Estimation of Thomsen's anisotropy parameter δ and ε using EO gathers

Pavan Elapavuluri and John .C. Bancroft

ABSTRACT

A measure of the different anisotropic parameters is essential in order to extend the seismic processing techniques to anisotropic media. The purpose of this study is to estimate the Thomsen's P-wave anisotropy parameters namely ε and δ, for transversely isotropic (TI) media. An inversion techniques based on the shifted hyperbola NMO equation is used for the estimation. Equivalent Offset (EO) gathers give a better estimate of the anisotropy parameters than conventional CMP/CDP gathers. This method was tested on the data collected over a synthetic model. It has been shown that δ can be estimated very accurately while estimation of ε is not as exact as δ. The anisotropic parameters of various formations of interest were estimated over the Blackfoot field in western Canada using this method.

INTRODUCTION

In order to study and understand the complex Earth, exploration geophysicists make many assumptions. One of them is that the earth is perfectly isotropic when it is fundamentally anisotropic (Thomsen 1986). Postma (1955) proved that an isotropic-layered earth behaves as an anisotropic medium if the layers were in finer scale than the wavelength of the seismic waves (Long-wave anisotropy). Postma in this paper considered only periodic layering of two types of rocks. Backus (1962) also worked on the same concept of long-wave anisotropy. He extended the work done by Postma(1955) to media containing three or more kind of rocks.

Alkalifah and Tsvankin (1994) proposed an anisotropic non-hyperbolic NMO equation and also a technique to invert it for the estimation Thomsen's anisotropic parameters.

Determination of δ (the short offset effect) is relatively easy but ε (long offset effect) is difficult and needs a measure of horizontal velocity, which is difficult to measure. In this study the long offset moveout information is used for ε estimation. Usually Dix type NMO correction at long offsets is not very accurate, and it even worsens when there is anisotropy present. Shifted hyperbola NMO equation is more accurate at longer offsets than Dix NMO equation (Castle 1994). Therefore by using shifted hyperbola NMO (SNMO) equation to correct long offset data, we get a better estimation of RMS velocity (therefore better interval velocity) and a better estimation of δ. The shift parameter 'S' is related to 'interval velocity type' formulation to the parameter ε.

THEORY

The most common measure of P-wave anisotropy is the ratio between the horizontal and vertical P-wave velocities, typically between 1.05 to 1.1 and is often as large as 1.2 (Sheriff 1991).

Thomsen introduced a more effective and scientific measure of anisotropy in 1986. He introduced the constants ε, γ, and δ as effective parameters for measuring anisotropy. According to Thomsen, δ is the most critical measure of anisotropy and it doesn't involve the horizontal velocity at all in its definition. Therefore measuring δ is very important for processes like depth imaging. According to Toldi et al (1999) "The depth effects are carried by the parameter δ which must therefore be measured with help of well control."

Normal Moveout (NMO)

NMO can be defined as " The additional time required for energy to travel from source to a flat reflecting bed and back to geophone at some distance from the source point compared with the time it takes to return to the geophone at the source point" (Sheriff. 1984).

The normal moveout equation used commonly to shift events at non-zero offsets to their equivalent zero offset time is given by Equation (1)

$$t = \sqrt{t_0 + \frac{x^2}{V_{NMO}^2}}\,, \tag{1}$$

where t is the traveltime at offset x, t_0 is the zero-offset (normal incidence) traveltime and V_{NMO} is the stacking velocity (Dix, 1955). This is a short offset (2 term) approximation of the Taylor series expansion of traveltime as the function of offset as given Taner and Koehler (1969) over an isotropic horizontally layered media (Appendix 1).

Shifted hyperbola NMO (SNMO) Equation

Castle in 1994 published an alternative NMO equation to the Dix NMO equation using the principles of '*reciprocity, finite slowness and exact constant velocity limit*'. For *"reasonable"* offsets, his approximation, termed as the shifted hyperbola equation, is given as:

$$t = t_0\left(1 - \frac{1}{S}\right) + \sqrt{\left(\frac{t_0}{S}\right)^2 + \frac{x^2}{SV_{NMO}^2}}\,. \tag{2}$$

In the above equation, the *"shift parameter"*, *S*, is a constant and is described as:

$$S = \frac{\mu_4}{\mu_2^2}\,, \tag{3}$$

where μ_2 and μ_4 are the second and fourth order moments of the velocity distribution.

Although, the shifted hyperbola equation with a constant '*S*' fits the larger offsets better than Dix NMO formula, Castle (1994) showed that by varying the *"shift parameter"* with offset, one could obtain an exact fit of the traveltime curve. Figure (1) shows different versions of SNMO curves with varying shifts from 0.1 to 0.9.

The most general form of the shifted hyperbola equation is written as

$$t = \tau_s(x) + \sqrt{\tau_0^2(x) + \frac{x^2}{v^2(x)}} \tag{4}$$

where the parameters τ_S, τ_0 and v are functions of the source-receiver offset (x) as

$$\tau_0(x) = \frac{t_0}{S(x)}, \tag{5}$$

$$\tau_S(x) = t_0\left[1 - \frac{1}{S(x)}\right], \tag{6}$$

$$v(x) = \sqrt{S(x)}V_{nmo}, \tag{7}$$

and the offset dependent shift parameter, $S(x)$, is defined as

$$S(x) = \frac{\frac{x^2}{V_{NMO}^2} - 2t_0(t - t_0)}{(t - t_0)^2} \tag{8}$$

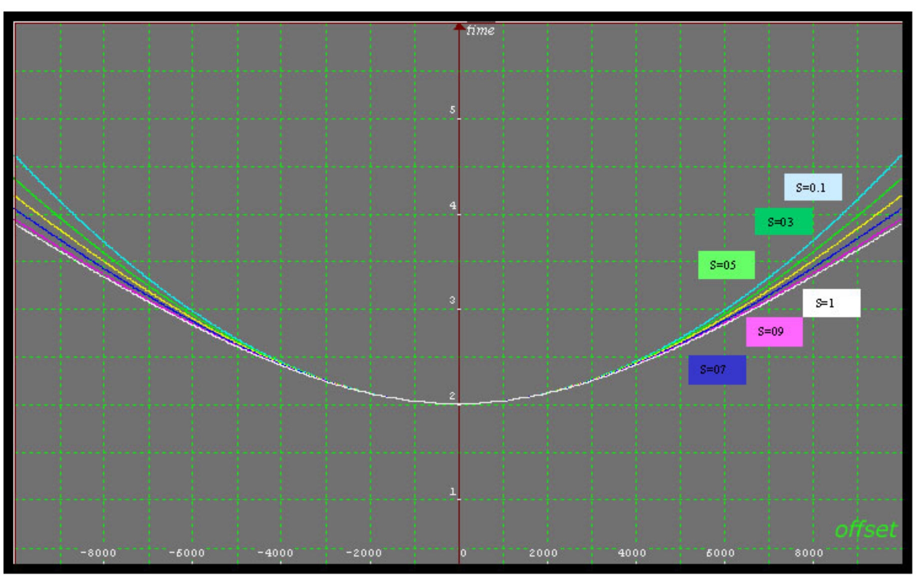

FIG.1. The plot showing shifted hyperbolas with varying shift.

Shift parameter *'S'* and the anisotropy parameters

The shift parameter S can be used to estimate the anisotropy parameters as given by the Equations (9) and (10). Their derivations are discussed in detail in Appendix1.

$$\delta_n = \frac{1}{2}\left(\frac{V_{NMOn}^2}{V_{0n}^2} - 1\right) \tag{9}$$

$$\varepsilon_n = \delta_n - \left(\frac{H_N\left(1-k^2\right)}{8V_0^4\left(1-k^2+2\delta_n\right)}\right) \tag{10}$$

EO Gathers

A common scatter point gather is a pre-stack migration gather that collects all the input traces that contain energy from a vertical array of scatter points.

The distance from the surface location of the scatter point to the source and receiver defines the offsets in a EO gather, but not the source receiver offset. The EO gather is similar in appearance to a common mid point (CMP) gather. They both define a subsurface location, and are sorted by offsets. Hence all the traces in the prestack migration aperture, regardless of the source or receiver position, may be used to form a EO gather.

The offsets in a EO gather are defined as given by Equation (11) (Bancroft, et.al, 1998)

$$h_e = x^2 + h^2 - \frac{4x^2h^2}{t^2v_{rms}^2} \tag{11}$$

Where x is distance between CMP and the EO gathers location and h is half the source receiver offset, V_{rms} is the RMS velocity.

Advantages of EO gather

An EO gather is characterized by its very high fold which increases the SNR, this in turn makes velocity analysis more accurate. Generally EO gathers have larger offsets when compared to CMP gathers.

The semblance plot of the EO gather shows tighter clustering of energy, which enables (and requires) more accurate picking of velocities. It has been shown by Bancroft (1996) that NMO velocities estimated using a EO gather are more accurate than velocities estimated over a CMP gather.

OUTLINE OF THE METHOD

In order to apply this method to the data, basic processing flow is preferably applied to the data. Equivalent offset is calculated and the data was then sorted into common

Equivalent offset (EO) gathers. The analysis was performed on both CMP and EO gathers. The estimation can be concisely described using the flowing steps:

Using the Monte Carlo inversion SNMO is fitted to the moveout curves and V_{nmo} and S are obtained at each interval.

Equation (A.14) is used to estimate the '*interval NMO velocities*'

The value of V_0, the vertical velocity is determined from the VSP data/sonic logs.

Equation is (9) used to calculate δ_n.

Equation is (10) used to calculate ε_n.

Estimation of S and V_{nmo}

As SNMO equation is a non-linear problem so linear inversion techniques (for example least square inversion) fail. A random walk technique like Monte-Carlo inversion would serve the purpose of inverting the moveout Equation (2) for both '*S*' and V_{nmo}.

Monte-Carlo Inversion

Monte-Carlo methods are random search methods in which the models are drawn randomly from the whole model space and tested against the data. The best model depending on the acceptance criteria is considered as the solution to the inversion problem.

The method can be described by the following Equation (12) for a model parameter set $\mathbf{m}(S, V_{nmo})$

$$m_i^{new} = m_i^{\min} + (rn)\left[m_i^{\max} - m_i^{\min} \right] , \qquad (12)$$

where m_i is the model parameter, m^{min} and m^{max} are the minimum and maximum values of the model parameter specified, and *rn* is a random number drawn from a uniform distribution [0,1].

The generated models $\mathbf{m}^{new}$ are tested iteratively. The generated model that best fits the data with a minimum misfit is accepted.

CASE STUDY

The anistropic parameter estimation technique will now be tested over synthetic anisotropic data generated using NORSAR2D software. Norsar 2D is a ray tracing program based on the ray theory by Cerveny (1985).

A layered geologic model (Figure 1) was built with 9 flat layers in it. The model is built in the depth domain and is 6 kms deep. The thinnest layer is of thickness 0.5 km and its velocity is 1000m/s. A 40hz zerophase ricker wavelet was used for the generation of seismograms without violating the raytracing assumptions. There are 101 shots in total and 300 receivers per shot. The shot spacing was 40m and receiver spacing was 20m.

Figure (1) shows the layered model with Table 1 showing the values of the material properties viz.

- P-wave velocity
- S-wave velocity
- Density
- ε, and
- δ.

Table 1. material properties of the model used

Interface	P-velocity	S - velocity	Density	ε	δ
1	1000	500	1.1	0	0.2
2	1200	600	1.2	0.05	0.25
3	1500	750	1.3	0.1	0.3
4	2000	1000	1.5	0.15	0.1
5	2500	1250	1.7	0.2	0.15
6	3000	1500	1.9	0.25	0.2
7	4000	2000	2.2	0.3	0.25
8	5000	2500	2.4	0.2	0.3

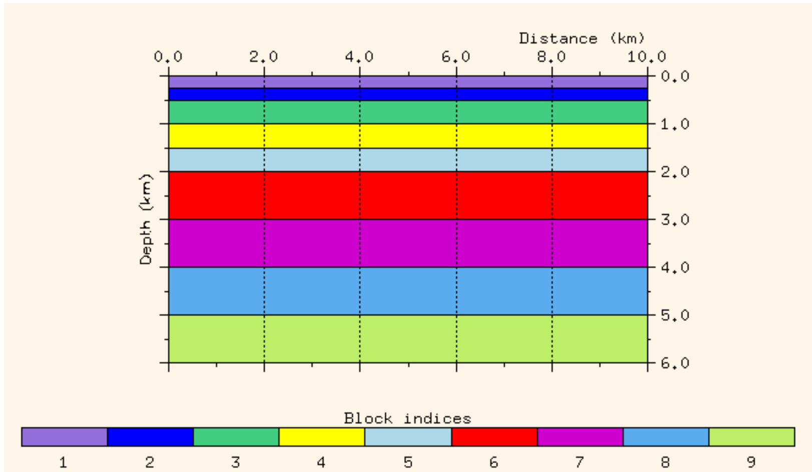

FIG. 2. The Geological model

This model data was used to test the above method. CMP gather and EO gather were calculated at a CMP at 5KM in the model shown in Figure 2. The analysis discussed was performed on both CMP and EO gathers.

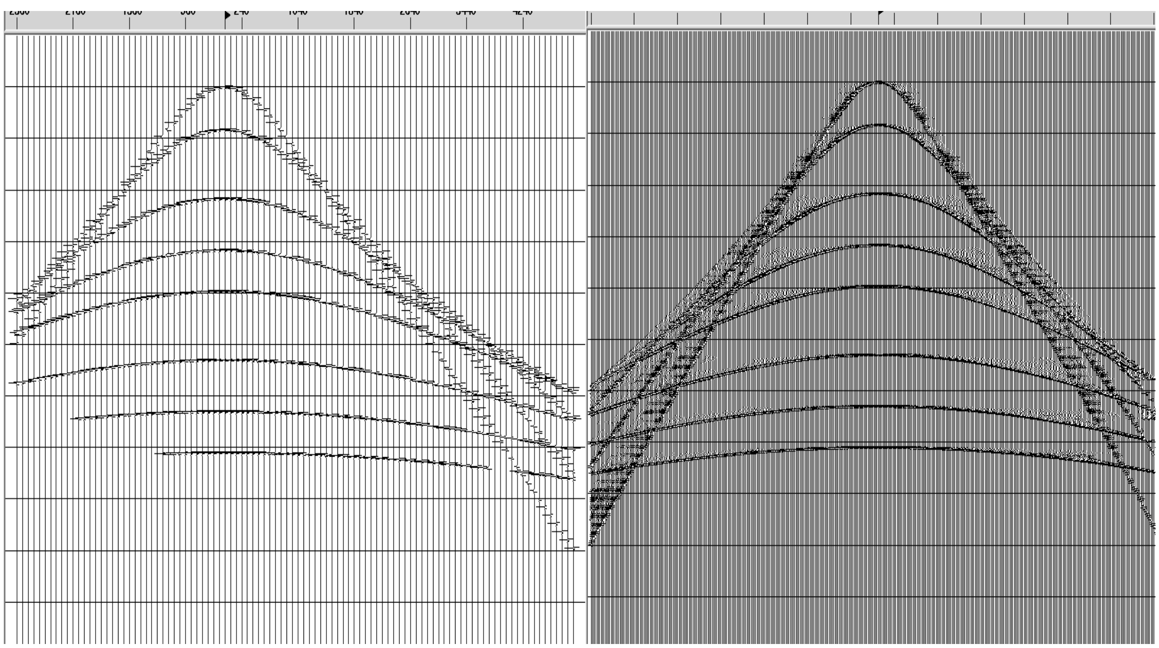

FIG.3. Comparison between CMP gather and EO gather at a CMP at 5 KM.

CMP gathers

Table 2 shows the V_{nmo} and S estimated over the CMP gathers.

Table.2. The Values of V_{nmo} and S estimated over the CMP gathers.

Intervals	V_{nmo} (M/Sec)	Shift 'S'
1	1237	0.5784
2	1336	0.3456
3	1828	0.6734
4	2290	0.6234
5	2908	0.7345
6	3655	0.9234
7	5059	0.5647
8	6259	0.8768

EO gathers

Table 3 shows the V_{nmo} and S estimated over the EO gathers.

Table.3. The values of V_{nmo} and S estimated over the EO gathers.

Intervals	V_{nmo} (M/Sec)	Shift 'S'
1	968	0.8134
2	1118	0.7999
3	1319	0.7868
4	1504	0.6942
5	1702	0.7198
6	1814	0.8517
7	2432	0.6863
8	2868	0.7107

CMP vs. EO Gathers

The values of δ were estimated using Equation (9) on both CMP and EO gathers.

The Figure (4) shows the plot of these values. It is evident from the plot that the values estimated from EO gather match perfectly with the model values while the CMP values show considerable mismatch. The reason for this mismatch is that the estimation of both ε and δ depends on the accuracy of estimation of RMS velocities. To test the dependency of accuracy of RMS velocity on parameter estimation, an error analysis was performed..

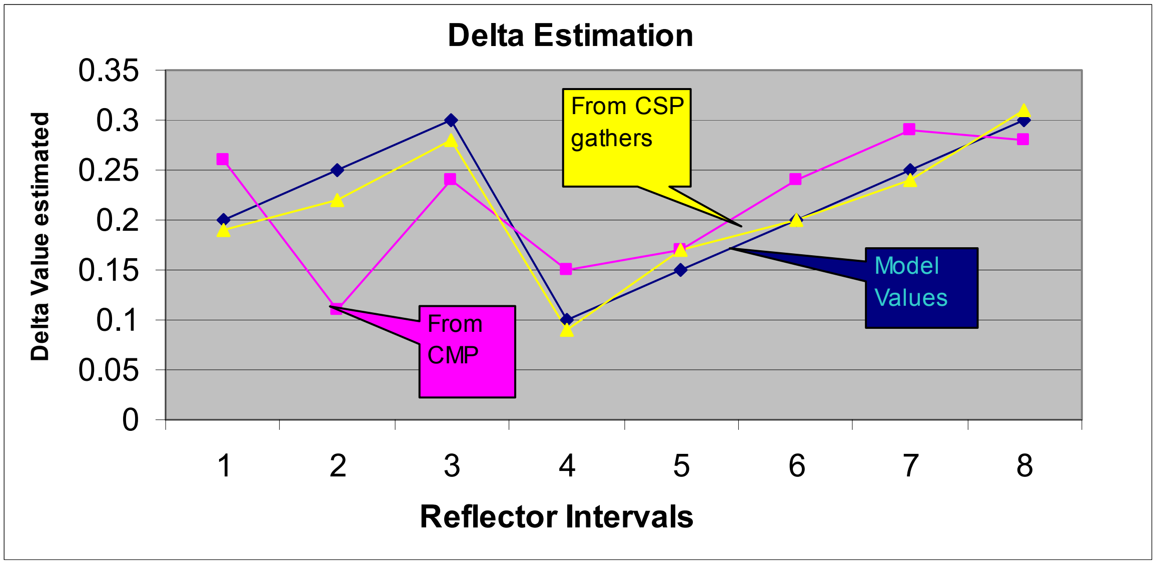

FIG.4. The values of ε estimated from EO and CMP gathers.

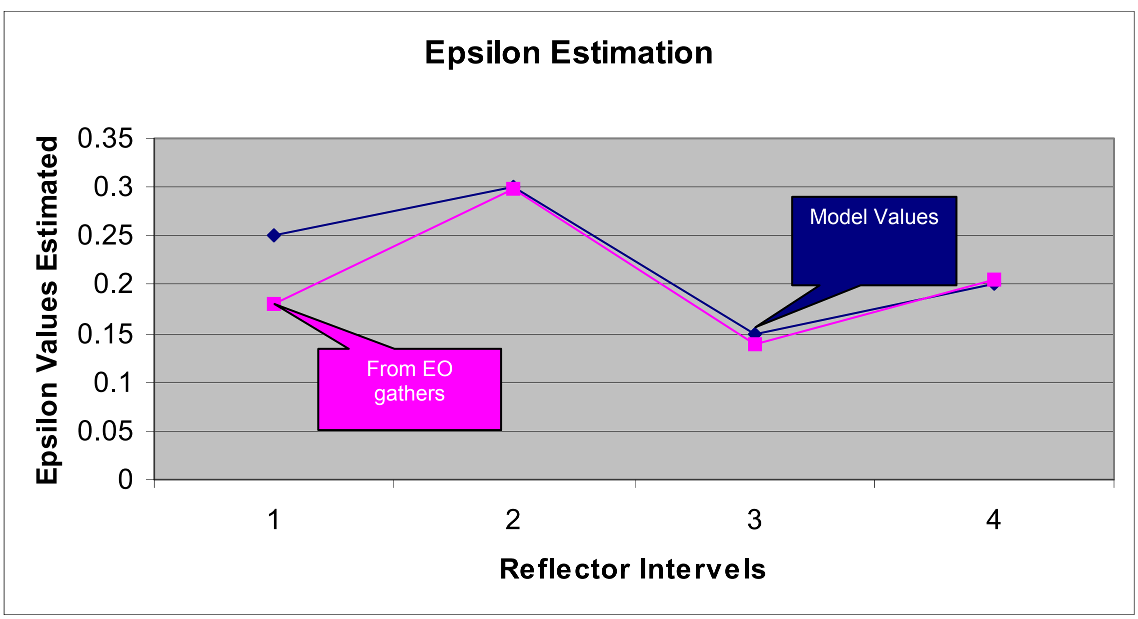

FIG. 5. The values of ε estimated from EO gathers.

Error analysis

The estimated values of the parameters are heavily dependent on the velocities obtained for velocity analysis. A simple error analysis was done by introducing an error into the NMO velocities estimated from the EO gather. The results are shown in the Figure (6). For a range of 0-10% error in estimated RMS velocities, a range of 10-150% error was encountered in the values of δ. Therefore, the velocities estimated from an EO gather are more accurate, δ and ε can be estimated with better confidence using the EO velocities.

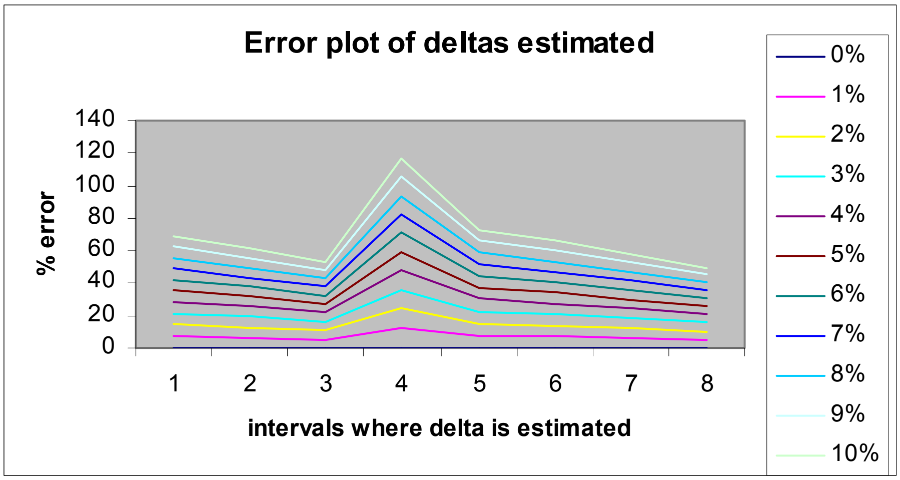

FIG. 6. Errors in δs measured due to the errors in the estimated velocities.

Discussion

The technique for anisotropic parameter estimation discussed earlier has been applied over a synthetic anisotropic seismic data. It was tested on both CMP and EO gathers. The error analysis performed showed the dependence of parameters estimated on the inverted NMO velocities. Due to the accuracy of velocity estimation on EO gathers, these gathers were used for the estimation.

FIELD DATA

The method proposed in this paper is now applied to the seismic data collected over the Blackfoot Field. Blackfoot field is near Strathmore, Alberta and is operated by EnCana petroleum. A 3C-3D data was acquired by CREWES in 1997.

The Blackfoot Field is located in Township 23, Range 23, West of 4^{th} meridian, in south central Alberta.

Geology

The geology of Blackfoot field has been discussed in detail by Miller et. al (1995). This is a very brief review of the lithology of the formations of interest to this work. The reservoir rocks in this field are Glauconitic incised valleys in the Lower Manville group of lower Cretaceous. Coals, Viking formation and base of fish scales shales overlie these reservoir rocks. Figure 7 shows the stratigraphy in Blackfoot region.

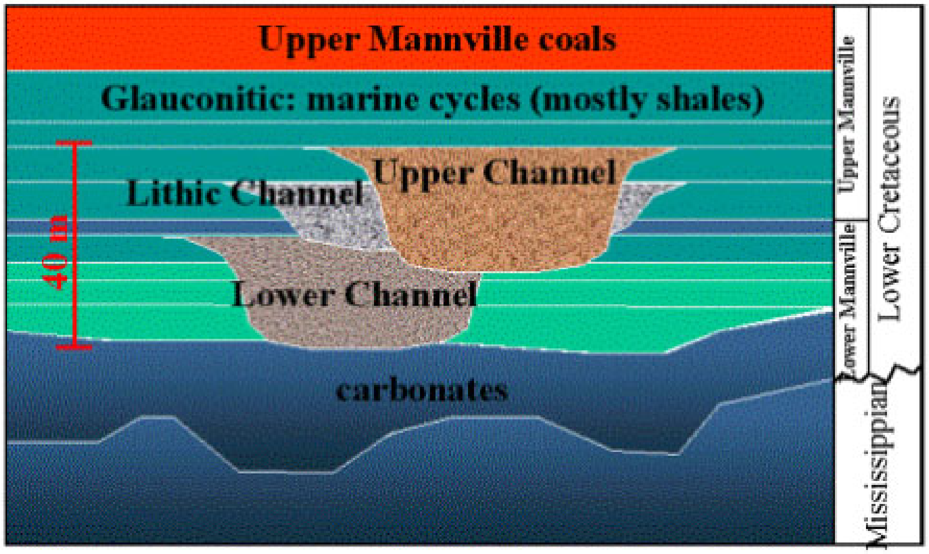

FIG. 7. Blackfoot Stratigraphy

A line numbered '20M *vertical*' that has a well (#09-08) located very near to it was chosen to test this method. The Figure 8 shows the position of the well on the CDP plot. Figure 9shows the position of the well on the stacked section. Having proved that CMP gathers are not very accurate, the anisotropic analysis will be performed only on EO gathers. EO gathers are the formed at CDP 149, which is nearest to the well location. Monte-Carlo velocity analysis was then performed on the EO gather. The velocities estimated were used in the algorithm discussed above to estimate the values of δ and ε.

The position of the well

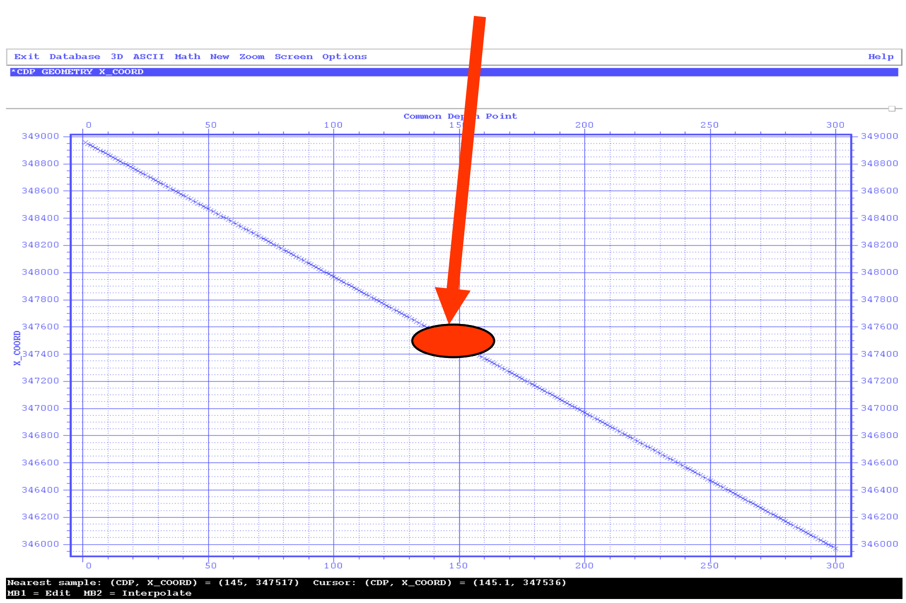

FIG.8. Position of the well 09-08 on the CDP plot

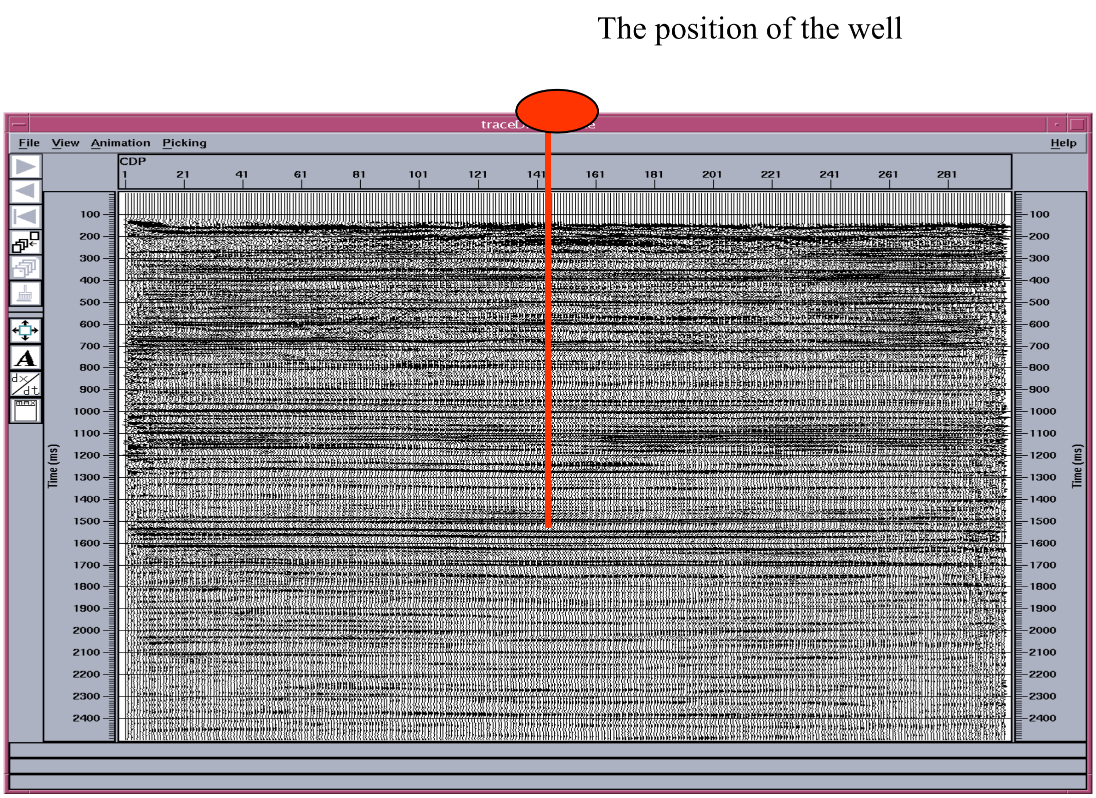

FIG.9. Position of the well 09-08 on the stacked section

The inversion technique based on the Monte-Carlo technique discussed above, is applied to the moveout curves in the EO gather. The inverted values of V_{nmo} and S are tabulated in Table 5

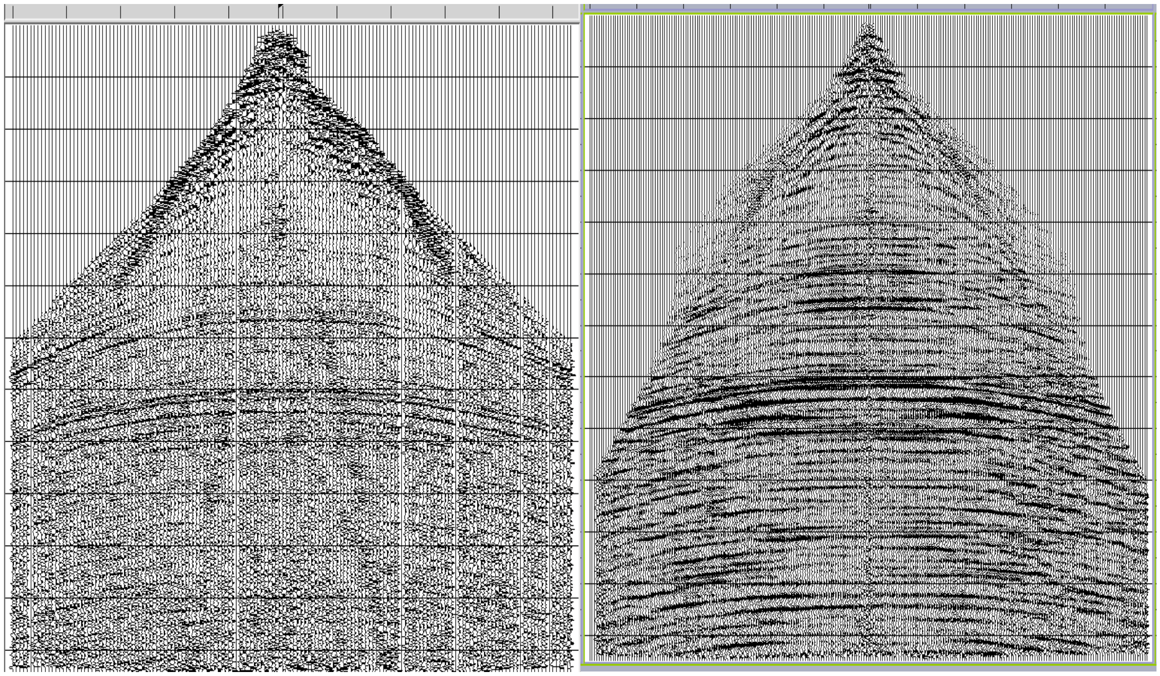

FIG.3. Comparison between CMP gather and EO gather at a CMP at 5 KM.

Table 5. Formation naming conventions.

Abbreviation	Unit Name
BFS	Base of Fish Scales Zone
MANN	Blairmore-Upper Mannville
COAL	Coal Layer
GLCTOP	Glauconitic Channel porous Sandstone unit
MISS	Shunda Mississippian

Table 6. δs calculated using CMP gathers.

Formation	V_{nmo}	Shift (S)	$V_{0,i}$
BFS	4002	0.6987	3300
MANN	4148	0.9388	3990
COAL	4755	0.6145	3900
GLCTOP	4460	0.8604	3860
MISS	5998	0.7256	6000

Table 7. δs calculated using EO gathers.

Formation	δ (estimated)	ε (estimated)
BFS	0.23	0.06
MANN	0.04	0.008
COAL	0.24	0.12
GLCTOP	0.06	0.006
MISS	0.00	0.001

DISCUSSION AND CONCLUSIONS

In this paper a method to estimate the Thomsen's anisotropy parameters (ε and δ) for TI media is described. An inversion technique for the estimation of 'S' is also proposed. The δ estimation is highly dependent on the estimation NMO velocity. The error analysis of δ's estimated proves how important the estimation of accurate NMO velocities is. It has also been showed that δ estimated using EO gathers are more accurate than that of those estimated using CMP gathers.

Extending this analysis to the real field data, we found that the shales and coals show very significant anisotropy. The vertical velocities estimated from a sonic log were used in this study. The velocities from the sonic log are greater than the seismic velocities. Vertical interval velocities estimated from VSP data would give more accurate estimates in this area. The inversion can be made more robust.

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APPENDIX-1

Taner and Koelhler (1969) gave the following generalized equation for NMO:

$$t^2 = c_1 + c_2 x^2 + c_3 x^2 + \dots \qquad \text{(A.1)}$$

Conventional NMO of Dix truncates the above series to the second power of *x* (source-receiver offset). This can be written as Equation (A.2)

$$t^2 = c_1 + c_2 x^2 , \qquad \text{(A.2)}$$

where the coefficients are given by Equations (A.3) and (A.4)

$$c_1 = t_0^2 \qquad \text{(A.3)}$$

$$c_2 = \frac{1}{V_{nmo}^2} . \qquad \text{(A.4)}$$

On the other hand Castles's NMO equation (2), which is a fourth order approximation of Taner and Koehler travel timeseries can be written as Equation (A.5) .

$$t^2 = c_1^s + c_2^s x^2 + c_3^s x^4 + \dots , \qquad \text{(A.5)}$$

coefficients in the Taylor's series are given by the following equations.

$$c_1^S = t_0^2 \qquad \text{(A.6)}$$

$$c_2^S = \frac{1}{V_{nmo}^2} \qquad \text{(A.7)}$$

$$c_3^s = \frac{1}{4} \frac{(1-S)}{t_0^2 V_{NMO}^2} \qquad \text{(A.8)}$$

Tsvankin and Thomsen (1994) described a NMO equation for TI media in terms of Thomsen's parameters. Their equation can be re-written in the form of Taner and Koehler's Taylor's series (1969), as in Equation (A.5), to yield the following Taylor series coefficients (denoted with a superscript *T*):

$$c_1^T = t_0^2 \tag{A.9}$$

$$c_2^T = \frac{1}{V_0^2(1+2\delta)} \tag{A.10}$$

$$c_3^T = \frac{\left(\sum_i V_{2i}^2 \Delta t_i\right)^2 - t_0 \sum_i \left(H_i + V_{2i}^4\right)\Delta t_{pi}}{4\left(\sum_i V_{2i}^2 \Delta t_i\right)} \tag{A.11}$$

where H is given by

$$H = 8V_0^4(\varepsilon - \delta)\left[1 + \frac{2\delta}{(1-k^2)}\right] \tag{A.12}$$

V_0 is the vertical velocity and k is ratio $\frac{V_p}{V_s}$.

Equating the co-efficient c_2^S (A.7) with c_2^T (A.10), we get the following relationship for δ:

$$\delta_n = \frac{1}{2}\left(\frac{V_{NMOn}^2}{V_{0n}^2} - 1\right) \tag{A.13}$$

Where V_{NMOn} is the interval RMS velocity for a particular layer and V_{0n}^2 is the vertical velocity obtained from the check shots.

Using Dix-type differentiation interval properties can be determined. "Dix (1955) formula makes it possible to recover the interval velocity for any particular layer from short spread moveout velocity, in flat-layered isotropic media." (Alkhalifah and Tsvankin, 1995)

The interval NMO velocity V_{NMOn} for the Nth layer may be recovered using the following Equation (A.14).

$$V_{NMOn}^2 = \frac{V_{NMO}^2(N)t_0(N) - V_{NMO}^2(N-1)t_0(N-1)}{t_0(N) - t_0(N-1)} \tag{A.14}$$

Using the same analogy, Equation (A.11)can be written as

$$\frac{1}{t_0(N)}\sum_{i-1}^{N}\left(V_{2i}^4 + H_i\right)\Delta t_i = V_2^4(N)\left[1 - 4A_4(N)t_0^2(N)V_2^4(N)\right] \tag{A.15}$$

(Alkhalifah and Tsvankin, 1995)

The Equation (A.15) can be written as

$$F(N) = V_2^4(N)\left[1 - 4c_3(N)t_0^2(N)V_2^4(N)\right] \tag{A.16}$$

(Alkhalifah and Tsvankin, 1995)

where $F(N)$ is thus a known function of the Taylor series coefficients for the reflection from the Nth boundary. $c_3(N)$ can be calculated using the following Equation (A.17).

$$c_3(N) = \frac{1}{4}\frac{(1-S)}{t_0^2 V_{NMO}^2} \tag{A.17}$$

Now, using the values of $F(N)$ and $F(N-1)$, H_N can be calculated as follows

$$H_N = \frac{F(N)t_0(N) - F(N-1)t_0(N-1)}{t_0(N) - t_0(N-1)} \tag{A.18}$$

But we know that H_N is given by the Equation (A.12)

The Equation (A.12) can be re written as the Equation (A.19)

$$\varepsilon_n = \delta_n - \left(\frac{H_N\left(1-k^2\right)}{8V_0^4\left(1-k^2+2\delta_n\right)}\right) \tag{A.19}$$

Using Equations (A.13)and (A.19) ε and δ can be estimated at each of the layers.