Modelling and simulation of seismic reflectivity

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ABSTRACT

We decompose the reflectivity series obtained from a seismic well log using statistical time series techniques. The resulting model, consisting of noise and systematic components, is used to simulate such reflectivity series for the development and testing of seismic imaging techniques.

INTRODUCTION

Seismic reflectivity series and similar "spiky" phenomenon have been aggressively modelled in recent years. Much work has also gone towards modelling time series of physical phenomenon as 1/f, self-affine signals (see, for example, Mandelbrot (1998); Riedi et al. (1999); Stefani and De (2001)). In the POTSI project, we routinely use simulated reflectivities as inputs to our seismic imaging algorithms, in order to quickly test ideas and develop new methods of imaging. We have become aware, however, that our standard methods of simulation could be deficient, and we are seeking better methods that more accurately simulate typical reflectivities one might encounter in a true well log.

The first author has previously considered statistical time-series techniques in order to model seismic reflectivity, however the nature of the noise in the signal has precluded such models from being successful before now. In this paper, we present a time-series model for the reflectivity series that is both algorithmically simple and mathematically understood.

The paper is organized as follows: first, we examine a typical reflectivity series taken from a well log and consider some of its statistical summaries; we then introduce the proposed model and examine the resulting fit between the model and the observed series; finally, we then use the model in order to statistically simulate reflectivity. We conclude with discussion and possible extensions.

A TYPICAL REFLECTIVITY SERIES

A plot of a typical reflectivity series is given in Figure 1. Although the well-log data are not physically measured at equal time intervals, the data have been transformed to reflect measurements every millisecond. We will call this original time series "realcoeff" and will refer to the successive values in the time domain as $\{ X(t) \}$, $t = 1,2,3,...$

A plot of the autocorrelation function (ACF) for the series, which is a measure of the linear relationship between $X(t)$ and $X(t+k)$ for various values of shift k, is given in Figure 2. The dotted horizontal lines above and below ACF=0 represent the 95% confidence limits for a series of the same length as realcoeff but with zero autocorrelation. A third statistical summary, the plot of the partial autocorrelation function (PACF) for the series, which can be useful in determining the type of time series to model a process after, is given in Figure 3. Finally, the sample periodogram for the series, of which the first 50% is typically of interest, is shown in Figure 4.

These statistical summaries will be used to assess the model proposed in the following sections.

TIME-SERIES MODELLING

As the time series in Figure 1 does not appear to be weakly stationary (that is, both the mean and autocorrelation function of *{ X(t) }* do not appear to be independent of the time parameter), a direct time series approach to the data in Figure 1 has not proved to be successful to this point. However, a plot of the natural logarithm transformation on absolute values of *{ X(t) }*, which we will call *{ Y(t) }* does display a more stable variability. Figures 6 and 7 show the corresponding statistical summaries for *{ Y(t) }*, of the autocorrelation function and partial correlation function, as given in Figures 2 and 3 for the original reflectivity series *{ X(t) }*. Furthermore, the successive differences of absolute values of *{ X(t) }* from Figure 1 are shown in Figure 8 and bear a very good visual similarity with the original reflectivity series. This is the motivation for proceeding to model the log transformed series shown in Figure 5 with standard time series methods, as from there, a simple exponentiation and differencing step should give us an acceptable simulated series.

For this sample series, the best (as measured by the Akaike Information Criteria) time series model obtained for *{ Y(t) }* is a sixth order autoregressive model, with Yule-Walker coefficient estimates:

$$\begin{aligned} Y(t) + 3.62 = {} & 0.187*[Y(t-1) + 3.62] + 0.197*[Y(t-2) + 3.62] + 0.193*[Y(t-3) + 3.62] \\ & + 0.053*[Y(t-4) + 3.62] + 0.076*[Y(t-5) + 3.62] + 0.122*[Y(t-6) + 3.62] \\ & + e(t), \end{aligned}$$

where *e(t)* is a zero mean, finite variance white noise process. The value -3.62 is the mean of the observed samples *{ Y(t) }* The white noise assumption for residuals was tested using a cumulative periodogram (see, for example, Venables and Ripley (1997)), and no violations from that assumptions were seen.

In the next section we examine the simple algorithm for the resulting computer simulation of seismic reflectivity data.

SIMULATION

Based on the time series model for the log series *{ Y(t) }* that we have arrived at, we are now able to simulate a reflectivity series by first simulating an appropriate white noise series (if normal residuals are assumed, then we will have independent and identicially distributed residuals with mean zero and variance estimated from the observed residuals, as in, for example, Brockwell and Davis (1996)), and then applying our sixth order autoregressive model. A realization of this step (adjusted for the mean -3.62) is presented in Figure 9, along with summary statistic figures in Figures 10 and 11, corresponding to Figures 6 and 7 for the real log series. There is a remarkable similarity between Figures 10 and 11 and Figures 6 and 7, demonstrating the success of this simulation exercise.

Once the values for *{ Y(t) }* are simulated as above (for example, the S-Plus software has a single line command that can do this, as will most stardard statistical software packages), we proceed to take the exponent of the series and finally plot successive differences, at which point we should obtain a series statistically similar to Figure 8.

A realization of the simulated reflectivity series (actually, the difference series corresponding to Figure 8) is given in Figure 12, along with its periodogram in Figure 13. Recall that only the first 50% of the original periodogram (that is, Figure 4) is considered accurate.

CONCLUSIONS

Statistical time-series modelling for the series corresponding to Figure 8 appears very successful. However, recall that the series in Figure 8 is not the original reflectivity series, but a series that looks visually similar to a typical reflectivity series. Whether this is sufficient in describing and simulating the original reflectivity series corresponding to Figure 1 will be the subject of further study. Statistically, the series in Figure 8 (and the simulated series) and Figure 1 are not the same, for example their autocorrelation functions differ in sign. This may be a result of the differencing applied in obtaining the Figure 8 series. Further statistical properties of the autoregressive model can be applied in a simulation study, for example, the asymptotic distribution of the coefficients and mean are known to be jointly multivariate normal (Brockwell and Davis (1996)).

A next step will be to investigate the success of these models for a large sample of well-log data.

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FIGURES

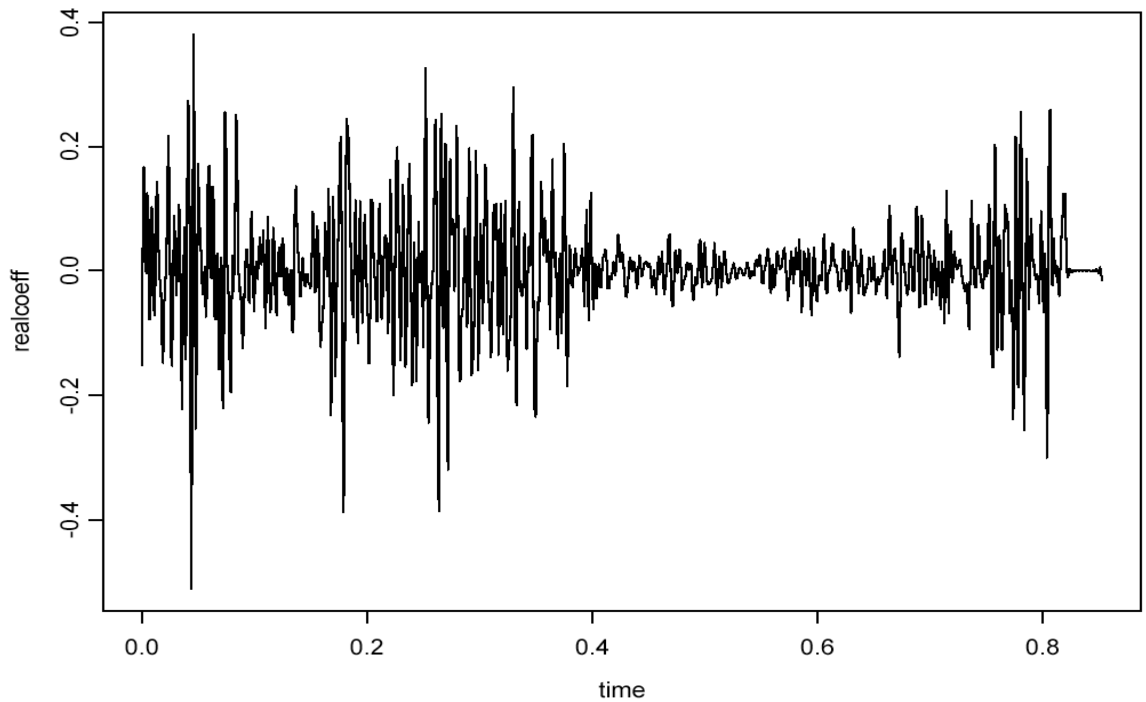

FIG. 1: The input reflectivity series *X(t)*, labelled realcoeff.

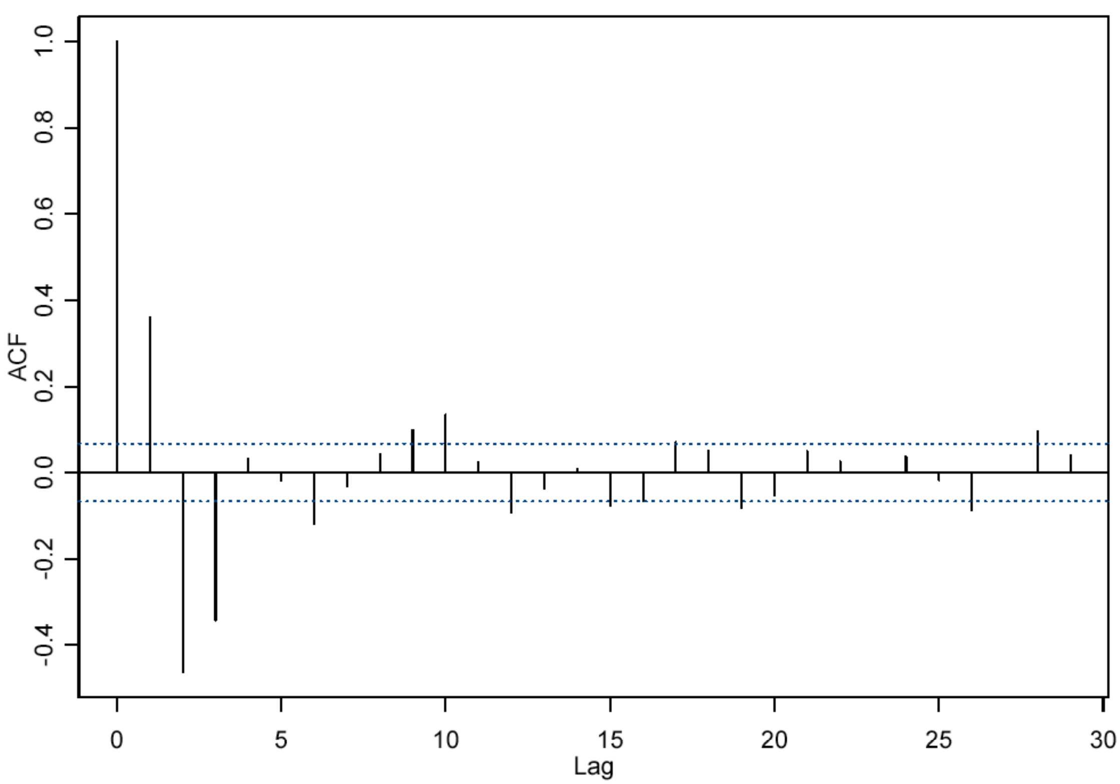

FIG. 2: Sample ACF for series realcoeff.

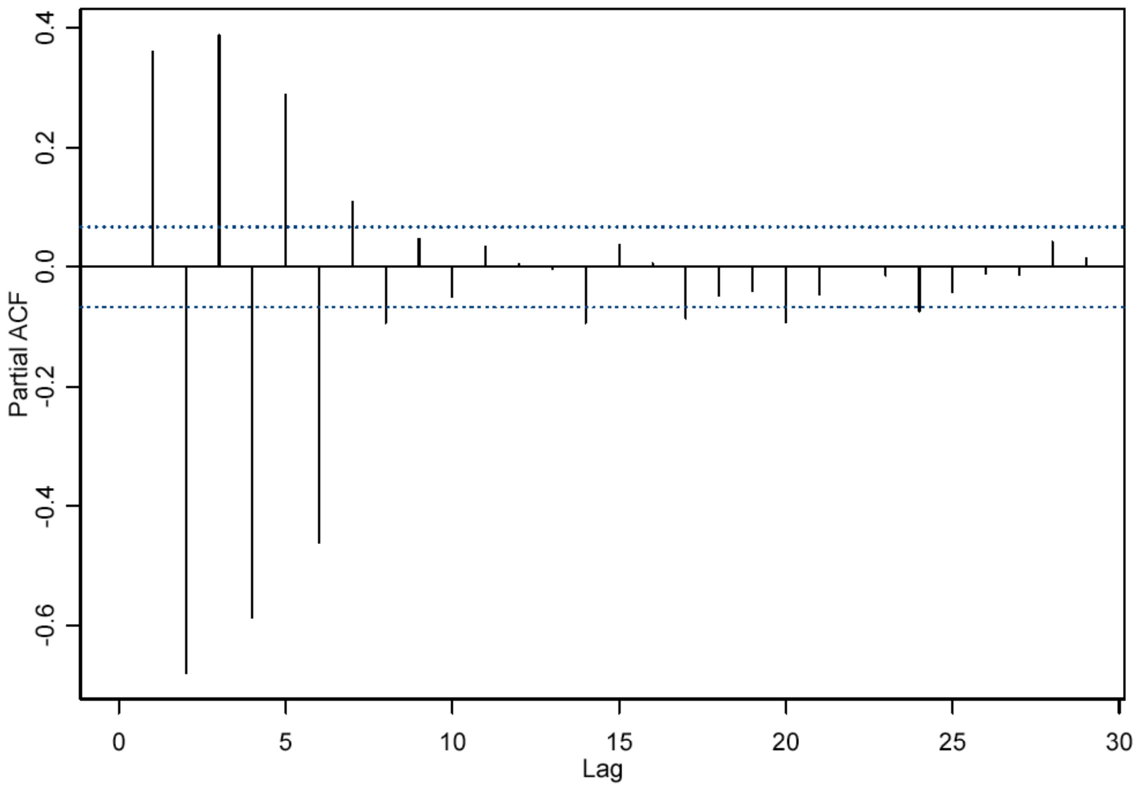

FIG. 3: Sample PACF for reflectivity series realcoeff.

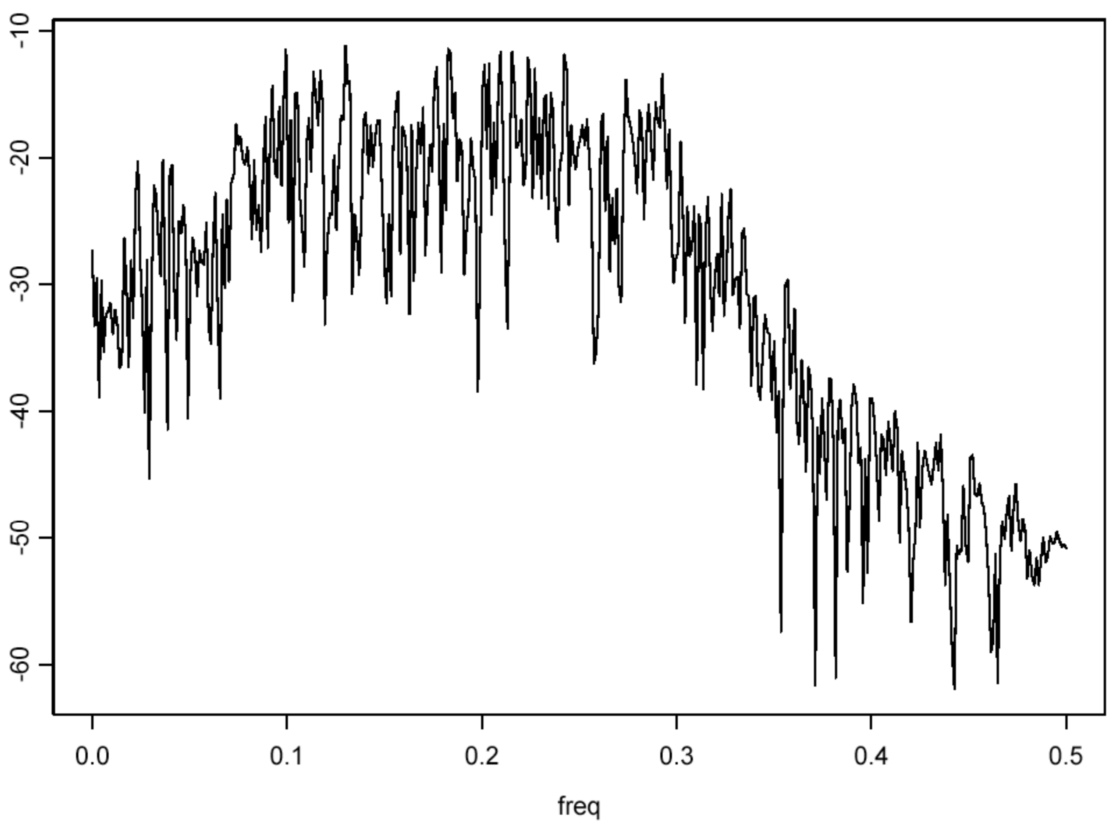

FIG. 4: Sample periodogram for reflectivity series realcoeff.

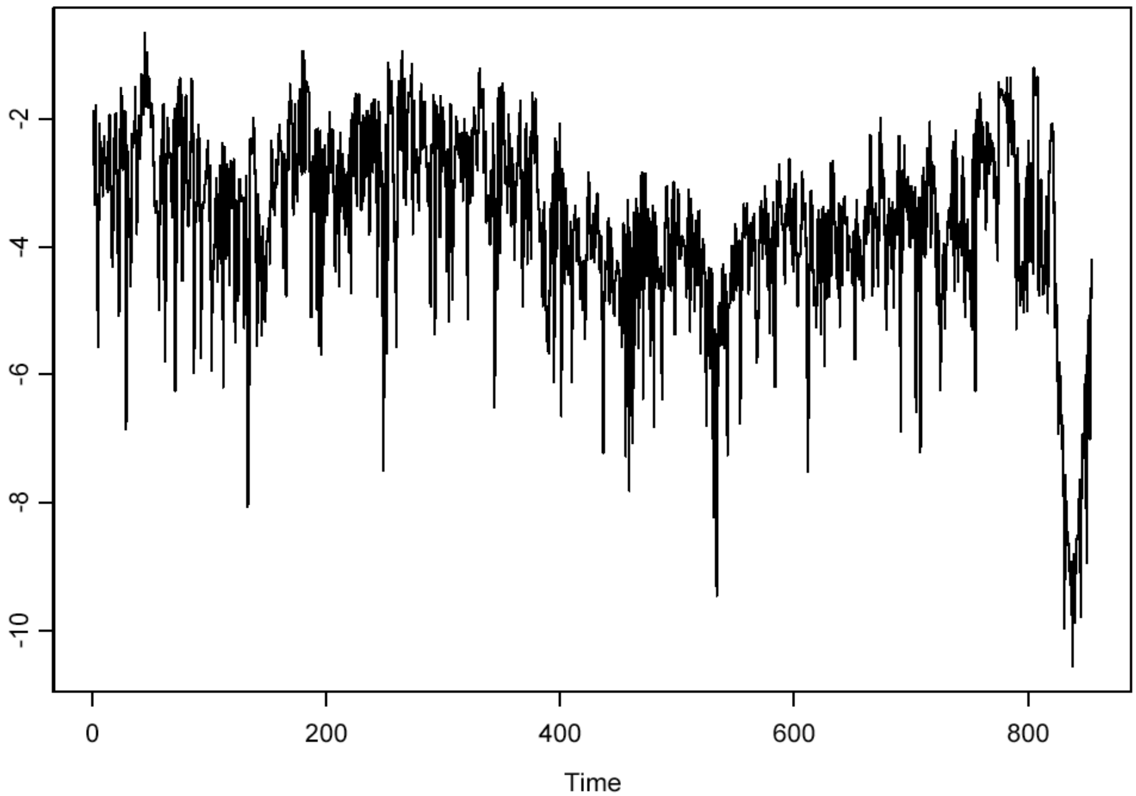

FIG. 5: Plot of the log transformation for the reflectivity series, *Y(t) = log* | *X(t)* |

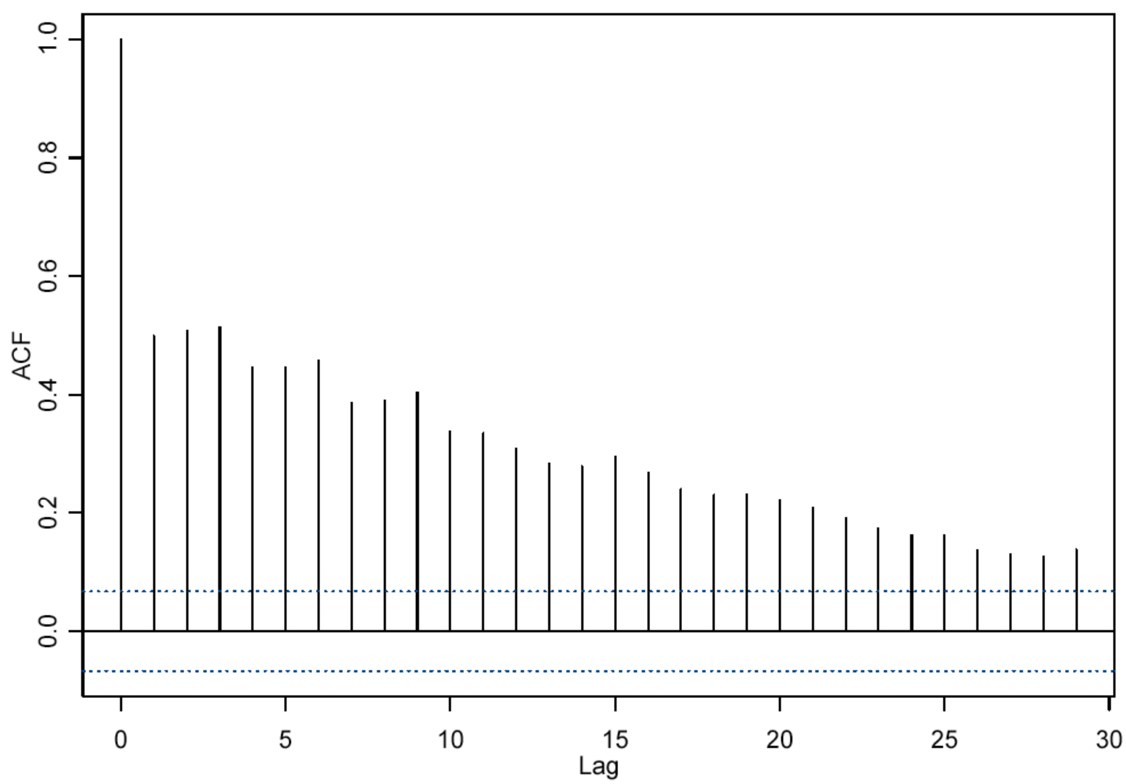

FIG. 6: Sample ACF for log series *Y(t)*

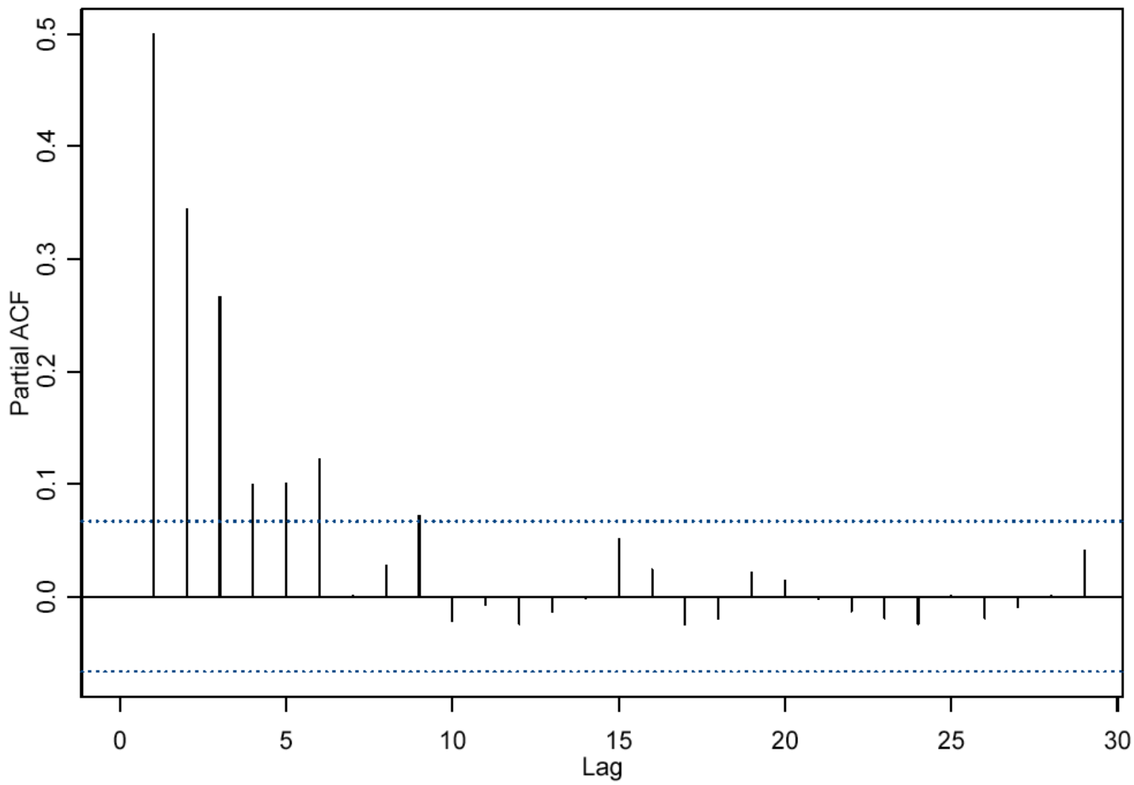

FIG. 7: Sample PACF for the log series *Y(t)*

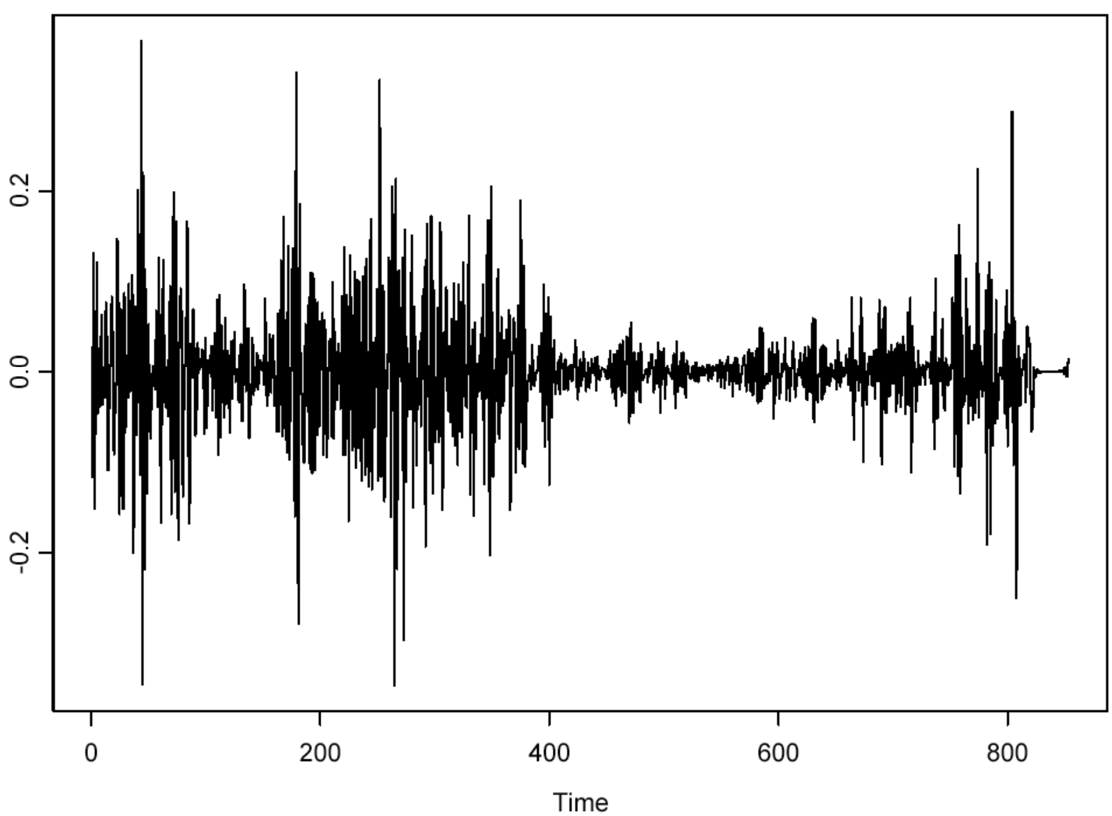

FIG. 8: Successive differences for the absolute values | *X(t)* |

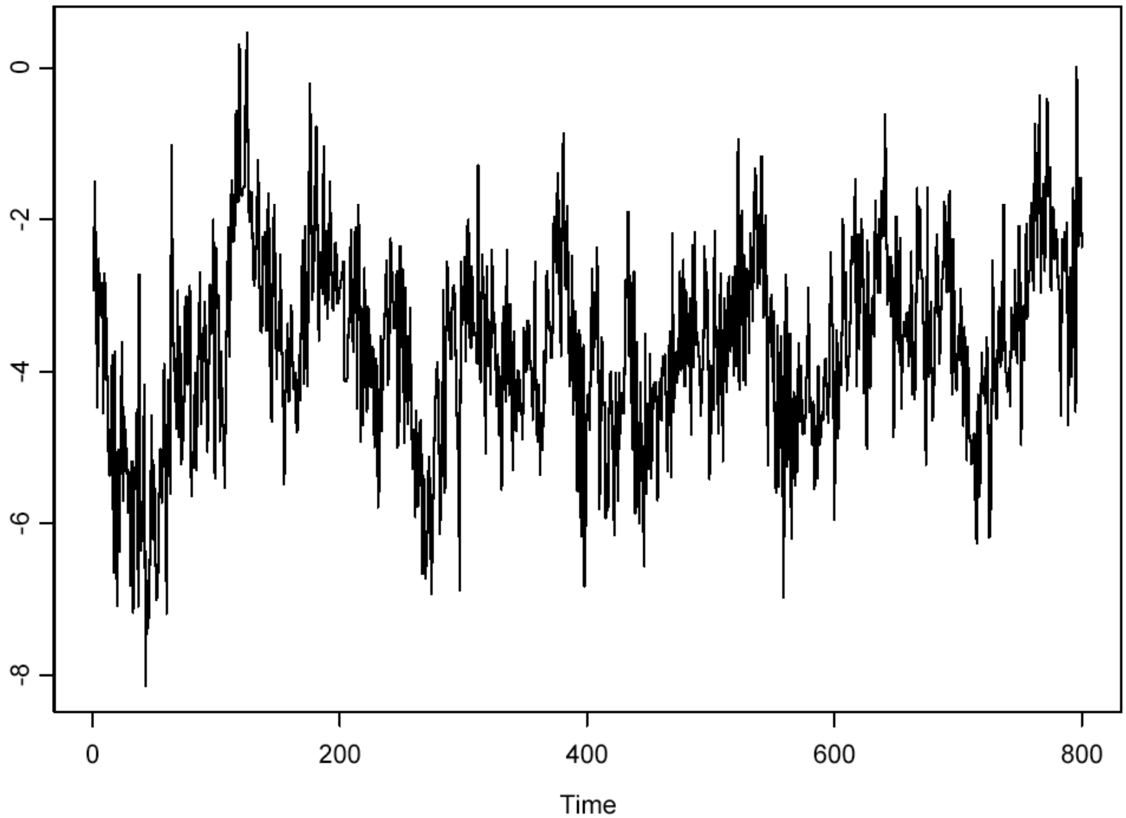

FIG. 9: Simulation of the log series *Y(t)*

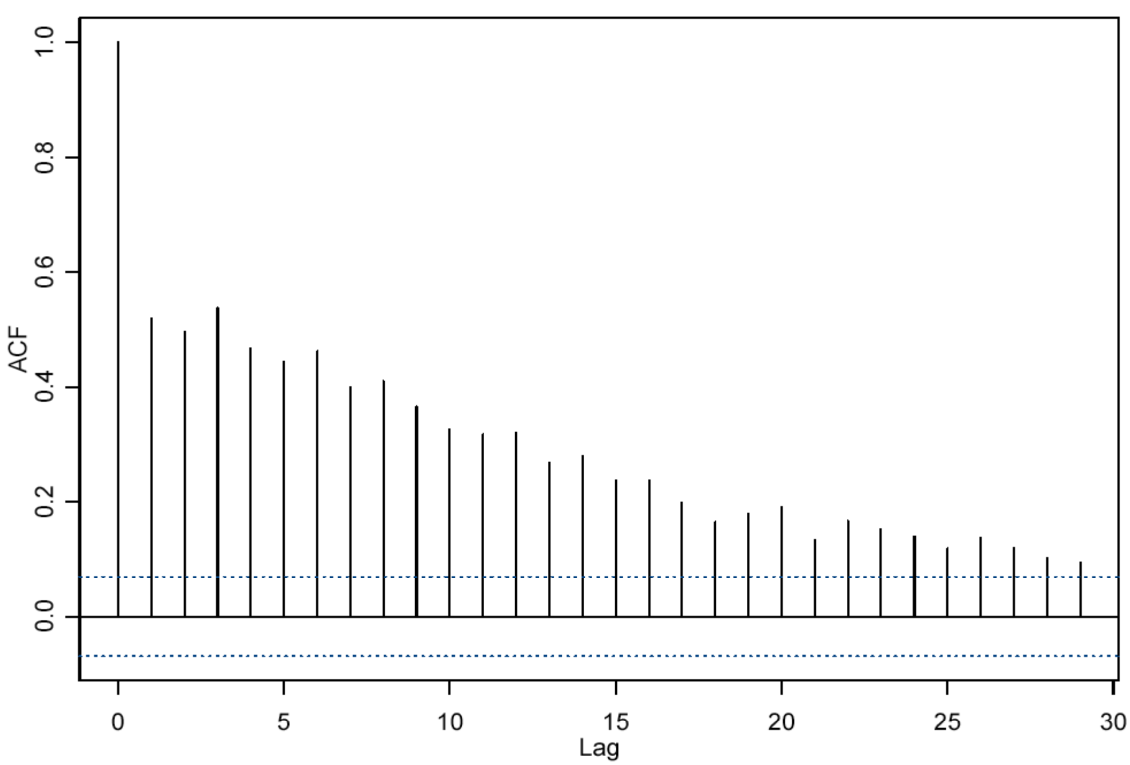

FIG. 10: ACF for a simulated log series *Y(t)*

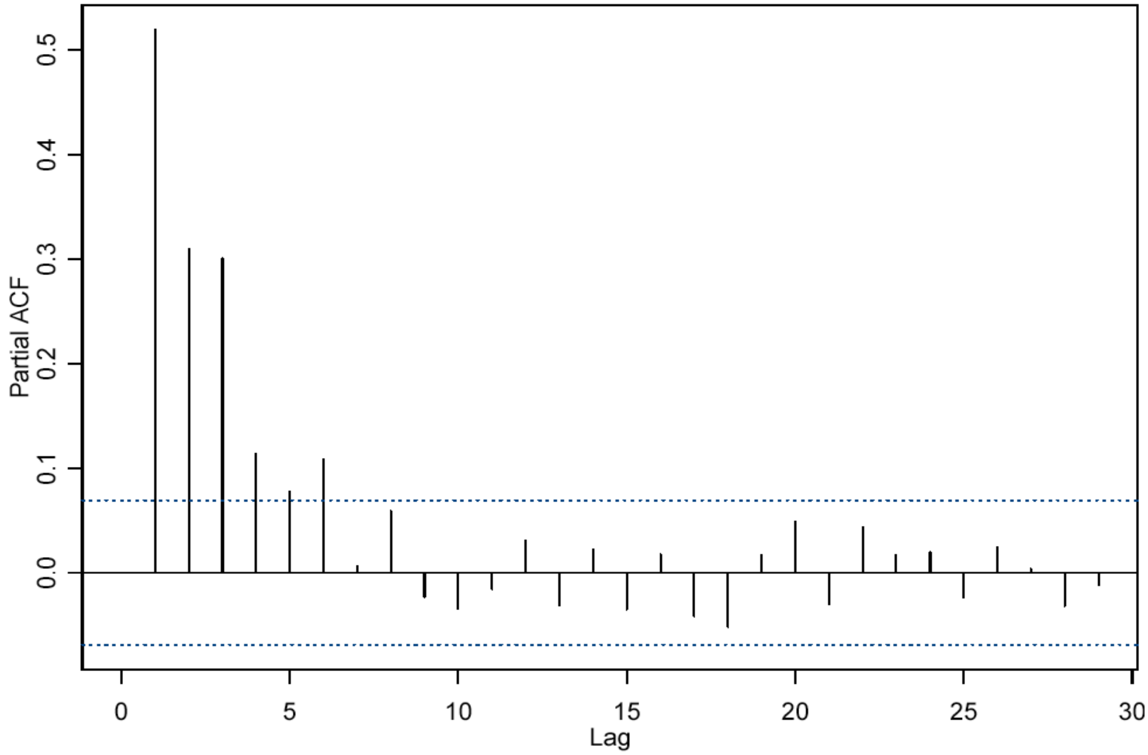

FIG. 11: PACF for simulated log series *Y(t)*

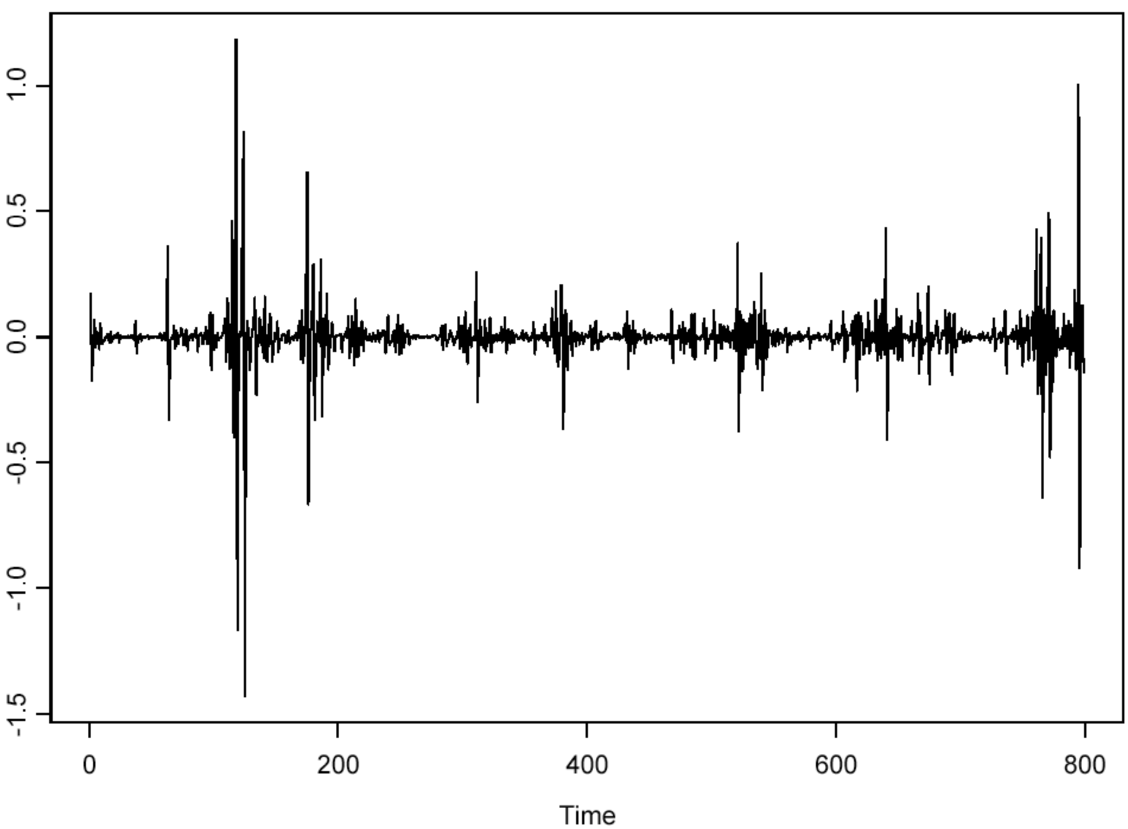

FIG. 12: Simulated reflectivity series, obtained by exponentiating, differencing *Y(t)*

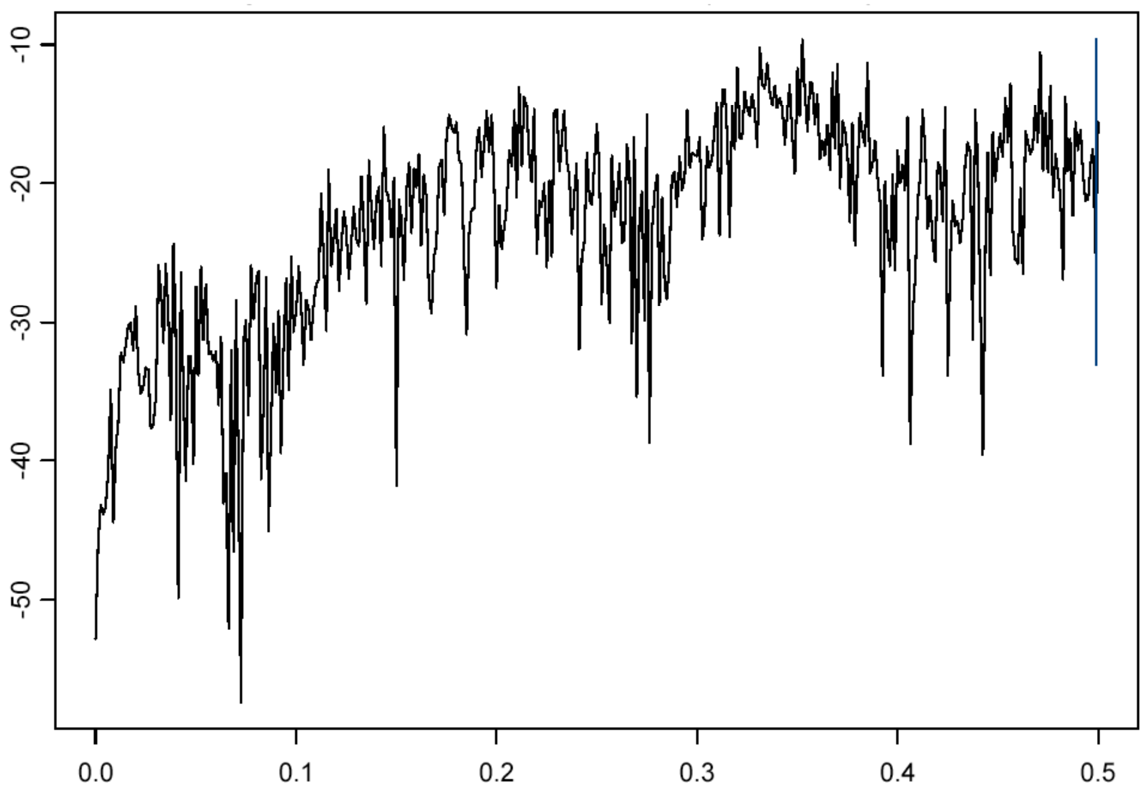

FIG. 13: Periodogram for simulated reflectivity series