Amplitude within the Fresnel zone for the zero-offset case

Shuang Sun and John C. Bancroft

ABSTRACT

Reflection energy from a linear reflector comes from an integrant over an aperture often described by the Fresnel zone. Within the Fresnel zone, the diffraction energy constructively builds the reflection energy. Recognition of the amplitude variation within the Fresnel zone will contribute to migration theory. This paper uses diffraction theory to explain the amplitude within the Fresnel zone and the reflector edge effect on amplitude. This recognition of the response near the edge of the reflector is required to when designing a minimum migration aperture.

INTRODUCTION

Geophysicists commonly recognize that a sizeable portion of a reflector is involved in causing a reflection, as seen on a seismic trace, but the area extent is usually not calculated and hence not appreciated. Commonly, concepts are simply transferred from classical physical optics and called Fresnel zone effects. The first Fresnel zone is the portion of a reflector from which reflected energy can reach a detector within the first one-half of the reflection; contributions from this zone add constructively to produce a reflection. There is a second Fresnel zone from which energy arrives, delayed one-half to one cycle, adding destructively to the energy from the first zone. Similarly, there is a third zone, which adds constructively, and so on. When the contributions of all zones are added together, a reflection from a plane reflector is one-half of the response of the first Fresnel zone, the effects of all subsequent zones cancelling each other, and the adjective 'first' is often dropped.

Attention to the Fresnel zone has increased as more efforts are devoted to improving seismic resolution. Fresnel zone considerations are the essence of horizontal resolution. Attention has also been increasing because of greater awareness of three-dimensional effects. Hagedoorn (1954) points out that the reflections area of the interface, and therefore vertical resolution can also be thought of as a Fresnel-zone problem.

The geometry for calculating the radius of the first Fresnel zone is shown in Figure 1 for coincident source and receiver and constant velocity using the Pythagorean theorem,

$$(z + \lambda/4)^2 = z^2 + R^2, \tag{1}$$

where z is the reflector depth, λ the wavelength, and R the radius of the first Fresnel zone. Solving for R gives,

$$R = (\lambda z / 2 + \lambda^2 / 16)^{\frac{1}{2}} \approx \sqrt{\lambda z / 2} \approx \sqrt{vz / 2f} , \qquad (2)$$

with the λ^2 term usually being small enough to be neglected. This can also be expressed in terms of two-way vertical traveltime t_0, velocity v, and wavelet period τ using the familiar relationships $z = vt_0/2$ and $\lambda = v\tau$. Thus

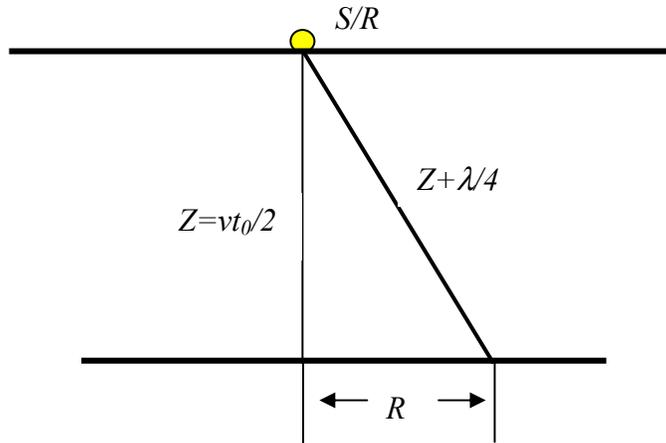$$R = (v/2)\sqrt{t_0 \tau} . \qquad (3)$$

FIG 1. Geometry for calculating radius of first Fresnel zone.

For example, if the subsurface has a constant velocity, the Fresnel zone varies with frequency and depth, as shown in Figure 2. With linear velocity, which can be found at a classic tertiary sedimentary basin, the Fresnel zone size varies with depth and frequency as shown in Figure 3.
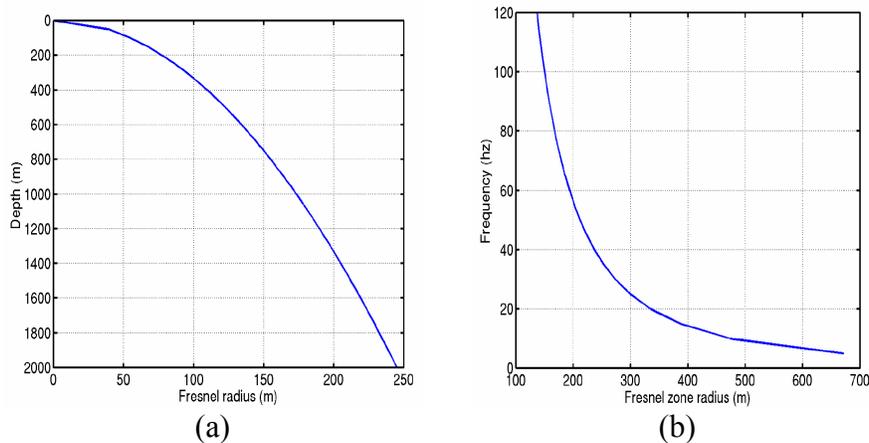
(a)   (b)

FIG 2. Fresnel zone varies with depth and frequency with constant velocity v=3000m/s.a) Fresnel zone varies with depth as frequency 50hz; b) Fresnel zone varies with frequency at depth 1500m.
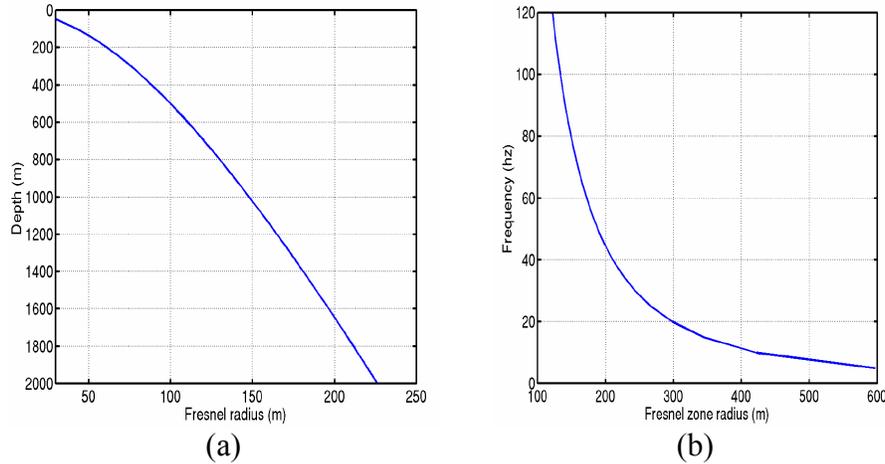
FIG 3. Fresnel zone varies with depth and frequency with linear velocity v=1800+0.8z. a) Fresnel zone varies with depth as frequency 50hz; b) Fresnel zone varies with frequency at depth 1500m.

FRESNEL ZONE AND SOURCE TYPE

The Fresnel zone effect described above is observed in optics using monochromatic light shining through holes of different sizes. But in seismology the source type is broadband wavelet. In the next section the Fresnel zone will be discussed as it relates to source type.

To investigate the relationships between Fresnel zone and the source type, the easiest horizontal circular reflector and zero-offset data are used as Figure 4 shows. The calculation was done in the time domain using the methods of Trorey (1970):

$$R(t) = \frac{1}{T_0} f(t - T_0) - \frac{z}{T\xi} f(t - T) . \tag{4}$$

where $R(t)$ is the reflected signal, $f(t)$ is the source wavelet which includes the reflection coefficient, z is the depth of the reflector, T_0 is two-way normal-incidence time to reflector, ξ is the distance from source to the edge of the reflector and T is two-way travel time to the edge of the reflector. From equation (4), the first term is the desired reflection, the source wavelet retarded by the time T_0, the second term is the polarity reversed source wavelet with retardation T, and from the edge of the reflector corresponding with a truncation effect caused by the finite integration area. For a reflector with infinite radius the truncation term vanishes, then $R(t)$ is exactly equal to the desired reflection.
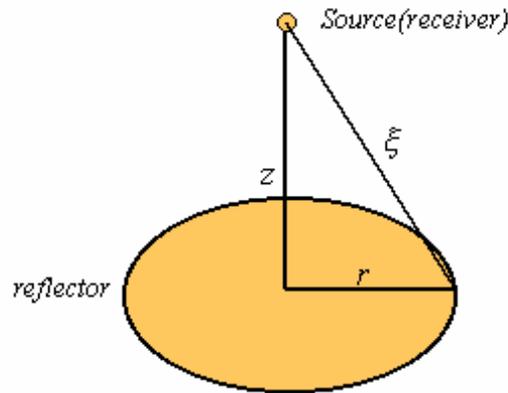
FIG 4. Easiest horizontal circular reflector example.

Monochromatic wavelet

Using equation (4) the energy of the reflected signal is a function of the reflector radius can be computed. For a monochromatic source, like sinusoid function, how the reflected energy relates to reflector size is shown in Figure 5.

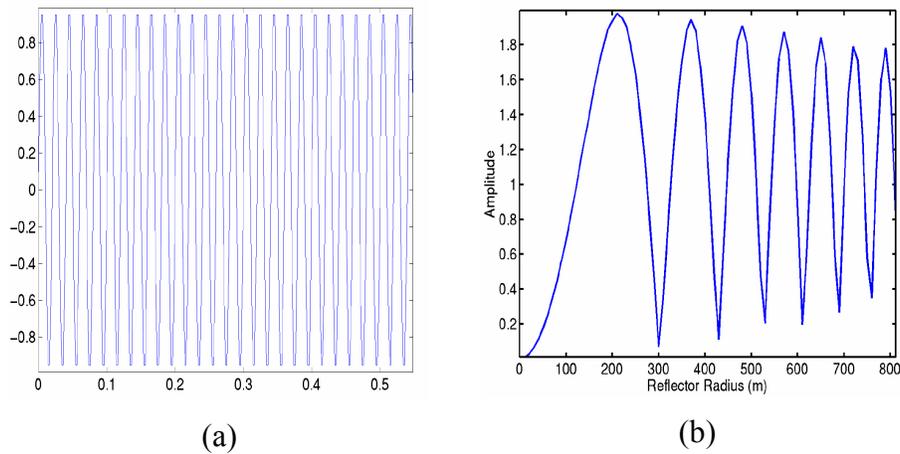(a)   (b)

FIG 5. Sinusoid source function and reflected energy. (a) The sinusoid source function; (b) The reflected energy with the reflector radius.

For a monochromatic signal, such as Figure 5a, the energy is oscillating. The first maximum of the curve defines the boundary of the first Fresnel zone, the other extreme ties correspond with the boundaries of the higher order Fresnel zones.

Ricker wavelet source

The Ricker wavelet used here is expressed as equation (5) (Sheriff, 1995),

$$f(t) = (1 - 2\pi^2 \upsilon^2 t^2) e^{-\pi^2 \upsilon^2 t^2} . \tag{5}$$

where υ is the dominant frequency and t is time. For this kind of source wavelet, the reflected energy is shown in Figure 6.
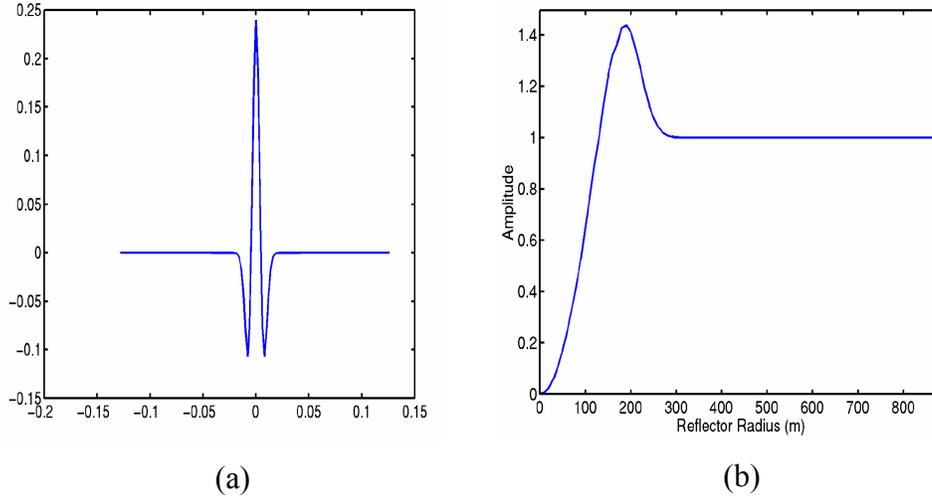

| (a) | (b) |

FIG 6. Ricker source wavelet and reflected energy. (a) Ricker source wavelet; (b) Reflected energy with reflector radius.

The energy in a Ricker source wavelet builds up to a maximum as it does also for the other signals, but it quickly stabilizes. Thereafter, the energy decreases monotonically after it reaches to maximum is because of the vanishing amplitude factor of the truncation term in equation (4).

Delta function source

With the Delta source function, the energy shows more oscillating than the Ricker wavelet, as Figure 7 shows. With a different frequency band, the Fresnel zone, which is the energy reaches its maximum, has a different size. The more broad the frequency slope, the smaller the size of the Fresnel zone.
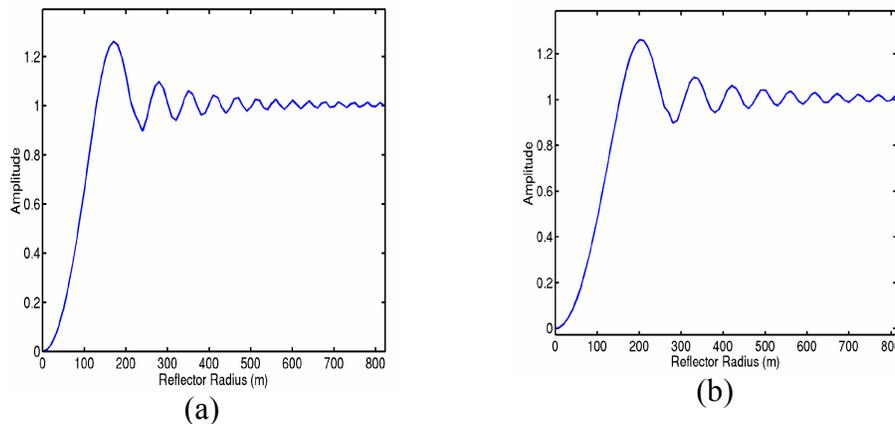

| (a) | (b) |

FIG 7. Reflected energy relates to reflector radius with Delta source. (a) Frequency is 0-150hz;

(b) Frequency is 0-120hz.

In summary, with the broadband data as with seismology, the Fresnel zone corresponds to the reflected energy reaching its maximum. Restricting the reflector radius to smaller then the Fresnel zone will result in a change of the reflected wavelet with respect to the input wavelet. The figures of reflected energy show that for removing the truncation effect (second term in equation 4) or edge effect, the reflector radius should be larger than the Fresnel zone at some point of the energy getting stable. In the next sectiont this phenomena will show again with diffraction theory that for true amplitude modelling, the Fresnel zone effect, i.e., the edge effect, must be considered.

AMPLITUDE VARIATION WITHIN THE FRESNEL ZONE IN THE ZERO-OFFSET CASE

To investigate the relationship between the Fresnel zone and the reflection strength, the easiest model, i.e., zero-offset 2D case data, is used. According to diffraction theory (Berryhill, 1977), the diffraction amplitude at zero shot-geophone distance due to the termination of a planar reflector can be calculated by convolving the reflection wavelet with a time-domain operator called the normalized diffraction response. The diffraction response at zero-offset can be written in the forms

$$p_n(t) = f(t) * D_0(t) U(t - t_e), \tag{6}$$

and

$$D_0(t') = \frac{\cos\theta_e}{\pi} \frac{t_e^2}{(t'+t_e)^2} \frac{d}{dt'}[\arctan(\sqrt{t'(t'+2t_e)}/(t_e\sin\theta_e))]. \tag{7}$$

in which

t_e, is the minimum two-way travel time to the edge of the reflector;

$t'=t-t_e$, the time measured after onset time t_e ;

θ_e, the angle between the normal to the reflector and the raypath of the minimum traveltime to the edge of the reflector;

$f(t)$, the source wavelet function and

$U(t-t_e)$, the unit step function to maintain zero value prior to t_e.

D_0 operator

Using equation (7) the D_0 operator calculation requires knowledge of only two parameters, t_0 and θ_0. In practical applications, D_0 is evaluated numerically with t' taking values which are multiples of some discrete time sample interval. The derivative term in equation (7) then becomes a difference, and the result is suitable for digital convolution with a seismic wavelet. Figure 8 presents the results of evaluating D_0 at one value of $t_e=1.0$ for four different choices of θ_0.
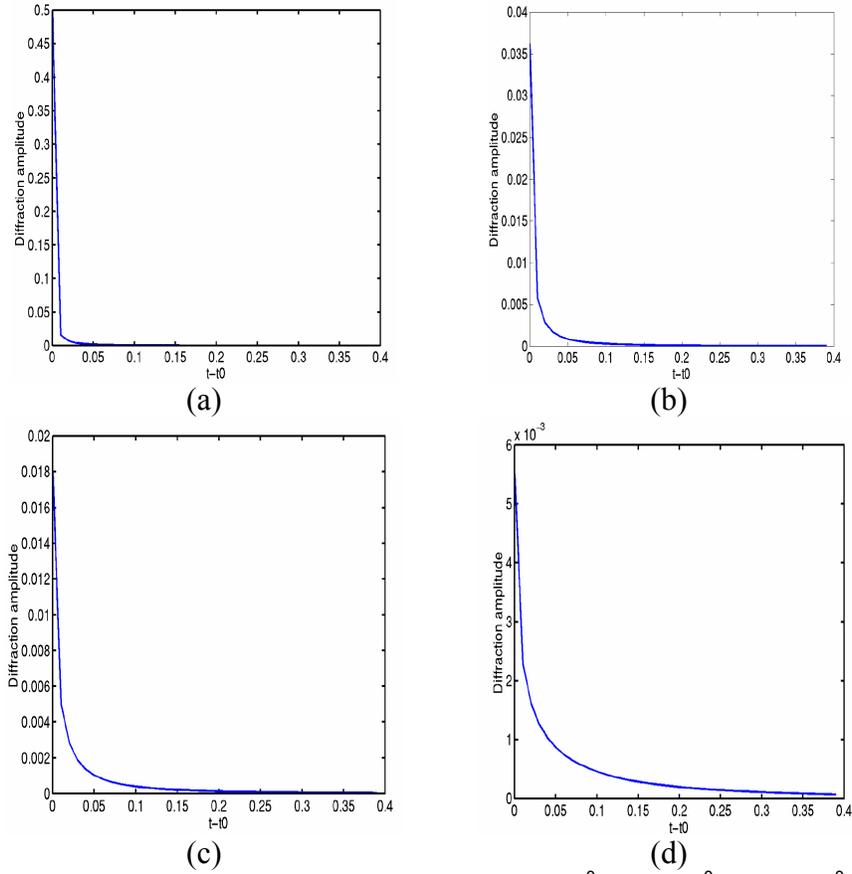
FIG 8. D_0 operator with same $t_e=1.0$ but different θ_0. (a) $\theta_0=0^0$; (b) $\theta_0=5^0$; (c) $\theta_0=10^0$; (d) $\theta_0=30^0$.

When θ_0 is 0 degrees, D_0 is a one-sample spike with a magnitude equal to exactly one-half. Convolving this with any wavelet simply reproduces the wavelet with one-half its original amplitude. As θ_0 increases, i.e. the reflector increases, and as the distance increases from the source to the edge of the reflector, the height of the initial peak decreases quickly. In other words, when the reflector reaches its certain size, the effect from the reflector edge can be ignored.

2D reflector model

Trorey created the 2D model in 1970 considering the diffraction at the edge of the reflector, in this paper also called the edge effect. V is reflection and D_1 and D_2 are diffraction from each side of the edge. The total response, i.e. reflection together with diffraction is D. Forward modelling this 2D model, the Ricker wavelet was convolved with the response D, and after convolution the amplitude corresponding to reflection coefficient changed its value to *0.024.* For this kind of 2D model, there are three typical cases which need to be investigated: 1. The reflector size is smaller than the Fresnel zone; 2. The reflector size equals to the Fresnel zone; 3. The reflector size is larger than the Fresnel zone.
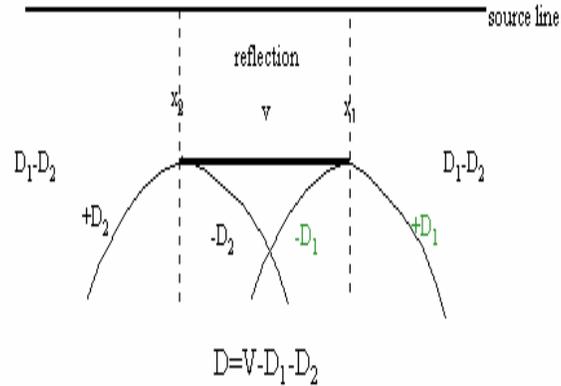
FIG 9. 2D model for test was created by Trorey (1970).

Reflector size smaller than the Fresnel zone

The edge effect is greatest on the amplitude when the reflector is smaller than the Fresnel zone as shown in Figure 10. The reflector is *100m* long and locates from *2400-2500m* along the surface coordinates *x*. The reflected and diffracted responses of the reflector with zero-offset is displayed in Figure 11, assuming the sources along the surface.
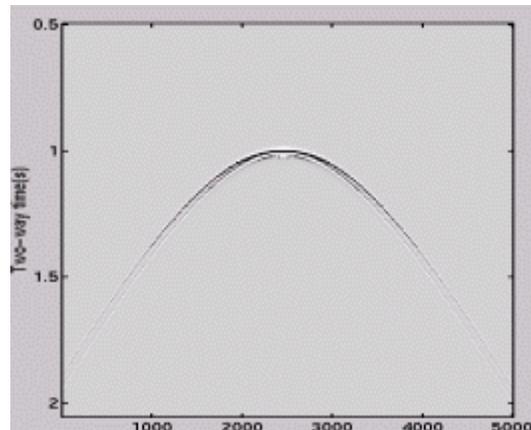

FIG 10. Reflected and diffracted responses of reflector assuming source along the surface.

It is hard to tell if it is a diffraction point or a reflector. Thus, when the reflector is smaller than the Fresnel zone, in seismic section the resolution is totally destroyed. In this case, not only is the response smeared but also the amplitude of the response is neither reflection nor diffraction. Figure 11 displays the amplitude response.
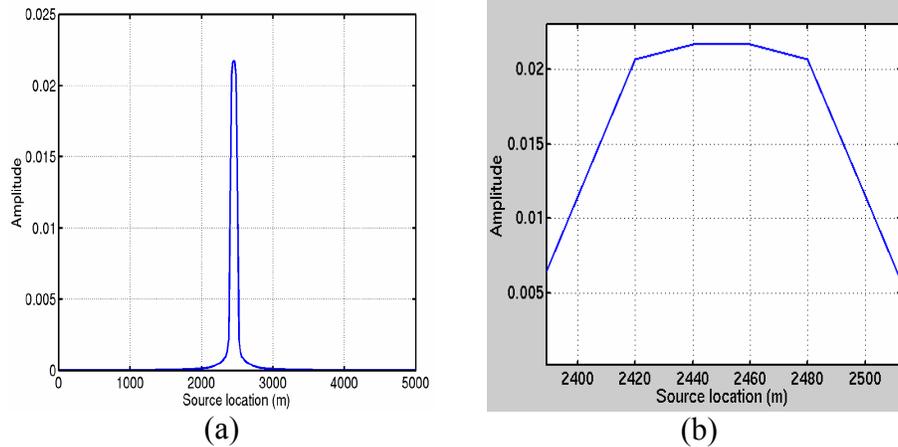
FIG 11. The amplitude response of a reflector smaller than the Fresnel zone. a) Amplitude in total; b) zoomed section.

Reflector equals the Fresnel zone

When the reflector equals the Fresnel zone, the edge is further from the source, compared to the case above, which can lead to a better image or better resolution. In fact, for a larger reflector the diffraction effect is insignificant, especially in the middle of the reflector. The reflector is clearer, as in Figure 12.
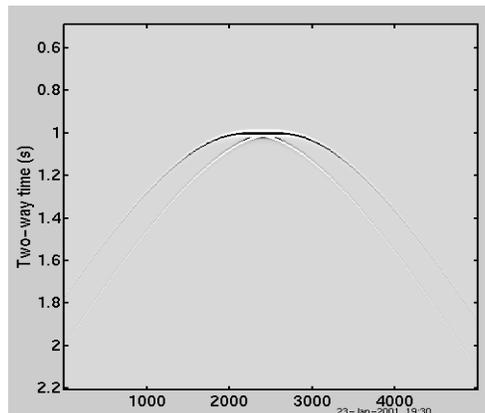

FIG 12. Response of reflection together with diffraction of reflector equal to the Fresnel zone.

The amplitude of diffraction effected on reflection can be seen as amplitude reaches its maximum in the middle of the reflector, which is the furthest point from the edge. The maximum at this point is the result of diffraction constructively contributing to the reflection, and it is greater than the reflection effect. Also, the response, which is near the reflector edges, is decreased as in the above case and the result is that the diffraction destructively contributes to reflection. Figure 13 displays this in detail.
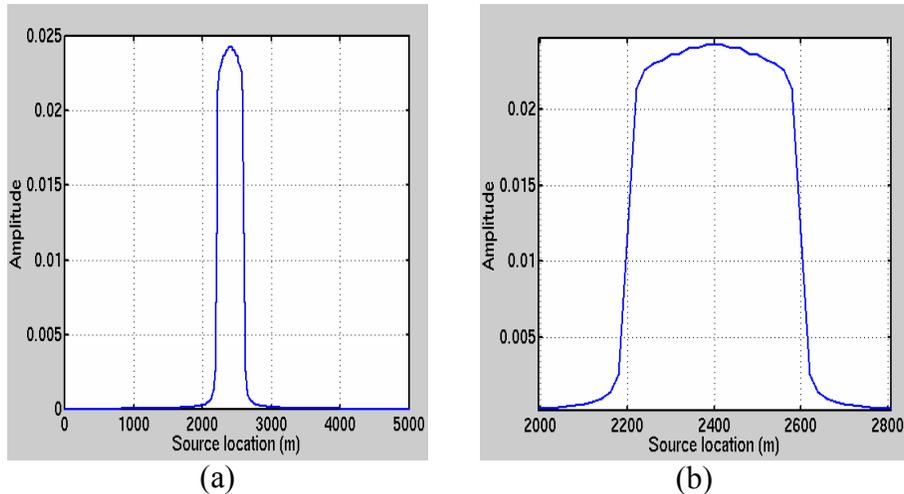
(a)   (b)

FIG 13. The amplitude response of the reflector equal to the Fresnel zone. a) Whole view; b) zoomed view.

Reflector larger than the Fresnel zone

In most cases, the reflector in the subsurface is larger than the Fresnel zone but still the diffraction exists at the edge of the reflector. Investigation of the amplitude response has a practical use when imaging the subsurface. For migration the diffraction should be eliminated. Therefore recognition of the response near the edge of the reflector is important in order to design the migration aperture and to eliminate the diffraction effect.
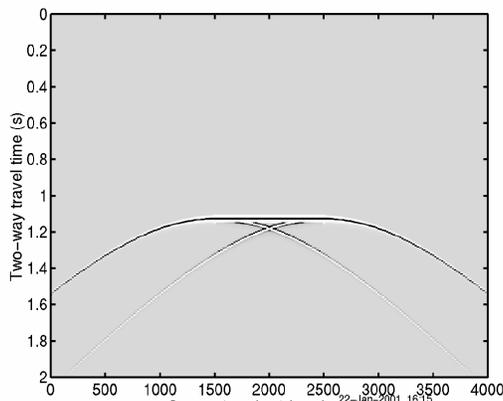

FIG 14. Response of the reflector larger than the Fresnel zone.

From amplitude response in Figure 15, the diffraction constructively contributes to reflection when the reflection point is at the distance equal to Fresnel zone from the edge. Nearer amplitude is smaller than reflection. When the distance between the reflection points and reflector edge is further than the Fresnel zone, the amplitude response has no diffraction effect on them, thus the amplitude is the reflection strength. These phenomena suggest that the amplitude near the diffraction is not reflection or diffraction, and more attention should be paid to getting rid of the diffraction.
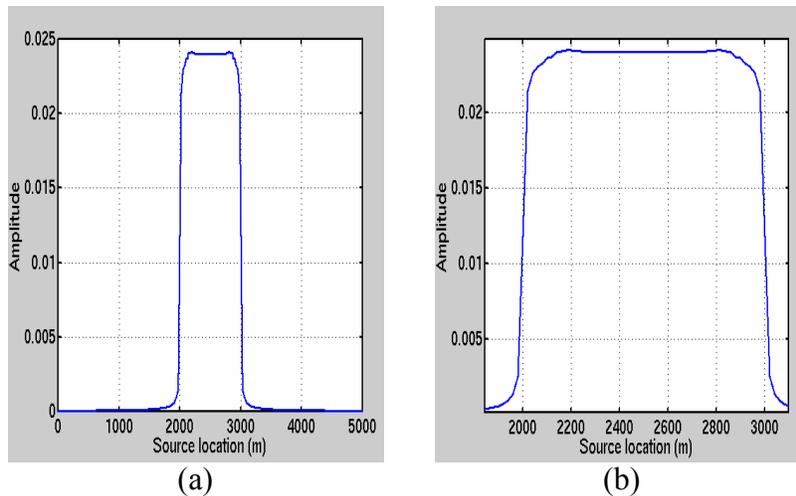
FIG 15. Amplitude response of the reflector larger than Fresnel zone. a) Whole view; b) zoomed view.

CONCLUSIONS

Fresnel zone size is determined by dominant frequency, depth, velocity and the bandwidth of the signal.

For a reflector larger than the Fresnel zone we can get the reflectivity only if the source is located out of the Fresnel radius from the reflector edge.

For a reflector smaller than the Fresnel zone the reflectivity deteriorates due to the diffraction effect from the reflector edge.

REFERENCES

Berryhill, J.R., 1977, Diffraction response for nonzero separation of source and receiver: Geophysics, 42, 1158-1176.
Claerbout, J.F., 1985, Imaging the Earth's Interior: Blackwell Scientific Publications, Available over Internet.
Hagedoorn, J.G., 1954, A process of seismic reflection interpretation: Geophysics Prosp., v.2, 85-127
Sheriff, R.E. and Geldart, L. P., 1995, Exploration Seismology: Cambridge University Press.
Sheriff, R.E., 1980, Nomogram for Fresnel-zone calculation: Geophysics, V 45, P. 968-972.
Trorey, A.W., 1970, A simple theory for seismic diffraction: Geophysics, 35, 762-784.

ACKNOWLEDGEMENTS