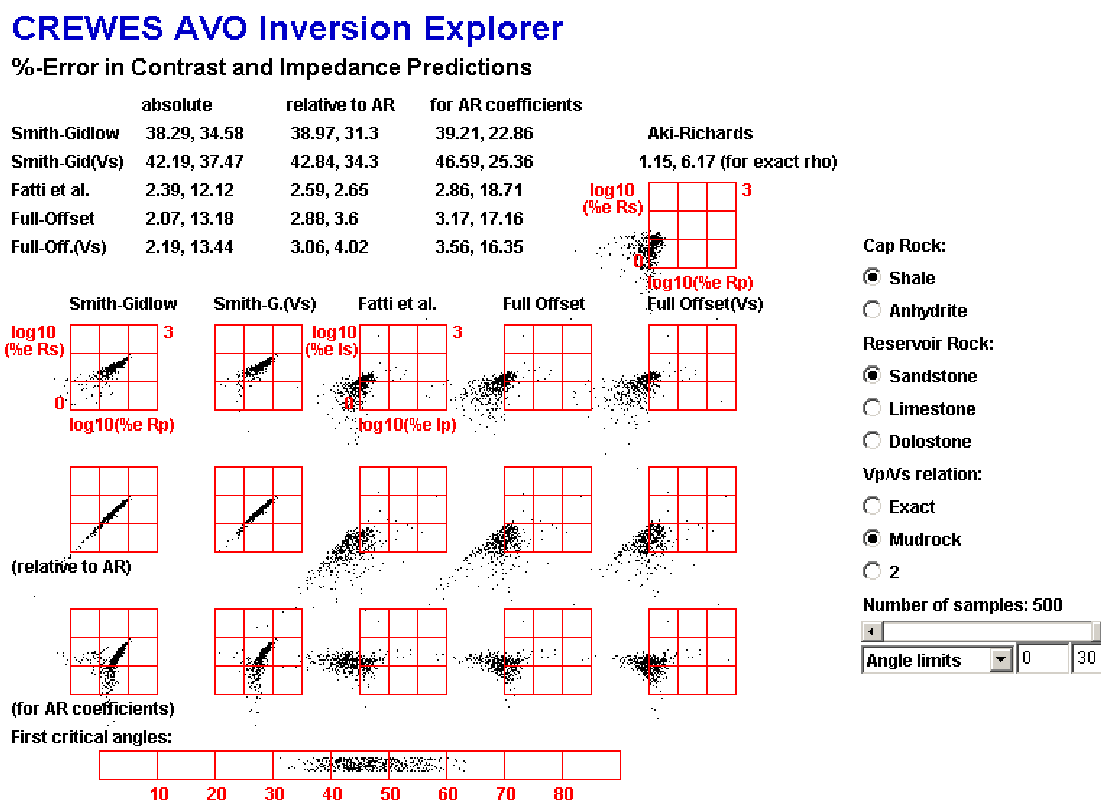

FIG. 3 An example of inversion results for the shale/sandstone lithology presented as a screen capture from the CREWES AVO Inversion Explorer. The inversion process employed Castagna's mudrock relation and an angle range of 0 to 30 degrees. 500 different sets of earth parameters typical of the lithology were averaged over, and also displayed individually in the scatter plots.

This is the type of output screen from which data in Tables 2-4 were collected. It features an interactive control panel on the right and a plot of first critical angles at the bottom. The %-errors of $\Delta\alpha/\alpha$, $\Delta\beta/\beta$ (or I_P, I_S) are presented as both averages and scatter plots. The label “absolute” refers to the %-errors recorded in Tables 2-4. The label “relative to AR” is the %-error relative to the Aki-Richards limit. The label “for A-R coefficient” is another way to compare to the Aki-Richards limit. In it we employ the reflection coefficients from the Aki-Richards approximation in the synthetic data, rather than the exact coefficients, but then to calculate the percent error deviation from the exact contrast or impedance. Comparison to exact values (“absolute”) indicates how well the linear inversion is expected to perform. Comparison to the Aki-Richards prediction, or to “exact” values obtained from the Aki-Richards coefficients, is expected to give some idea of how well analogous non-linear approximations would work.

This display provides a useful comparison of the various approximations. First it is of interest to see scatter plots for data in Tables 3 and 4. Turning to the Generalized Gardner Relation, we observe that the Smith-Gidlow and Full Offset methods are not significantly improved and perhaps slightly worsened by use of the new Gardner relation.

One result which is at variance with expectations is that the result relative to the A-R prediction and the result using A-R coefficients are not necessarily similar to each other, particularly for I_S. This calls into question which of these two A-R methods, if any, is likely to be indicative of results with full Zoeppritz inversion. In general the first method gives smaller %-errors than the second.

One notes that the degree of scatter is strongly dependent on lithology, and is generally greater for impedance methods. From observation of other lithologies (not shown) it also appears to be related to the scatter in critical angles.

Inversion for $\Delta\rho/\rho$

Although the Generalized Gardner Relation did not appear to be an improvement over the original Gardner Relation in the results of the previous subsection, one benefit of its form is that it allows a unified approach for generating inversion schemes for $\Delta\rho/\rho$, such as Equations (17)-(20). We now present results of inversion for density contrast, and compare with some results from Table 2.

Table 5. Typical average %-errors of $\Delta\rho/\rho$ predicted using Equations (17)-(20) and compared to the Aki-Richards limits.

%-error ($\Delta\rho/\rho$)	Eq. (17)	Eq. (18)	Eq. (19)	Eq. (20)	(α-ρ, β-ρ)
shale/sandstone	512	512	4160	233	155, 18.4
shale/limestone	181	181	1344	118	258, 11.2
shale/dolostone	126	126	1477	143	376, 13.0
anhydrite/limestone	95.7	95.7	129	104	14.6, 0.51
anhydrite/dolostone	162	162	335	81.1	272, 8.64

We note that the same value of $\Delta\rho/\rho$ is obtained from Equations (17) and (18), which are both contrast-based methods. The impedance-based methods [Equations (19) and (20)] differ in their predictions. The bottom line of course is that $\Delta\rho/\rho$ is not particularly well predicted anywhere. However, it does not even approach the Aki-Richards limit, so that there still is plenty of room for improvement within the context of linear inversion.

FUTURE DIRECTIONS

This study suggests several directions of interest for future work. It will certainly be valuable to extend the utility to treat cases of three-parameter inversion and joint inversion. New $\Delta\rho/\rho$ inversion approaches are clearly of interest in the linear inversion framework, but the inherent limitations of the Aki-Richards approximation also motivate extension to non-linear inversion.

Even given the utility in its current state, it would be useful to correlate subsets of the parameter distribution with different regions of the scatter plots, to help discern when approximations are most accurate. Other representations of the data may be helpful, such as values of the parameters in addition to %-errors. There are also intriguing questions regarding the origin of distinct patterns in the scatter plots, and perhaps understanding their origin would shed light on new directions for developing improved approximations.

CONCLUSIONS

This report presents results from a novel and useful tool, the CREWES AVO Inversion Explorer, which permits rapid assessment of AVO inversion approximations. As a web-based tool it is convenient to use in any computing environment. It presents graphically the wide range of values that can be obtained for any given lithological environment, and demonstrates the limits of accuracy available within the framework of linear inversion rooted in the Aki-Richards approximation.

Assessment of specific practical methods shows that the method of Fatti et al. is significantly more accurate than that of Smith and Gidlow, and that a new approach presented here, the Full Offset method, gains somewhat on that of Fatti et al. Introducing the Generalized Gardner Approximation does not seem to improve inversion results, but does open up a convenient route to density inversion methods. Comparison with the Aki-Richards limit suggests that considerable improvement should still be possible without resorting to non-linear inversion.

The assessment method in this report is based on simple synthetic offset gathers with spike wavelets. It thus provides a useful bound to the accuracy expected when one introduces the wavelet, noise, and larger offset binning of real data. Its greatest strength is in its ability to model a wide distribution of earth parameters for each lithology, rather than just a few test cases. Thus it is expected to become an important component of future AVO studies.

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