Approximations to seismic velocities in anisotropic media

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ABSTRACT

Vertical and horizontal transverse isotropy (VTI and HTI) provide good models for wave propagation in many rock media. Expressing the exact body-wave velocities in TI media requires five parameters (in addition to density); expressing just the exact P-wave velocity requires four. That is more than can usually be estimated from surface seismic data. In this report we review a recent paper by Fowler, who makes several practical approximations for P- and S-wave phase and group velocities, each using only three parameters. It is shown, among other things that, with a good choice of parameterization, P-wave velocities become nearly independent of v_{sz}, the vertical S-wave velocity. We propose some further potentially valuable extensions on Fowler's work

INTRODUCTION

Fowler (2202) carried out a sytematic study of several different approximations to the P- and S-wave velocities in VTI media, different ones of which may be more or less practical in different situations. We shall first review and clarify Fowler's (2002) work and then propose a number of potentially useful extensions of it.

Phase velocity and polarization of body waves propagating in VTI media can be obtained from the Kelvin-Christoffel equations (e.g. Musgrave, 1970). Solving these equations for the plane of propagation ($n_2 = 0$) yields:

$$\begin{vmatrix} c_{11}n_1^2 + c_{44}n_3^2 - \rho v^2 & 0 & (c_{13}+c_{44})n_1 n_3 \\ 0 & c_{66}n_1^2 + c_{44}n_3^2 - \rho v^2 & 0 \\ (c_{13}+c_{44})n_1 n_3 & 0 & c_{44}n_1^2 + c_{33}n_3^2 - \rho v^2 \end{vmatrix}. \quad (1)$$

Finding the eigenvalues of this determinant leads to phase-velocities for P, SV and SH modes. Here, c_{ij} are the stiffness and ρ the density. Then

$$v_{SH}^2(\theta) = a_{66}\sin^2\theta + a_{44}\cos^2\theta \quad (2)$$

where $a_{ij} = c_{ij}/\rho$, and

$$2v_{P,SV}^2(\theta) = a_{11}\sin^2\theta + a_{33}\cos^2\theta + a_{44} \pm \sqrt{\left[(a_{11}-a_{44})\sin^2\theta - (a_{33}-a_{44})\cos^2\theta\right]^2 + (a_{13}+a_{44})^2\sin^2 2\theta}\cdot \quad (3)$$

Here weak anisotropy is assumed and some practical limitations are set on P and S phase velocities. In practice, $\sqrt{2} \le v_{Pz}/v_{Sz} \le 4$; $0 \le \varepsilon \le 0.2$, and $-0.05 \le \delta \le 0.1$ (Fowler, 2002), where v_{Pz} is the vertical P-wave velocity, v_{Sz} the vertical S-wave velocity and ε

and δ are Thomsen's (1986) anisotropy parameters, defined in the next section.

VTI PARAMETERIZATION FOR SYMMETRY PLANES

In equations (2) and (3), solving for P-wave phase velocity for $\theta = 90°$ and $\theta = 0°$:

$$v_{px}^2 = a_{11}; \text{ and } v_{pz}^2 = a_{33}. \tag{4}$$

Solving now for SV-wave phase velocity for $\theta = 90°$ and $\theta = 0°$:

$$v_{sx}^2 = v_{sz}^2 = a_{44}. \tag{5}$$

Thomsen (1986) introduced the dimensionless parameters ε, δ and γ for VTI media. They are defined in terms of the stiffness by:

$$\varepsilon = \frac{a_{11} - a_{33}}{2a_{33}} \tag{6}$$

$$\delta = \frac{(a_{13} + a_{44})^2 - (a_{33} - a_{44})^2}{2a_{33}(a_{33} - a_{44})} \tag{7}$$

and

$$\gamma = \frac{a_{66} - a_{44}}{2a_{44}}. \tag{8}$$

Tsvankin and Thomsen (1994) also define the related parameter σ as:

$$\sigma = \left(\frac{V_{P0}}{V_{S0}}\right)^2 (\varepsilon - \delta) = \frac{(a_{33} - a_{44})(a_{11} - a_{44}) - (a_{13} + a_{44})^2}{2a_{44}(a_{11} - a_{44})} \tag{9}$$

and Alkhalifah and Tsvankin (1995) introduced the parameter:

$$\eta = \frac{\varepsilon - \delta}{1 + 2\delta}. \tag{10}$$

From equation (6) and Fowler's notation:

$$v_{px}^2 = (1 + 2\varepsilon) v_{pz}^2 \tag{11}$$

We also have this relation for the small-offset P-wave normal-moveout velocity:

$$v_{pn}^2 = (1 + 2\delta) v_{pz}^2 \tag{12}$$

$$v_{sn}^2 = (1 + 2\sigma) v_{sz}^2 \tag{13}$$

$$v_{px}^2 = (1 + 2\eta) v_{pn}^2 \tag{14}$$

Considering the definition of δ one gets:

$$
\begin{aligned}
&(a_{13}+a_{44})^2 = 2\delta a_{33}(a_{33}-a_{44}) + (a_{33}-a_{44})^2 && \text{(a)}\\
&(a_{13}+a_{44})^2 = (a_{33}-a_{44})((1+2\delta)a_{33}-a_{44}) && \text{(b)}\\
&(a_{13}+a_{44})^2 = (a_{33}-a_{44})(v_{pn}^2-a_{44}) && \text{(c)}\\
&(a_{13}+v_{sz}^2)^2 = (v_{pz}^2-v_{sz}^2)(v_{pn}^2-v_{sz}^2). && \text{(d)}
\end{aligned}
\qquad (15)
$$

Equations (15) are not the only possible factorization. In general:

$$(a_{13}+v_{sz}^2)^2 = (v_{p1}^2-v_{sz}^2)(v_{p2}^2-v_{sz}^2)\,. \qquad (16)$$

For different possible v_{p1} one can calculate v_{p2} as:

$$v_{p2}^2 = \frac{(a_{13}+v_{sz}^2)^2}{(v_{p1}^2-v_{sz}^2)} + v_{sz}^2 \qquad (17)$$

$$v_{p2}^2 = \frac{(v_{pz}^2-v_{sz}^2)^2(v_{pn}^2-v_{sz}^2)^2}{(v_{p1}^2-v_{sz}^2)} + v_{sz}^2\,. \qquad (18)$$

Some possible values for v_{p1}^2 are:

$$
\begin{gathered}
v_{p1}^2 = v_{px}^2\,, \quad v_{p1}^2 = v_{pz}^2\,, \quad v_{p1}^2 = a_{13}+2v_{sz}^2\,, \quad v_{p1}^2 = \frac{(v_{pz}^2+2v_{px}^2)}{2}\,,\\
v_{p1}^{-2} = \frac{(v_{pz}^{-2}+2v_{px}^{-2})}{2}\,, \quad v_{p1}^2 = v_{px}v_{px}\,,
\end{gathered}
\qquad (19)
$$

With the above notation the exact P and SV phase velocities become:

$$
\begin{aligned}
2v_{P,SV}^2(\theta) &= v_{px}^2\sin^2\theta + v_{pz}^2\cos^2\theta + v_{sz}^2\\
&\pm\sqrt{\left[(v_{px}^2-v_{sz}^2)\sin^2\theta + (v_{pz}^2-v_{sz}^2)\cos^2\theta\right]^2 + (a_{13}+v_{sz}^2)^2\sin^2 2\theta}
\end{aligned}
\qquad (20)
$$

Figure 1a (after Fowler, 2002) shows $v_p(\theta)$ from equation (20). The various curves represent different values of v_{sz}. Clearly the P-wave velocity depends strongly on v_{sz}. Figure 1b shows, however, that using factorization (15d) and replacing a_{13}, this dependence on the value of v_{sz} is nearly eliminated.

P-WAVE PHASE VELOCITY APPROXIMATION

Equation (20) for P-wave velocity may be rewritten as:

$$
\begin{aligned}
2\left[v_P^2(\theta)-v_{sz}^2\right] &= \left(v_{px}^2-v_{sz}^2\right)\sin^2\theta + \left(v_{pz}^2-v_{sz}^2\right)\cos^2\theta\\
&+\Big\{\left[\left(v_{px}^2-v_{sz}^2\right)\sin^2\theta + \left(v_{pz}^2-v_{sz}^2\right)\cos^2\theta\right]^2\\
&+\left[\left(a_{13}+v_{sz}^2\right)^2 - \left(v_{px}^2-v_{sz}^2\right)\left(v_{pz}^2-v_{sz}^2\right)\right]\sin^2 2\theta\Big\}^{1/2}
\end{aligned}
\qquad (21)
$$

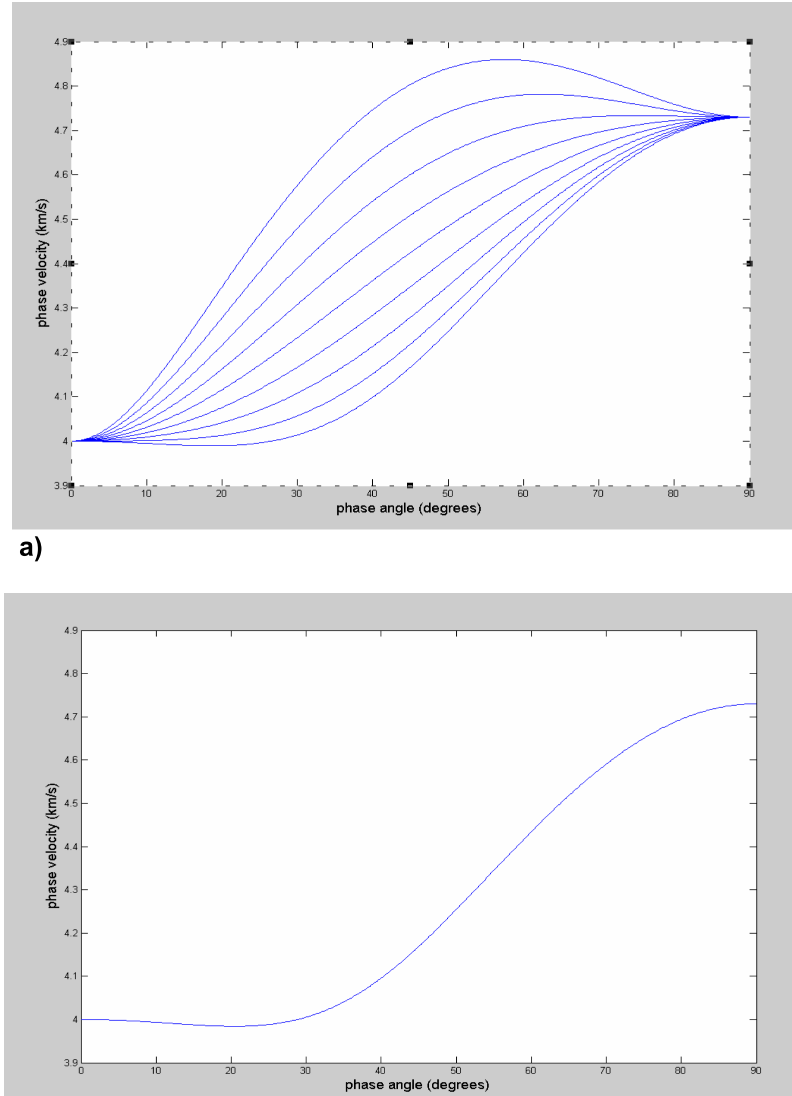

FIG. 1. P-wave phase-velocity curves for varying values of v_{sz}. The parameters v_{pz} = 4 km/s, ε = 0.2, and δ = −0.05 are kept fixed, while v_{sz} is allowed to vary from 1.0 km/s to 2.6 km/s. In (a) the P-wave phase velocity is parameterized by the fixed value of a_{13} = 13.18 km^2/s^2, whereas in (b) a parameterization with a fixed value of v_{pn} = 3.79 km/s is used instead. The second parameterization removes nearly all the variation with v_{sz}.

Replacing a_{13} using factorization (16):

$$\begin{aligned} 2\left[v_P^2(\theta) - v_{sz}^2\right] = & \left(v_{px}^2 - v_{sz}^2\right)\sin^2\theta + \left(v_{pz}^2 - v_{sz}^2\right)\cos^2\theta \\ & + \left\{\left[\left(v_{px}^2 - v_{sz}^2\right)\sin^2\theta + \left(v_{pz}^2 - v_{sz}^2\right)\cos^2\theta\right]^2 \right. \\ & \left. + \left[\left(v_{p1}^2 - v_{sz}^2\right)\left(v_{p2}^2 - v_{sz}^2\right) - \left(v_{px}^2 - v_{sz}^2\right)\left(v_{pz}^2 - v_{sz}^2\right)\right]\sin^2 2\theta\right\}^{1/2} \end{aligned} \tag{22}$$

or:

$$
\begin{aligned}
2v_P^2(\theta)\left[1-\frac{v_{sz}^2}{v_P^2(\theta)}\right] &= v_{px}^2\left(1-\frac{v_{sz}^2}{v_{px}^2}\right)\sin^2\theta + v_{pz}^2\left(1-\frac{v_{sz}^2}{v_{pz}^2}\right)\cos^2\theta \\
&+ \left\{\left[v_{px}^2\left(1-\frac{v_{sz}^2}{v_{px}^2}\right)\sin^2\theta + v_{pz}^2\left(1-\frac{v_{sz}^2}{v_{pz}^2}\right)\cos^2\theta\right]^2\right. \\
&+ \left[v_{p1}^2 v_{p2}^2\left(1-\frac{v_{sz}^2}{v_{p1}^2}\right)\left(1-\frac{v_{sz}^2}{v_{p2}^2}\right)\right. \\
&- \left.\left. v_{px}^2 v_{pz}^2\left(1-\frac{v_{sz}^2}{v_{px}^2}\right)\left(1-\frac{v_{sz}^2}{v_{pz}^2}\right)\right]\sin^2 2\theta\right\}^{1/2}
\end{aligned}
\qquad (23)
$$

Fowler (2002) chose a constant reference velocity v_{pr} in the range of $v_P(\theta)$, so as to allow replacement of $v_P^2(\theta)$, v_{pz}^2, v_{px}^2, v_{p1}^2 and v_{p2}^2 with v_{pr}^2. This substitution is reasonable if shear velocity is relatively small and P-wave anisotropy is not too great; and also if $v_{p\min}^2 < v_{p1}^2 < v_{p\max}^2$ and $v_{p\min}^2 < v_{p2}^2 < v_{p\max}^2$. Then applying this to equation (22) one gets:

$$
\begin{aligned}
2v_P^2(\theta)\left[1-\frac{v_{sz}^2}{v_{pr}^2(\theta)}\right] &= v_{px}^2\left(1-\frac{v_{sz}^2}{v_{pr}^2}\right)\sin^2\theta + v_{pz}^2\left(1-\frac{v_{sz}^2}{v_{pr}^2}\right)\cos^2\theta \\
&+ \left\{\left[v_{px}^2\left(1-\frac{v_{sz}^2}{v_{pr}^2}\right)\sin^2\theta + v_{pz}^2\left(1-\frac{v_{sz}^2}{v_{pr}^2}\right)\cos^2\theta\right]^2\right. \\
&+ \left[v_{p1}^2 v_{p2}^2\left(1-\frac{v_{sz}^2}{v_{pr}^2}\right)\left(1-\frac{v_{sz}^2}{v_{pr}^2}\right)\right. \\
&- \left.\left. v_{px}^2 v_{pz}^2\left(1-\frac{v_{sz}^2}{v_{pr}^2}\right)\left(1-\frac{v_{sz}^2}{v_{pr}^2}\right)\right]\sin^2 2\theta\right\}^{1/2} .
\end{aligned}
\qquad (24)
$$

All terms involving the reference velocity now cancel out to give the simpler three-parameter approximation:

$$
\begin{aligned}
2v_P^2(\theta) &= v_{px}^2\sin^2\theta + v_{pz}^2\cos^2\theta \\
&\pm \sqrt{\left[v_{px}^2\sin^2\theta + v_{pz}^2\cos^2\theta\right]^2 + \left(v_{p1}^2 v_{p2}^2 - v_{pz}^2 v_{px}^2\right)\sin^2 2\theta} \\
&= v_{pe}^2(\theta) + \sqrt{v_{pe}^4(\theta) + \left(v_{p1}^2 v_{p2}^2 - v_{pz}^2 v_{px}^2\right)\sin^2 2\theta}
\end{aligned}
\qquad (25)
$$

where $v_{pe}^2(\theta) = v_{px}^2\sin^2\theta + v_{pz}^2\cos^2\theta$.

Fowler (2002) then catalogues several different linear approximations, many of them previously published.

For $v_{p1} = v_{pz}$ and $v_{p2} = v_{pn}$, equation (25) becomes:

Approximation P1:

$$\begin{aligned} 2v_P^2(\theta) &= v_{pe}^2(\theta) + \sqrt{v_{pe}^4(\theta) + v_{pz}^2(v_{pn}^2 - v_{px}^2)\sin^2 2\theta} \\ &= v_{pe}^2(\theta)\left[1 + \sqrt{1 + \frac{v_{pz}^2(v_{pn}^2 - v_{px}^2)\sin^2 2\theta}{v_{pe}^2(\theta)}}\right] \end{aligned} \quad (26)$$

(Stopin, 2001; Alkhalifah, 1998).

Approximation P2:

Linearizing the radical:

$$\begin{aligned} v_P^2(\theta) &= v_{pe}^2(\theta) + \frac{v_{pz}^2(v_{pn}^2 - v_{px}^2)}{v_{pe}^2(\theta)}\sin^2\theta\cos^2\theta \\ v_P^2(\theta) &= v_{pe}^2(\theta)\left[1 + \frac{v_{pz}^2(v_{pn}^2 - v_{px}^2)}{v_{pe}^4(\theta)}\sin^2\theta\cos^2\theta\right] \end{aligned} \quad (27)$$

(Dellinger et al., 1993; Klie and Toro, 2001).

Approximation P3:

Taking the square root of both sides and linearizing the radical:

$$v_P(\theta) = v_{pe}(\theta) + \frac{v_{pz}^2(v_{pn}^2 - v_{px}^2)}{2v_{pe}^3(\theta)}\sin^2\theta\cos^2\theta\,. \quad (28)$$

Taking $v_{pe}(\theta) = v_{pz}$ (for near offsets) yields:

Approximation P4:

$$\begin{aligned} v_P^2(\theta) &= v_{pe}^2(\theta) + (v_{pn}^2 - v_{px}^2)\sin^2\theta\cos^2\theta \\ &= v_{pe}^2(\theta)\cos^2\theta + v_{pn}^2\sin^2\theta\cos^2\theta + v_{px}^2\sin^4\theta \end{aligned} \quad (29)$$

(Harlan, 1995; Stopin, 2001).

Approximation P5:

Linearizing (29) one step further:

$$v_P(\theta) = v_{pe}(\theta) + \frac{(v_{pn}^2 - v_{px}^2)}{2v_{pe}(\theta)} \sin^2\theta \cos^2\theta \, . \tag{30}$$

Approximation P6:

Replacing v_{px}^2 by v_{pn}^4 / v_{px}^2 yields:

$$v_P^2(\theta) = v_{pe}^2(\theta) + \frac{v_{pz}^2 (v_{pn}^2 - v_{px}^2)}{v_{pz}^2 \cos^2\theta + \dfrac{v_{pn}^4}{v_{px}^2} \sin^2\theta} \sin^2\theta \cos^2\theta \tag{31}$$

(Tsvankin and Thomsen, 1994).

Approximation P7:

Again linearizing equation (31) gives:

$$v_P(\theta) = v_{pe}(\theta) + \frac{v_{pz}^2 (v_{pn}^2 - v_{px}^2)}{2v_{pe}(\theta)\left[v_{pz}^2 \cos^2\theta + \dfrac{v_{pn}^4}{v_{px}^2} \sin^2\theta \right]} \sin^2\theta \cos^2\theta \, . \tag{32}$$

From the exact P-wave phase velocity, by linearizing the radical, one gets:

$$\begin{aligned} 2v_p^2(\theta) &= (v_{pe}^2(\theta) + v_{sz}^2) + (v_{pe}^2(\theta) - v_{sz}^2) \\ &\quad + \sqrt{1 + \frac{(a_{13} + v_{sz}^2)^2 - (v_{pz}^2 - v_{sz}^2)(v_{px}^2 - v_{sz}^2)}{(v_{pe}^2(\theta) - v_{sz}^2)^2} \sin \ 2\theta} \, . \end{aligned} \tag{33}$$

Linearizing the radical:

$$\begin{aligned} 2v_p^2(\theta) &= (v_{pe}^2(\theta) + v_{sz}^2) + (v_{pe}^2(\theta) - v_{sz}^2) \\ &\quad + \left\{ 1 + \frac{(a_{13} + v_{sz}^2)^2 - (v_{pz}^2 - v_{sz}^2)(v_{px}^2 - v_{sz}^2)}{2(v_{pe}^2(\theta) - v_{sz}^2)^2} \sin^2 2\theta \right\} . \end{aligned} \tag{34}$$

Using the substitution $(a_{13} + v_{sz}^2)^2 = (v_{pz}^2 - v_{sz}^2)(v_{pn}^2 - v_{sz}^2)$:

Approximation P8:

$$v_p^2(\theta) = v_{pe}^2(\theta) + \frac{(v_{pz}^2 - v_{sz}^2)(v_{pn}^2 - v_{px}^2)}{v_{pe}^2(\theta) - v_{sz}^2} \sin^2\theta \cos^2\theta \qquad (35)$$

Linearizing one more time yields:

Approximation P9:

$$v_p(\theta) = v_{pe}(\theta) + \frac{(v_{pz}^2 - v_{sz}^2)(v_{pn}^2 - v_{px}^2)}{2v_{pe}(\theta)[v_{pe}^2(\theta) - v_{sz}^2]} \sin^2\theta \cos^2\theta . \qquad (36)$$

Comparing approximation P2 and P8, one sees that they are the same, except that in P2 Fowler used the general factorization form: $(a_{13} + v_{sz}^2)^2 = (v_{p1}^2 - v_{sz}^2)(v_{p2}^2 - v_{sz}^2)$ whereas in P8 he used the substitution $(a_{13} + v_{sz}^2)^2 = (v_{pz}^2 - v_{sz}^2)(v_{pn}^2 - v_{sz}^2)$. Expanding equation (33) around v_{pz} gives:

$$v_p^2(\theta) = v_{pz}^2 \left[1 + \left(\frac{v_{pn}^2}{v_{pz}^2} - 1 \right) \sin^2\theta \cos^2\theta + \left(\frac{v_{px}^2}{v_{pz}^2} - 1 \right) \sin^4\theta \right]. \qquad (37)$$

Taking the square root of each side and linearizing the result:

Approximation P10:

$$v_p(\theta) = v_{pz} \left[1 + \frac{1}{2} \left(\frac{v_{pn}^2}{v_{pz}^2} - 1 \right) \sin^2\theta \cos^2\theta + \frac{1}{2} \left(\frac{v_{px}^2}{v_{pz}^2} - 1 \right) \sin^4\theta \right] \qquad (38)$$

$$2v_p(\theta) = v_{pz} \left(1 + \cos^2\theta + \frac{v_{pn}^2}{v_{pz}^2} \sin^2\theta \cos^2\theta + \frac{v_{px}^2}{v_{pz}^2} \sin^4\theta \right) \qquad (39)$$

and

$$v_p(\theta) = v_{pz} \left(1 + \delta \sin^2\theta \cos^2\theta + \varepsilon \sin^4\theta \right). \qquad (40)$$

The approximation P10 is the approximation suggested by Thomsen (1986) and it is less accurate at wide-angle propagation because the expansion was carried out around v_{pz} .

VTI PARAMETERIZATION FOR NONSYMMETRY PLANES

The dimensionless anisotropy parameters are particularly suitable for simplifying the P-wave phase-velocity function in the limit of weak anisotropy. For the planes as symmetry as shown above, it is sufficient just to adapt the known expression for weak transverse isotropy. Approximating the P-wave phase velocity outside the symmetry planes is obtained in Tsvankin (1997) by linearizing the exact P-wave phase-velocity equations:

$$v_P^2(\theta) = v_{pz}^2\left[1 + 2n_1^4\varepsilon^{(2)} + 2n_2^4\varepsilon^{(1)} + 2n_1^2n_3^2\delta^{(2)} + 2n_2^2n_3^2\delta^{(1)} + 2n_1^2n_2^2(2\varepsilon^{(2)} + \delta^{(3)})\right] \quad (41)$$

It is convenient to replace the directional cosines, n_j, of the phase velocity vector by the polar (θ) and azimuthal (φ) phase angles,

$$n_1 = \sin\theta\cos\varphi, \quad n_2 = \sin\theta\sin\varphi, \quad n_3 = \cos\theta\,. \quad (42)$$

Taking the square root of equation (41) yields the phase velocity exactly in the same form as in VTI media for symmetry planes, but with azimuthally dependent coefficients ε and δ:

$$v_p(\theta,\varphi) = v_{pz}\left(1 + \delta(\varphi)\sin^2\theta\cos^2\theta + \varepsilon(\varphi)\sin^4\theta\right) \quad (43)$$

where:

$$\begin{aligned}\delta(\varphi) &= \delta^{(1)}\sin^2\varphi + \delta^{(2)}\cos^2\varphi \\ \varepsilon(\varphi) &= \varepsilon^{(1)}\sin^4\varphi + \varepsilon^{(2)}\cos^4\varphi + (2\varepsilon^{(2)} + \delta^{(3)})\sin^2\varphi\cos^2\varphi\end{aligned} \quad (44)$$

GROUP-VELOCITY APPROXIMATION

In general, the group velocity can be found from the phase velocity and phase angle via the relations:

$$\begin{aligned}V^2(\phi) &= v^2(\theta) + \left[\frac{dv(\theta)}{d\theta}\right]^2 \\ \tan(\phi - \theta) &= \frac{1}{v(\theta)}\frac{dv(\theta)}{d\theta}\end{aligned} \quad (45)$$

Just for the SH-wave we have an explicit equation for group velocity:

$$V_{SH}^2(\phi) = v_{pz}^2\cos^2\phi + v_{px}^2\sin^2\phi \quad (46)$$

But for exact P or SV waves in VTI, there is no such simple explicit solution, so group angle and velocities must be derived numerically from phase angles and velocities. Muir and Dellinger (1985) and Harlan (1995) have suggested using substitution of group slowness for phase velocities to obtain reasonable approximations for P and SV waves. They pointed out that under this substitution:

$$\begin{aligned}v^{-2}(\theta) &= V^{-2}(\phi) + \left[\frac{dV^{-1}(\phi)}{d\phi}\right]^2 \\ \tan(\theta - \phi) &= V(\phi)\frac{d[V^{-1}(\phi)]}{d\phi}\,.\end{aligned} \quad (47)$$

Making this substitution directly into different P-approximations yields the group-velocity approximations. In the group-velocity approximations we used $V_{pe}^{-2} = v_{px}^{-2}\sin^2\phi + v_{pz}^{-2}\cos^2\phi$. They are:

Approximation P1:

$$2V_P^{-2}(\phi) = V_{pe}^{-2}(\phi) + \sqrt{V_{pe}^{-4}(\phi) + v_{pz}^{-2}(v_{pn}^{-2} - v_{px}^{-2})\sin^2 2\phi} \tag{48}$$

Approximation P2:

$$V_P^{-2}(\phi) = V_{pe}^{-2}(\phi) + \frac{v_{pz}^{-2}(v_{pn}^{-2} - v_{px}^{-2})}{v_{pe}^{-2}(\phi)}\sin^2\phi\cos^2\phi \tag{49}$$

Approximation P3:

$$V_P^{-1}(\phi) = V_{pe}^{-1}(\phi) + \frac{v_{pz}^{-2}(v_{pn}^{-2} - v_{px}^{-2})}{2v_{pe}^{-3}(\phi)}\sin^2\phi\cos^2\phi \tag{50}$$

Approximation P4:

$$V_P^{-2}(\phi) = V_{pe}^{-2}(\phi) + (v_{pn}^{-2} - v_{px}^{-2})\sin^2\phi\cos^2\phi \tag{51}$$

Approximation P5:

$$V_P^{-1}(\phi) = V_{pe}^{-1}(\phi) + \frac{(v_{pn}^{-2} - v_{px}^{-2})}{2v_{pe}^{-1}(\phi)}\sin^2\phi\cos^2\phi \tag{52}$$

Approximation P6:

$$V_P^{-2}(\phi) = V_{pe}^{-2}(\phi) + \frac{v_{pz}^{-2}(v_{pn}^{-2} - v_{px}^{-2})}{v_{pz}^{-2}\cos^2\phi + \dfrac{v-_{pn}^{4}}{v_{px}^{-2}}\sin^2\phi}\sin^2\phi\cos^2\phi \tag{53}$$

Approximation P7:

$$V_P^{-1}(\phi) = V_{pe}^{-1}(\phi) + \frac{v_{pz}^{-2}(v_{pn}^{-2} - v_{px}^{-2})}{2v_{pe}^{-1}(\phi)\left[v_{pz}^{-2}\cos^2\phi + \dfrac{v_{pn}^{-4}}{v_{px}^{-2}}\sin^2\phi\right]}\sin^2\phi\cos^2\phi \tag{54}$$

Approximation P8:

$$V_p^{-2}(\phi) = V_{pe}^{-2}(\phi) + \frac{(v_{pz}^{-2} - v_{sz}^{-2})(v_{pn}^{-2} - v_{px}^{-2})}{V_{pe}^{-2}(\phi) - v_{sz}^{-2}}\sin^2\phi\cos^2\phi \tag{55}$$

Approximation P9:

$$V_p^{-1}(\phi) = V_{pe}^{-1}(\phi) + \frac{(v_{pz}^{-2} - v_{sz}^{-2})(v_{pn}^{-2} - v_{px}^{-2})}{2V_{pe}^{-1}(\phi)[V_{pe}^{-2}(\phi) - v_{sz}^{-2}]}\sin^2\theta\cos^2\theta \tag{56}$$

Approximation P10:

$$2V_p^{-1}(\phi) = v_{pz}^{-1}\left(1 + \cos^2\phi + \frac{v_{pn}^{-2}}{v_{pz}^{-2}}\sin^2\phi\cos^2\phi + \frac{v_{px}^{-2}}{v_{pz}^{-2}}\sin^4\phi\right) \tag{57}$$

PROPOSAL FOR FUTURE WORK

Fowler (2002) stated that it is often a challenge to determine good subsurface velocities for prestack migration of steep-dip reflection data. For computational efficiency, it is usually necessary to use simpler expressions with fewer parameters than those for exact transverse isotropy. The various approximations listed above – as well as many more for SV waves, plus further sets for P-SV dispersion relations and traveltimes – were all worked out by Fowler (2002). However, it remains for us to test the various approximations and determine which are more appropriate for what circumstances.

We envisage many different such tests that one could run on the phase-velocity expressions. There is also a need for accurate group-velocity approximations for cases where the exact group-velocity expression cannot be determined analytically from the phase velocity. Testing here is also anticipated. In addition, this entire analysis could be extended, in a fairly straightforward way, to the case of horizontal transverse isotropy (HTI) and, in probably a less straightforward way, to the orthorhombic case.

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