Constructing adaptive, nonuniform Gabor frames from partitions of unity

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ABSTRACT

Adaptive molecular decompositions are useful for analyzing nonstationary signals or images. They can be built up from maximally redundant, uniform partitions of unity, the latter being generated by translations of a single atom to each coordinate sample. These decompositions are adaptive in the sense that they respect a given measure of stationarity in an optimal way. The result is a collection of molecules that generate a one-parameter family of analysis and synthesis frames for the Gabor transform. Each such pair of frames ensures an overall amplitude and energy preserving transformation to the Gabor domain and back. Moreover, for a given atom and stationarity criterion, the redundancy of these frames is minimized, and so the computation time for the Gabor transform is optimized.

INTRODUCTION

Gabor theory sets the stage for nonstationary analysis of a signal, "in which time and frequency play symmetrical parts, and which contains 'time analysis' and 'frequency analysis' as special cases" (Gabor, 1946). Our experience with sound – as in music – calls for a unified mathematical treatment of time and frequency analysis. Indeed, the human ear, combined with the brain's processing abilities, interprets acoustic amplitude information as temporally localized packets of bandlimited spectral information. Conversely, in following a musical score, a musician transforms a time-frequency representation of a signal into temporally varying acoustic amplitude data. Classical Fourier theory fails to explain what we intuitively know: that "frequency content" changes with time. "The reason is that the Fourier-integral method considers phenomena in an infinite interval, and this is very far from our everyday point of view" (Gabor, 1946). These fundamental observations imply that we ought to be modelling the localized time and frequency characteristics of a signal simultaneously; Gabor's milestone theory provides us with the appropriate tools.

The subject of time-frequency analysis is largely based on the short-time Fourier transform (STFT), namely

$$\left[\mathrm{STFT}(f)\right](x,\xi)=\int_{\mathbb{R}^d} f(y)\overline{g(y-x)}e^{-2\pi i\xi\cdot y}dy, \qquad (1)$$

since "most other time-frequency representations can be expressed in terms of the STFT" (Grochenig, 2001). In particular, the (discrete) Gabor transform of a signal (or image) is a discrete sampling - in the time-frequency plane - of the STFT of the signal.

In our applications, the window, *g,* is always real-valued and symmetric about the origin. The *x*-dependence of the STFT of *f* thus corresponds to the central coordinate of the translated atom, $g(t-x)$, which in turn acts by windowing the signal before Fourier transforming. Thus for each *x*, we obtain a *localized spectrum* of the signal. The suite of these spectra, STFT(*f*), is a representation of *f* in the time-frequency plane.

The structure of the STFT of f depends on the choice of window: larger windows achieve better frequency resolution at the cost of reduced time resolution, while smaller windows achieve better time resolution at the cost of reduced frequency resolution. This is a fundamental obstruction to the concept of instantaneous frequency and to time-frequency analysis in general (Grochenig, 2001). The formal expression of this fact is known as Heisenberg's uncertainty principle. The only window that balances this resolution trade-off is the Gaussian. However, the Gaussian is nowhere zero, so in many applications, it is more efficient to use "Gaussian-like" windows of finite extent.

We begin by fixing some notation and introducing the required background from Gabor analysis. This is followed by a discussion of Gabor frames, culminating in a criterion (Theorem 1) for generating such frames. The remainder of the paper is mainly concerned with the construction of a one-parameter family of Gabor frames from a maximally redundant uniform partition of unity. These preliminaries set the stage for our ultimate objective of constructing of a family of adaptive Gabor frames.

Notation and background

Given a (possibly complex-valued) function *g(x)* on the real line, and constants $a,b \in \mathbb{Z}^d$, the *translation operator,* T_{na}, and the *modulation operator*, M_{mb} are defined by

$$\left(T_{na} f\right)(x) = f\left(x - na\right) \quad and \quad \left(M_{mb} f\right)(x) = e^{2\pi imb\cdot x} f\left(x\right), \tag{2}$$

for any $f \in L^2\left(\mathbb{R}^d\right)$. Here, $L^2\left(\mathbb{R}^d\right)$ is the Hilbert space of square-integrable functions, with inner product

$$\left\langle f, h \right\rangle = \int_{\mathbb{R}^d} f\left(x\right) \overline{h}\left(x\right) dx, \tag{3}$$

and norm (or energy)

$$\left\| f \right\| = \left\langle f, f \right\rangle^{1/2} = \left(\int_{\mathbb{R}^d} \left| f\left(x\right) \right|^2 dx \right)^{1/2}. \tag{4}$$

A function f belongs to $L^2\left(\mathbb{R}^d\right)$ if and only if it has finite measurable energy: $\left\| f \right\| < \infty$. Functions in $L^2\left(\mathbb{R}^d\right)$ will be referred to as *signals*, (where $d = 1$) although the theory we develop applies equally well to *images* ($d = 1, 2,$ or 3). In fact, the generalization to any dimension is straightforward, but we consider only a single dimension for simplicity. So the real variable x represents time or space, depending on the desired application, and the corresponding Fourier *dual variable*, ξ, accordingly represents temporal or spatial frequency. The *time-frequency plane* is the continuum of all pairs $(x,\xi) \in \mathbb{R}^{2d}$; in quantum mechanics, these are the position and momentum coordinates of *phase space* ($d = 3$).

We consider also the Hilbert space $\ell^2(\mathbb{Z}\times\mathbb{Z})$, consisting of square-summable sequences of complex numbers, $\left\{c_{m,n}\right\}$, indexed by the integers $m,n \in \mathbb{Z}$. A sequence $\left\{c_{m,n}\right\}$ belongs to $\ell^2(\mathbb{Z}\times\mathbb{Z})$ if and only if

$$\sum_{m,n\in\mathbb{Z}} \left| c_{m,n} \right|^2 < \infty. \tag{5}$$

A *partition of unity* (POU) is a collection $\{\psi_j : j \in \mathbb{Z}\}$ of functions that sums to 1:

$$\sum_{j\in\mathbb{Z}} \psi_j(x) = 1, \quad \text{for each } x \in \mathbb{R}. \tag{6}$$

With respect to such a POU, any function $f \in L^2(\mathbb{R})$ can be represented as the superposition of its *windowed components*,

$$f_j(x) = \psi_j(x) f(x), \tag{7}$$

as follows:

$$f(x) = \sum_{j\in\mathbb{Z}} \psi_j(x) f(x) = \sum_{j\in\mathbb{Z}} f_j(x). \tag{8}$$

Given any collection of functions, $\{\varphi_j : j \in \mathbb{Z}\}$ satisfying

$$0 < A \le \sum_{j\in\mathbb{Z}} \varphi_j(x) \le B < \infty, \quad (\text{for each } x \in \mathbb{R}) \tag{9}$$

for some constants $A, B > 0$, the collection can always be normalized to give a POU. Indeed, by setting

$$\psi_j(x) = \frac{1}{\sum_{i\in\mathbb{Z}} \varphi_i(x)} \varphi_j(x), \tag{10}$$

we have

$$\sum_{j\in\mathbb{Z}} \psi_j(x) = 1, \quad \text{for each } x \in \mathbb{R}. \tag{11}$$

Fundamentals of Gabor frames

Before discussing nonuniform Gabor frames, we provide an outline of the relevant results from Gabor frame theory. The presentation is largely based on that of Grochenig (2001), and further details may be found in Feichtinger and Strohmer, (1998) and the insightful paper of Denis Gabor (1946).

Given $a, b > 0$, a system $\{g_{m,n}\} = \{M_{mb} T_{na} g\}$, $(m, n \in \mathbb{Z})$ is a *Gabor frame*, or *Weyl-Heisenberg frame* for $L^2(\mathbb{R})$ if there exist constants $A, B > 0$ for which

$$A\|f\|^2 \le \sum_{m,n\in\mathbb{Z}} \left|\langle f, g_{m,n}\rangle\right|^2 \le B\|f\|^2, \quad \left(f \in L^2(\mathbb{R})\right). \tag{12}$$

The function *g* is called a *Gabor atom* (or window). If $A = B$, then $\{g_{m,n}\}$ is called a *tight frame*. Any Gabor frame $\{g_{m,n}\}$ has the property that the *analysis mapping*, or *Gabor transform*

$$V_g : L^2(\mathbb{R}) \to \ell^2(\mathbb{Z}\times\mathbb{Z}), \tag{13}$$

defined by

$$\left(V_g f\right)(m,n) = \langle f, g_{m,n}\rangle = \int_{\mathbb{R}} f(x)\, g(x - na)\, e^{-2\pi i m b x} dx, \tag{14}$$

and its adjoint, the *synthesis mapping*, or *Gabor expansion*:

$$V_g^*:\ell^2\left(\mathbb{Z}\times\mathbb{Z}\right)\to L^2\left(\mathbb{R}\right) \tag{15}$$

given by

$$V_g^*\left\{c_{m,n}\right\}(x)=\sum_{m,n\in\mathbb{Z}} c_{m,n}\, g_{m,n}(x), \tag{16}$$

are continuous linear operators. Consequently, the Gabor *frame operator*, S_g, defined by

$$S_g=V_g^*V_g:L^2\left(\mathbb{R}\right)\to L^2\left(\mathbb{R}\right) \tag{17}$$

$$S_g f=\sum_{m,n\in\mathbb{Z}}\left\langle f,g_{m,n}\right\rangle g_{m,n}, \tag{18}$$

is also a continuous linear operator. Given any Gabor frame, $\left\{g_{m,n}\right\}$, the associated frame operator (18) is always invertible (as implied by condition (12)). The most important consequence of this fact is that any $f\in L^2\left(\mathbb{R}\right)$ can be reconstructed from its Gabor coefficients as

$$f=\sum_{m,n\in\mathbb{Z}}\left\langle f,g_{m,n}\right\rangle S_g^{-1}g_{m,n}, \tag{19}$$

which follows from applying S_g^{-1} to both sides of equation (18). The resulting system, $\left\{\gamma_{m,n}=S_g^{-1}g_{m,n}\right\}_{m,n\in\mathbb{Z}}$, is also a Gabor frame, called the *dual frame*. It is generated by translations and modulations of the *dual atom,* or *synthesis* window, $\gamma=S_g^{-1}g$. Note that the choice of synthesis window is not unique in general. Many other windows can be substituted for γ that will allow reconstruction of the signal from its Gabor coefficients, but the dual atom happens to be the synthesis window of least energy.

There has been a great deal of research into the existence of Gabor frames, particularly in finding sufficient conditions on g, a, and b for which the frame inequalities (12) can be satisfied (see Grochenig, 2001, and Feichtinger and Strohmer, 1998). However, as the following theorem will show, this is not an issue in the applied setting.

Theorem 1: (Daubechies et al., 1986) *Suppose that* $g\in L^\infty\left(\mathbb{R}^d\right)$ *is supported on the cube* $Q_L=[0,L]^d$. *If* $a\le L$ *and* $b\le 1/L$, *then the frame operator is the multiplication operator*

$$Sf(x)=\left(\frac{1}{b^d}\sum_{n\in\mathbb{Z}^d}\left|g_n(x)\right|^2\right)f(x), \tag{20}$$

where $g_n(x)=\left(T_{na}g\right)(x)$. *Consequently,* $\left\{g_{m,n}\right\}$ *is a Gabor frame, with frame bounds* $A=\alpha/a^d$ *and* $B=\beta/b^d$, *if and only if*

$$0<\alpha\le\sum_{n\in\mathbb{Z}}\left|g_n(x)\right|^2\le\beta \text{ a.e.} \tag{21}$$

Moreover, $\left\{g_{m,n}\right\}$ *is a tight frame if and only if* $\sum_{n\in\mathbb{Z}}\left|g_n(x)\right|^2=constant$ *a.e.*

The sufficient conditions for $\{g_{m,n}\}$ to be a frame are easily satisfied in practice. Indeed, any continuous function *g* on a closed interval is bounded, so it belongs to $L^{\infty}\left(\mathbb{R}^{d}\right)$. Also, the distance *a* between window centres cannot exceed the width, *L*, of the window: otherwise, part of the signal being analyzed would be neglected. Moreover, since we will be using the FFT, we have that $b = 1/L$. These remarks, and the criterion (21), will be further discussed below.

Nonuniform Gabor frames from maximal POU frames

Implementation of Gabor transforms with FFT's

In applications, efficiency is gained by implementing the Gabor transform with the fast Fourier transform (FFT). This specializes the definition of the Gabor transform – to the extent that the sampling interval, *b,* (of the frequency coordinate) is predetermined by the FFT and the window's length.

Window selection

As we have mentioned, the Gaussian window is optimal in the sense that it achieves the best possible time-frequency resolution. For numerical efficiency, however, we recommend using a finite-length window that has a similar shape to the Gaussian. As shown in Bale et al., (this volume) the Gaussian decays to below machine precision after only several halfwidths, so a suitably truncated Gaussian could be used. The window should also be sufficiently smooth, to avoid artefacts such as spectral ringing; and it should be at least wide enough so that its sampled version faithfully represents it.

Generation of maximally redundant POU Gabor frames

Consider any signal $f \in L^{2}\left(\mathbb{R}\right)$, and select a window $\tilde{g}$ according to the above prescription. Suppose the signal is sampled at M points, with a uniform sample spacing of Δx units, and that $\tilde{g}$ has N points, with the same sample spacing. Then the sampling interval in the frequency domain is given by

$$b = \frac{2}{\left(N-1\right)} f_{Nyquist} = \frac{1}{\Delta x\left(N-1\right)}. \tag{22}$$

Next, consider the family of translates:

$$\left\{\tilde{g}_{n} = T_{na}\tilde{g} : n = 0,1,2,\ldots,M-1\right\}, \tag{23}$$

with $a = \Delta x$. This is a suite of windows, one centred at each sample point. By construction, it satisfies the inequalities (9). Thus, by normalizing as in (10) if necessary, we can replace the family (23) with a POU, namely:

$$\left\{g_{n} : n = 0,1,2,\ldots,M-1\right\}, \text{ where}$$

$$g_{n} = \frac{\tilde{g}_{n}}{\sum_{m=0}^{M-1}\tilde{g}_{m}}. \tag{24}$$

Remark. Note that although this defines a partition of unity, it will not be *uniform* near the edges of the domain, $[x_0, x_{M-1}]$, because the sum is over a finite set. One way around this problem is to pad the input data with zeros, to displace these edge effects away from the original domain.

The area of a cell in the corresponding sampling grid in the time-frequency plane is thus

$$ab = \frac{1}{(N-1)}. \tag{25}$$

In this case, we chose the distance a between windows to be Δx, but in general, we would have:

$$\Delta x \le a \le (N-1)\Delta x. \tag{26}$$

Multiplying (26) by expression (21) for *b*, we see that the area, *ab* is bounded by

$$\frac{1}{N-1} \le ab \le 1. \tag{27}$$

The *redundancy* is defined as $1/ab$, and the system is said to be *oversampled* by a factor of $1/ab$, or, in this case, N-1. This is the maximum possible oversampling rate, and for this reason we call the POU *maximally redundant*.

Evidently, the hypotheses of the first part of Theorem 1 are satisfied for this maximally redundant system (24). To prove that our system is in fact a Gabor frame, it remains to verify condition (21). First, note that $g(x) \le 1$, so that $[g(x)]^2 \le g(x)$. Thus

$$\sum_{n=1}^{M} |g_n(x)|^2 \le \sum_{n=1}^{M} |g_n(x)| = 1 \equiv \beta. \tag{28}$$

For the lower bound, we need only check that $\sum_{n=1}^{M} |g_n(x)|^2 \ne 0$ for all x. This is clear, since otherwise there would be some $x_0 \in \mathbb{R}$, for which $|g_n(x_0)|^2 = 0 = g_n(x_0)$ for all n, implying the following contradiction:

$$1 = \sum_{n=1}^{M} |g_n(x_0)| = 0. \tag{29}$$

Thus (24) represents a maximally redundant POU Gabor frame.

One-parameter families of Gabor frames

Suppose $\{\varphi_n = T_{na}\varphi : n \in \mathbb{Z}\}$ is a partition of unity, and choose any $p \in [0,1]$. If we set $g(x) = [\varphi(x)]^p$, then the collection

$$\{g_{m,n} = T_{na} M_{mb} g : m,n \in \mathbb{Z}\} \tag{30}$$

evidently defines a Gabor analysis frame. We claim that a corresponding synthesis frame is given by

$$\{\gamma_{m,n} = T_{na} M_{mb} \gamma : m,n \in \mathbb{Z}\}, \tag{31}$$

where γ is defined as $\gamma(x)=\left[\varphi(x)\right]^{1-p}$. Indeed, let $f\in L^2\left(\mathbb{R}^d\right)$ be given, and consider the Gabor coefficients of f, namely:

$$c_{m,n}=\int f\left(\tilde{x}\right)g\left(\tilde{x}-na\right)e^{-2\pi imb\tilde{x}}d\tilde{x}\,. \tag{32}$$

Summing these coefficients against the frame (29) yields:

$$\begin{aligned}\sum_{m,n\in\mathbb{Z}}c_{m,n}\gamma(x-na)e^{2\pi imbx} &= \sum_{m,n\in\mathbb{Z}}\left(\int f\left(\tilde{x}\right)g\left(\tilde{x}-na\right)e^{-2\pi imb\tilde{x}}d\tilde{x}\right)\gamma(x-na)e^{2\pi imbx}\\ &=\int f\left(\tilde{x}\right)\sum_{n\in\mathbb{Z}}g\left(\tilde{x}-na\right)\gamma(x-na)\sum_{m\in\mathbb{Z}}e^{2\pi imb\left(x-\tilde{x}\right)}d\tilde{x}\\ &=\int f\left(\tilde{x}\right)\sum_{n\in\mathbb{Z}}g\left(\tilde{x}-na\right)\gamma(x-na)\delta\left(x-\tilde{x}\right)d\tilde{x}\\ &=f(x)\sum_{n\in\mathbb{Z}}g(x-na)\gamma(x-na)\\ &=f(x)\sum_{n\in\mathbb{Z}}\varphi^{p}(x-na)\varphi^{1-p}(x-na)\\ &=f(x),\end{aligned} \tag{33}$$

which is what we set out to prove, i.e., that (30) and (31) form a dual pair of Gabor frames. We have used the fact that the sum, $\sum_{m\in\mathbb{Z}}\exp(2\pi imb(x-\tilde{x}))$ under the integral in (31), has the same action as the Dirac-delta distribution concentrated at $\tilde{x}=x$. Indeed, the sum represents the inverse DFT, evaluated at $x-\tilde{x}$, of the constant function, 1.

Remarks

The choice of p assigns a relative balance between the mass distributions of the analysis and synthesis atoms. Since the atoms are members of a partition of unity, their amplitudes are bounded above by one. Thus, as the exponent $p\in[0,1]$ decreases, the atomic mass increases and spreads simultaneously.

From Theorem 1, it is easy to see that for $p=1/2$, the resulting frame is tight; that the converse also holds remains a conjecture. Evidently, this is also the only p for which the analysis and synthesis frames are identical.

In the case of a Gaussian of halfwidth σ, it is easily seen that exponentiation by p results in a new Gaussian, of halfwidth, $\sigma/\sqrt{p}$. A suite of such Gaussians, parameterized by $p\in[0,1]$, is displayed in Figure 1. Note that in the limit as $p\to 0$, the family degenerates to the constant function, $g(x)=1$. Note that the Gaussian can only form an approximate partition of unity. However, the error can be made as small as desired by increasing the ratio σ/a of the halfwidth to the spacing between translates. Margrave and Lamoureux (2001) showed that this error is dominated by a sinusoid with amplitude that decreases exponentially with the ratio σ/a. Alternatively, the suite of

Gaussians can be normalized, as in (10), at the cost of a deviation from optimal time-frequency resolution.

In the case of a compactly supported atom, for example a Lamoureux window of order k, (a bell-shaped polynomial spline of order C^k) a little caution is required. Figure 2 displays a suite of atoms generated by exponentiating a Lamoureux window of order four. As is evident by inspection of the endpoints of the windows, the order of differentiability of the resulting atom decreases as the exponent $p \in (0,1]$ decreases (any $p \leq 1/k$). However, in practice this is easily resolved by suitably adjusting the order of the Lamoureux window for a given choice of p.

Adaptive nonuniform Gabor frames

Any maximal POU can be adapted to yield a nonuniform partition of unity, in such a way that the essential nonstationarity of the particular problem is respected. The idea is to form *molecules,* or macro-windows, by summing neighbouring atoms over regions that meet a local stationarity measure. The result is an adaptive, nonuniform partition of unity, or *molecular decomposition.* These molecules can then be used to generate a one-parameter family of *nonuniform* Gabor analysis and synthesis frames, in the same way that they were generated above from atomic partitions of unity (equations (30) and (31)).

For example, a stationarity measure could be given by a fixed threshold, against which the deviation of a given velocity field from its mean over the current molecule is compared. Thus, if this deviation is less than the threshold, the current atom is conjoined to the current molecule. Each molecule gathers atoms until it encounters a large enough velocity anomaly.

A 1D example of such a molecular decomposition is shown in Figure 3. The illustrated velocity model is meant to be as general as possible, ranging from a simple constant to a nowhere-continuous random function. Note how the atoms cluster near the larger local variations in the velocity. Everywhere else, the atoms ‘bond’ to form molecules; and their size varies inversely with the magnitude of the local variation of *v*. Note also that the molecules sum to unity, as they should.

SUMMARY

We showed how to construct an adaptive molecular decomposition from a maximally redundant partition of unity, which in turn was generated by translations of a single atom to each coordinate sample. The decomposition was adaptive in the sense that it respected a given measure of stationarity. This collection of molecules then gave rise to a one-parameter family of analysis and synthesis frames for the Gabor transform. Each such pair of frames ensured an overall amplitude and energy preserving transformation to the Gabor domain and back. Moreover, the redundancy of these Gabor frames was minimized with respect to the choice of atom and the stationarity criterion, which in turn minimized the computation time for the Gabor transform.

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FIGURES

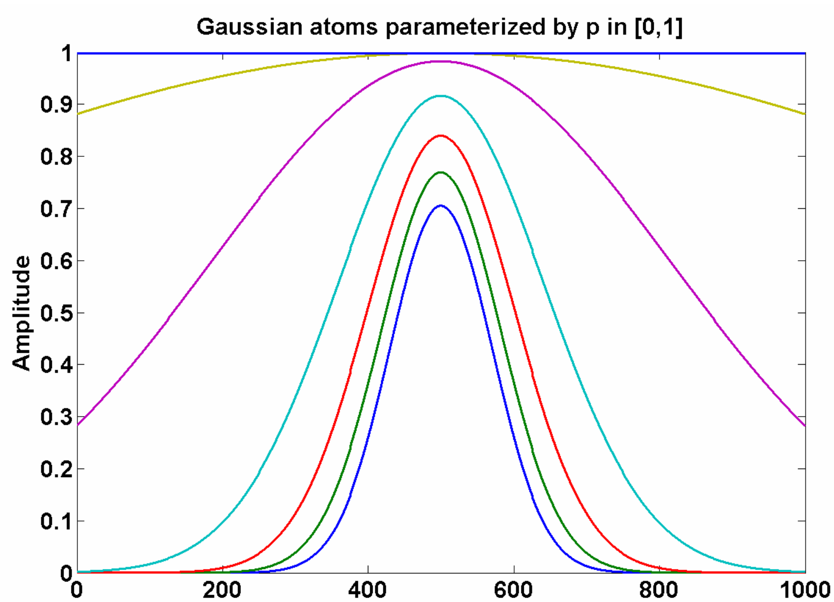

FIG. 1: A Gaussian atom of halfwidth σ, (lower blue) and the family of Gaussians, of halfwidth $\sigma/\sqrt{p}$, obtained by raising the initial Gaussian to the power $p \in [0,1]$. As p decreases, the atoms gain mass, and their mass distribution spreads. In the limit as $p \to 0$, the family converges to constant function, 1.

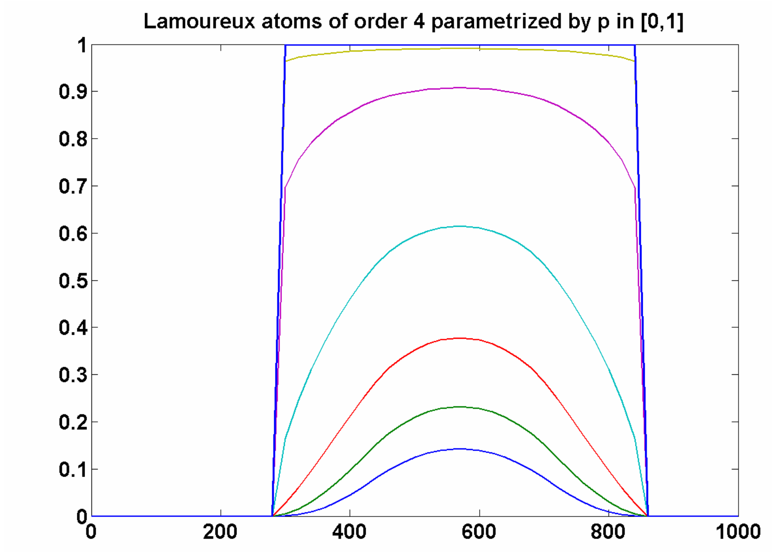

FIG. 2: A Lamoureux atom (lower blue) and the family of windows obtained by raising the initial atom to the power $p \in [0,1]$. As *p* decreases, the atoms gain mass and their spreading mass-distributions are confined to the original domain. At *p* = 1/4, (cyan) the window is no longer differentiable at the two boundaries. However, this can be resolved for any given *p* by starting with a sufficiently high order atom. In the limit as $p \to 0,$ the family converges to a boxcar.

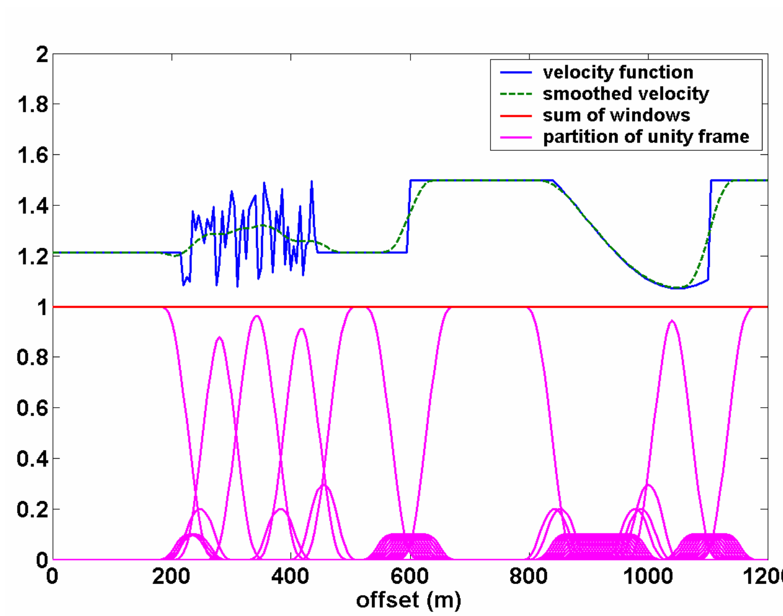

FIG. 3: A demonstration of how an adaptive, nonuniform partition of unity Gabor frame (magenta) can be obtained from a maximal partition of unity, (the result of translating any of the smallest windows to each offset sample-point) which reflects the local nonstationarity of a velocity model. The piecewise-defined velocity model (blue) represents all types of velocity variation: constant, random, a jump discontinuity between two constants, and a jump from smooth to constant. The smoothed velocity (dashed green), obtained by convolving the velocity model with a fundamental atom, (any of the smallest magenta windows) acts as a visual aid in comparing the molecules (magenta) to the local velocity variations.