Pore geometry and elastic moduli in sandstones

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ABSTRACT

Pore geometry is a difficult parameter to quantify, but it affects the elastic moduli in a fundamental way. The uncertainty in elastic moduli is to a large degree due to the uncertainty in pore geometry. The authors inverted the velocity measurements of ninety-seven sandstone samples for the pore aspect ratio spectra. It was found that the porosity of round pores is in direct proportion to total porosity, and that the porosity of cracks is small and varies randomly for a large set of samples from different geological backgrounds but may be linearly related to total porosity for a small subset from some specific area. This pore geometrical model is central to predicting the elastic properties in sandstones. The dry elastic moduli at a given porosity vary chiefly due to cracks and the variability increases with decreasing porosity. This causes the cross-plot of dry elastic moduli versus total porosity to scatter in a triangular form, and is also responsible for higher stress sensitivity of low-porosity sandstones. The linearity between total porosity and elastic moduli results from strong correlation between crack porosity and total porosity, and it becomes more pronounced at high effective pressure or at saturation with a less compressible fluid such as water. The Vp/Vs ratio in wet sandstones is affected by the Vp/Vs ratio of rock solid and pore geometry, but the former has far more influence, which may provide the rock physics basis for lithology identification using the Vp/Vs ratio. Vp and Vs are correlated very well for both dry and wet sandstones. The two straight lines are separate at low velocities but merging toward high velocities. In addition, clay content has been found with little relationship to pore geometry and with some connection to the elastic moduli of rock solid.

INTRODUCTION

The elastic moduli of rocks depend directly on the elastic moduli of rock solid, pore geometry (including pore shape and fractional volume) and the elastic moduli of pore fluids. Dolomite, sandstone and shale have different mineral compositions and thus have different elastic moduli and velocities. The porosity-velocity relationship (Castagna et al., 1985; Han et al., 1986; Kowallis et al., 1984; Tosaya and Nur, 1982) relates the variation of velocities to the variation of porosity. The variability of velocities at a given porosity is caused partly by varying pore shapes (Wang, 2001). Fluid substitution can result in considerable changes in bulk modulus, which is the basis for time-lapse seismic surveys and AVO analysis for fluid identification. Many other factors such as consolidation, cementation, temperature, pressure, frequency, age etc. are indirect in that they affect the elastic moduli through their effect on the three direct factors. For instance, changes in effective pressure lead to closing or opening cracks, which in turn increases or reduces the elastic moduli. Heating at high temperature may generate new cracks and decrease the bulk modulus of pore fluids, lowering the elastic moduli.

Pore geometry is a difficult parameter to quantify. The uncertainty in elastic moduli is to a large degree due to the uncertainty in pore geometry. In theoretical studies, the pore geometry in rocks is generally simplified to consist of a population of regular pores dispersed throughout the rock solid (Walsh and Grosenbaugh, 1979). Under this assumption, many authors in the past few decades (e.g., Budiansky and O'Connell, 1975; Kuster and Toksoz, 1975; Mavko and Nur, 1978; Eshelby, 1957; Walsh, 1965; Wu, 1966) have developed mathematical models to calculate the elastic moduli based on specified pore geometry. Of these works, the KT model (Kuster and Toksoz, 1974) is recognized as the most realistic and simplest since the specified pore geometry is comparable to lab measurements and the formulae are relatively less complicated. Another advantage of the KT model is that the pore geometry can be inverted from measurements of the elastic moduli and velocities. The methodology of inversion was summarized in the papers by Cheng and Toksoz (1979) and Toksoz et al. (1976).

Despite much work on mathematical modelling, there is no systematic study of pore geometry in rocks. A pore geometrical model would be an important tool to evaluate the elastic moduli. In this paper, based on inverse KT modelling (Cheng and Toksoz, 1979 and Toksoz et al., 1976), we inverted the velocity measurements of ninety-seven sandstone samples for the pore aspect ratio spectra, which are a series of pore aspect ratios and corresponding volume fractions. Then the pore aspect ratio spectra were analyzed for their relationship with total porosity. The resultant distribution of the pore aspect ratio spectra was employed to interpret the elastic properties in sandstones.

INVERSE MODELLING FOR THE PORE ASPECT RATIO SPECTRUM

Formulation of inverse problem

Estimating the pore aspect ratio spectrum of a rock from velocity measurements that were made at different effective pressures is an inverse KT modelling problem. We can choose a set of pore aspect ratios (α_m), assign the corresponding fractional volumes [$c(\alpha_m)$] and calculate theoretical velocities as a function of effective pressure according to formulae (see equations 10, 11, 12, 14 in the paper by Cheng and Toksoz, 1979). Then the pore aspect ratio spectrum is adjusted and the calculation is repeated until a good fit to the experimental data is obtained. This process, however, is time-consuming since we simultaneously resolve both the pore aspect ratios and their corresponding fractional volumes. In order to simplify the inverse problem, we used the distributions of pore aspect ratios for sandstones acquired by Cheng and Toksoz (1979) and Toksoz et al. (1976) as a guide. The distributions of pore aspect ratios obtained by them for a number of sandstones are similar. They differ chiefly in corresponding fractional volumes. It is therefore reasonable to choose one of the sets of pore aspect ratios to represent the distribution of pore aspect ratios for our samples. In this paper, the distribution of pore aspect ratios for the Navajo sandstone (see Table 1 of Cheng and Toksoz, 1979) was selected. Accordingly, the inverse problem reduces to estimating the fractional volumes for the pre-assigned pore aspect ratios.

With the pore aspect ratio distribution specified, the equations for inverse KT modelling were formulated with $c(\alpha_m)/\alpha_m$ as x_m, m being the pore aspect ratio index. At each effective pressure there are two equations, one for the bulk modulus and the other for the shear modulus. The number of all equations total $2*N$, N being the number of different effective pressure points. The equations can be written in matrix form as $Ax = b$. A is $2N\times m$ matrix and b is $2N\times 1$ vector. Two constraints are imposed on the equations:

that the sum of the fractional volumes of pores of all aspect ratios is equal to be total porosity, and the noninteraction assumption,

$$x_m \geq 0 \ (m = 1, 2, \ldots 11) \ and \ \sum_{m=1}^{M} x_m < 1 \ . \tag{1}$$

The second constraint may be relaxed to $0 \leq x_m < 1$ (Toksoz et al., 1976). The inverse problem reduces to solving the non-linear equations $Ax = b$ subject to the two constraints. The objective function may be constructed as:

$$S = (Ax - b)2 \, . \tag{2}$$

The solution is to minimize equation (2) in the feasible areas defined by the two constraints.

At any x, the gradient (g) of S is calculated as:

$$g = A^T Ax - A^T b \, , \tag{3}$$

where A^T is the transpose of A. Given an initial guess, the minimum can be found by a decent gradient method.

Elastic moduli of rock solid

The inverse KT modelling requires the elastic moduli of rock solid as input, which are generally acquired using the VRH model. Yet this method has limitations. First, not all minerals are stress-bearing (Marion et al., 1992). The minerals in the stress-free areas have little effect on the effective elastic moduli of rock solid. Dispersed clays in the large pores (Marion et al., 1992; Sams et al., 2001) away from the grain-supported framework do not affect the effective elastic moduli of rock solid. Clays at grain contacts, however, have a significant influence. It is therefore not reasonable to calculate the effective elastic moduli of rock solid by averaging the elastic moduli over the total volumes of individual minerals (VRH model). Second, the elastic moduli of clay are difficult to measure because of the unavailability of ‘pure clay’ that is large enough to perform tests on (Zimmerman, 1991). There are some data in literature (Woeber et al., 1963; Wang et al., 1980), but they are measured on an aggregate of clay minerals, which contains varying microporosity (Hurst and Nadeau, 1995; Khaksar et al., 1999; Kowallis et al., 1984).

In this paper we present a new approach to estimate the effective elastic moduli of the rock solid. Cracks or low-aspect-ratio pores will close at high effective pressure and the remaining open pores are largely spherical. Given velocity measurements at high effective pressure, the KT model can be used to solve for K_s and μ_s on the assumption of the existence of only round pores. If the velocities at high effective pressure are unknown, the velocity measurements at lower effective pressures can be used to extrapolate them to high effective pressures. With this scheme, we computed the effective elastic moduli of rock solid for ninety-seven sandstone samples. Figure 1 a and b demonstrate that clay content is not well correlated with the elastic moduli of the rock solid. The data points are scattered with a slight trend, which makes it difficult to predict the elastic moduli of rock solid from clay content.

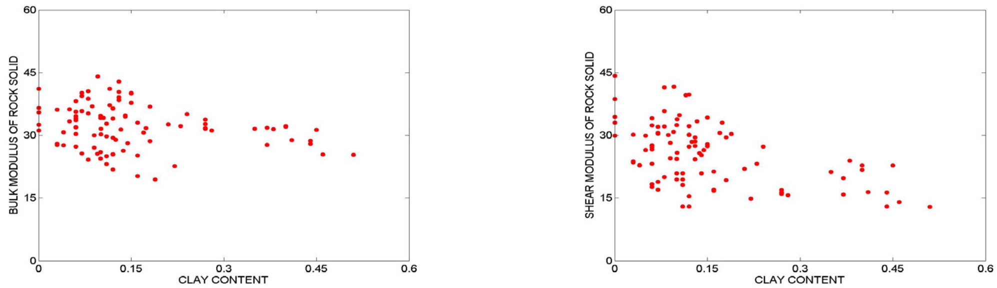


FIG. 1. Clay content versus elastic moduli (GPa) of rock solid.

Accuracy of inversion results

The velocities measured as a function of effective pressures for ninety-seven sandstones were inverted for the pore aspect ratio spectra. These sandstone samples come from a wide range of geological settings, with a variety of porosity and clay content (Han et al., 1986; Khaksar et al., 1999). There are a small subset of twenty-two samples, which were taken from sandstone reservoirs within the gas-producing fields in the southern Cooper Basin, South Australia (Khaksar et al., 1999). A detailed description of these samples and experimental conditions can be found in the papers by Han et al. (1986) and Khaksar et al. (1999). In order to test the quality of the inversion fit, the pore aspect ratio spectra were input into the formulae to calculate the elastic moduli and velocities as a function of effective pressures. The percentage errors between theoretical velocities and lab measured velocities are small, approximately 2% on average and not more than 4.5% in worst cases. Figure 2 shows the comparison of theoretical velocities and lab measured velocities for two samples. They fit well with only minor mismatch.

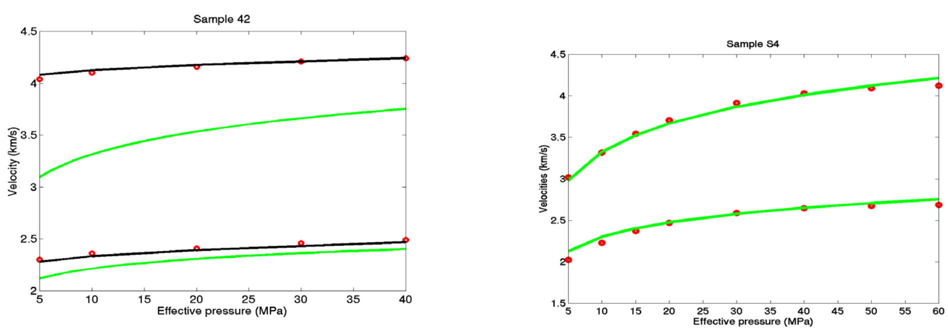


FIG. 2. Results of inversion for samples 42 and S4. The red solid circles are velocity measurements; the black and green solid curves are theoretical velocities calculated for water-saturated and dry sandstones respectively; upper curves are compressional wave velocities and lower curves are shear wave velocities.

PORE GEOMETRICAL MODEL

Variation of pore aspect ratio spectra with total porosity

The pore aspect ratio spectrum may be simplified into round pores and cracks. The latter refer to the pores with aspect ratio equal to or less than 0.1. It is also assumed in this study that pores do not have aspect ratios between 1 and 0.1. Total porosity is then broken into two parts, porosity of round pores and porosity of cracks. As shown in Figure 3 (left), the porosity of round pores is well correlated to total porosity with the correlation coefficient of 0.98. The regression line for the cross plot falls slightly below the line y=x, implying the existence of only a small volume of cracks for all total porosities. As seen in Figure 3 (right), the porosity of cracks averages only 4% and is not correlated with total porosity. The porosity of cracks with a single pore aspect ratio appears to be independent of total porosity, as seen in Figure 4. This observation is consistent with the observations of previous workers. Bernabe (1991) mentioned that most sandstones exhibited a honeycomb structure, in which two types of pores were identified: large intergranular equidimentional pores and narrow sheet-like pores situated in two-grain faces. The latter only accounts for a small portion of pore space. Sausse et al. (2001) and Sun and Goldberg (1997) took the crack porosity to be 4.5% in their calculations. Cerney and Carlson (1999) found the distribution of crack types was independent of total porosity. Despite being qualitative, these authors considered that the cracks occupy a small amount of space, which may not be correlated with total porosity.

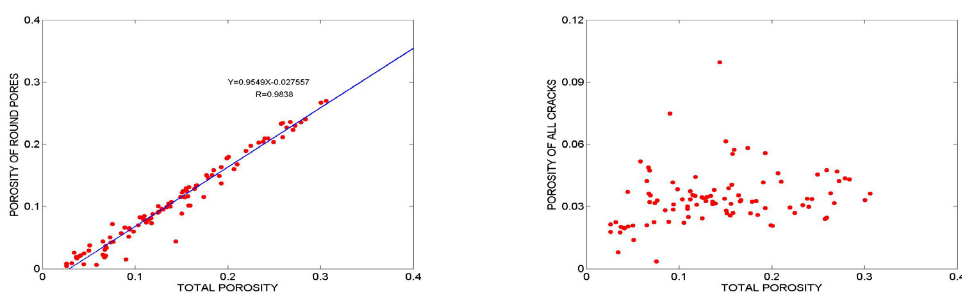


FIG. 3. Total porosity versus porosity of round pores (left) and cracks (right).

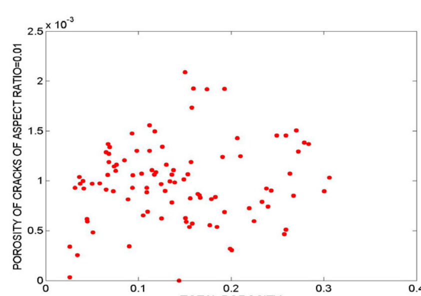


FIG. 4. Total porosity versus porosity of cracks (aspect ratio = 0.01).

The development of cracks in a rock is influenced by many factors. At sedimentation, crack abundance in sands depends on grain size, sorting, roundness etc.. In the subsequent burial, porosity is reduced by compaction and cementation. Compaction increases the number of grain contacts and changes contact types from tangential to concave-convex and sutured (Taylor, 1950). It may also be accompanied by grain fracturing. As a result, the crack porosity tends to be augmented with decreasing total porosity (Tosaya and Nur, 1982). Cements, however, fill in pore space and precipitate in the grain contacts, decreasing both crack porosity and total porosity (Kowallis et al., 1984). Compaction is very active in sandstones that contain abundant ductile lithic fragments (Blatt, 1982), while the evolution of cementation is controlled mainly by temperature, effective pressure, and geological time (Giles et al., 2000). These geological factors would vary with location and therefore porosity reduction could be dominated by compaction in one area and by cementation in the other. In Wyoming sandstones, chemical cements such as silica, calcite, anhydrite and feldspar, which account for no more than 7% of the solid material in any of the sandstones, play a minor role in porosity reduction (Taylor, 1950). Compaction is prevalent in this region. In upper Palaezoic Haushi Group sandstones, Sultanate of Oman, intense quartz cementation is the most important process reducing reservoir quality (Hartmann et al., 2000). Yet, on a statistical basis, these two types of diagenesis may be equally significant in porosity reduction and total porosity will not be correlated with crack porosity for a large set of samples from different geological settings. In a specific area, a correlation between total porosity and the porosity of cracks may exist due to dominance of compaction or cementation. In Figure 5, the porosity of cracks increases with increasing total porosity for a small subset of samples from a gas field in Australia.

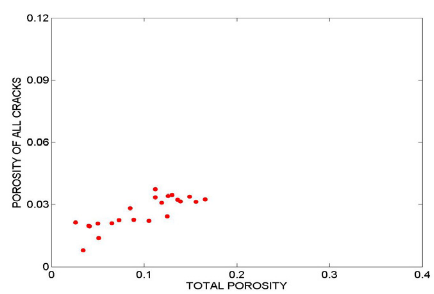
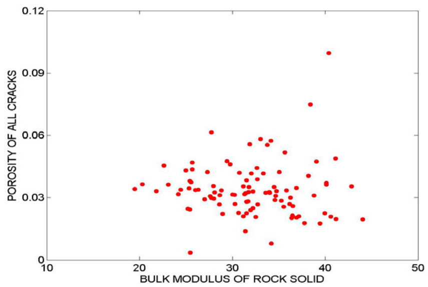


FIG. 5. Total porosity versus porosity of cracks from a small subset of samples.

Although this model of the variation of pore aspect ratio spectra with total porosity does not detail the quantitative distribution of cracks with varying pore aspect ratios, it is theoretically useful to predict the elastic properties.

Relationship of pore aspect ratio spectra to lithology and clay content

In theoretical modelling, rock solid and pore geometry are separate and independent. In reality, they may be linked in a subtle way. Compaction happens more easily in ductile fragments and, consequently, the amount of cracks may be related to lithology. Quartz cementation binds grains together and reduces the amount of both cracks and round pores. Once again the amount of cracks and round pores may be related to lithology. Many other factors such as grain size, sorting and roundness also have impact on the evolution of pore geometry. Tatham (1982) suspected that the observed association between lithology and Vp/Vs ratio is likely to be a result of an association between lithology and distribution of pore and crack shapes. Figure 6 (left) exhibits a poor correlation between the bulk modulus of rock solid and the porosity of cracks. However, the bulk modulus of rock solid varies inversely with the porosity of round pores in Figure 6 (right), which may result from stronger quartz cementation with decreasing total porosity.



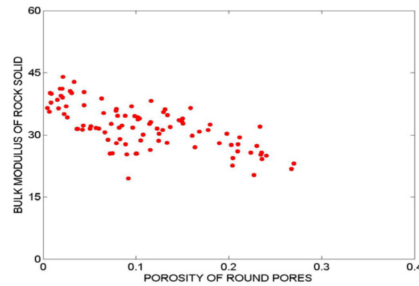


FIG. 6. Bulk modulus of rock solid versus porosity of cracks (left) and porosity of round pores (right).

Clay reduces the elastic moduli and velocities (Castagna et al., 1985; Eberhart-Phillips et al., 1989; Han et al., 1986; Kowallis et al., 1984; Miller and Stewart, 1990; Sams and Andrea, 2001; Tosaya and Nur, 1982) since clay has intrinsically softer elastic moduli (Kowallis et al., 1984) and microporosity (Hurst and Nadeau, 1995; Khaksar et al., 1999; Kowallis et al., 1984). As discussed previously, clay content does not have close association with the elastic moduli of rock solid. Its effect on the elastic moduli of rocks may be ascribed to its role in changing the pore aspect ratio spectrum. Xu and White (1995) set the pore aspect ratio to be 0.15 for sand-related pores and 0.04 for clay-related pores, attributing velocity reduction in clay-bearing rocks chiefly to flatter pores in clay. Kowallis et al. (1984) also assumed microporosity in clay is somewhat flatter than porosity. In contrast, Bryant and Raikes (1995) employed a model where pore shape is independent of clay. Scanning Electron Microscope (SEM) images show that clay aggregates look like a heap of books with both thin flat pores and equant pores (Khaksar et al., 1999). It appears that clay has more flat pores than other fragments and grains because of numerous flat pores between ‘pages’. However, these very thin flat pores are easily closed with small stress. Clay may not have impact on pore geometry. Figure 7 (a) indicates a weak correlation between the porosity of cracks and clay content. Figure 7 (b) may exhibit inverse correlation of the porosity of round pores with clay content.

Clay content weakens the elastic moduli of rock solid and increases the porosity of cracks. But the influence is not dominant, as evidenced by the low correlation coefficients. This observation may help explain why clay content is not a major factor in determining the elastic moduli and velocities of sandstones.

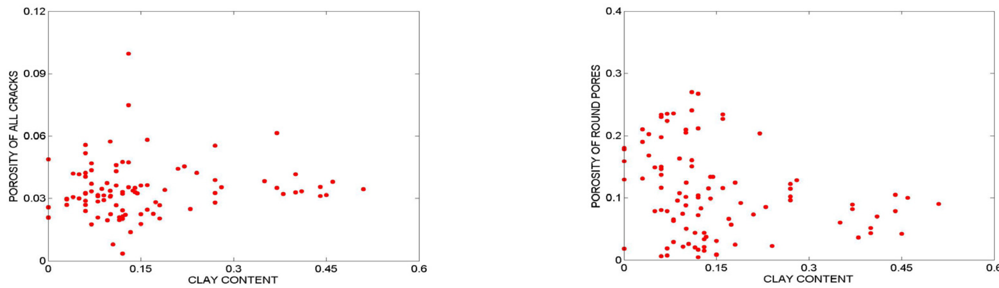


FIG. 7. Clay content versus porosity of cracks (left) and round pores (right).

INFLUENCE OF THE PORE GEOMETRICAL MODEL ON ELASTIC PROPERTIES

Dry elastic moduli

The dry bulk and shear moduli were calculated using the KT modelling based on the pore aspect ratio spectra inverted from velocity measurements and the elastic moduli of rock solid, and were then plotted against total porosity. As shown in Figure 8, the upper bound of the moduli increases with decreasing total porosity, with more scatter at low porosity. This triangular distribution may result from the variability in elastic moduli of rock solid or in pore geometry. The latter refers to the pore aspect ratio spectrum. In order to examine the effect of these two factors, it is necessary to separate them. First, the elastic moduli of rock solid set at K_s=37 GPa and μ_s=44 GPa for all samples, the dry elastic moduli were recalculated with the same pore aspect ratio spectra. Second, we tried to employ one parameter (called pore geometrical parameter) to represent the combined effect of all round pores and cracks on the elastic moduli. The fractional volumes of pores of all pore aspect ratios were weighted and summed. More weight is given to low aspect ratio pores (thin cracks) because low aspect ratio pores at a given fractional volume decrease the elastic moduli more than high aspect ratio pores. The sum of the weighted fractional volumes of all pores is one parameter and should uniquely determine the elastic moduli if the variation in the elastic moduli of rock solid is disregarded. The weight coefficients are 2.9 for round pores and the inverse of the pore aspect ratios for pores of lower aspect ratios (see Appendix for detail).

In Figure 9, we plotted the pore geometrical parameter against the dry elastic moduli calculated assuming constant K_s and μ_s. As predicted, the pore geometrical parameter is almost perfectly correlated with the dry bulk modulus and also strongly correlated to the dry shear modulus. The worse correlation for shear modulus is due to slightly different weight coefficients, which are neglected in this study.

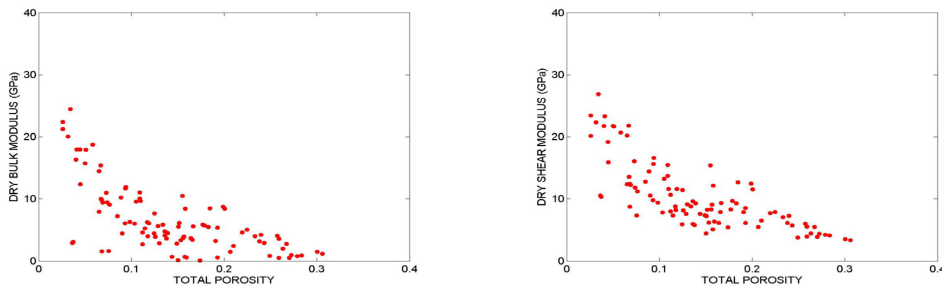


FIG. 8. Total porosity versus dry bulk (left) and shear (right) moduli.

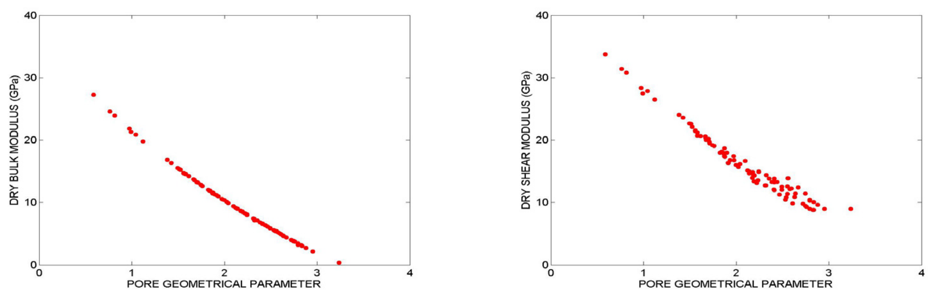


FIG. 9. Pore geometrical parameter versus dry bulk (left) and dry shear moduli assuming the constant elastic moduli of rock solid.

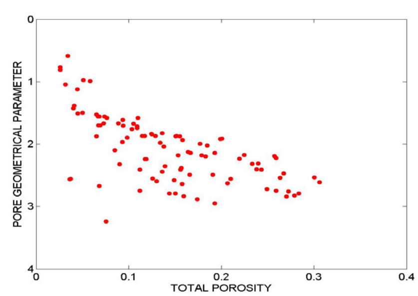


FIG. 10. Total porosity versus pore geometrical parameter.

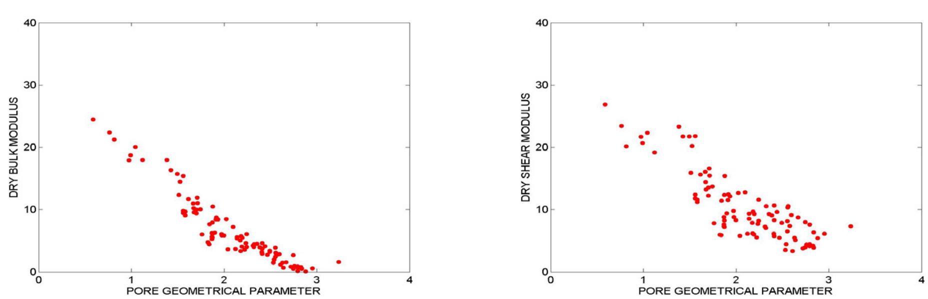


FIG.11. Pore geometrical parameter versus dry bulk (left) and dry shear (right) moduli.

Now we can determine which factor, pore geometrical parameter or elastic moduli of rock solid, is more significant in determining the triangular distribution in Figure 8. Figure 10 cross-plots total porosity versus pore geometrical parameter. The reverse of y-axis order is for convenience of comparison due to the inverse variation of the elastic moduli with the pore geometrical parameter. This diagram is very similar to Figure 8 especially for the bulk modulus. Moreover, the correlation coefficients between pore geometrical parameter and dry elastic moduli average 0.9, as indicated in Figure 11. On the other hand, the dry elastic moduli are not well correlated to the elastic moduli of rock solid with the approximate correlation coefficient of 0.6 as seen in Figure 12. It can be concluded that pore geometry is the more important than the elastic moduli of rock solid in determining the dry elastic moduli in sandstones, agreeing with the observations by other authors such as Zimmerman (1991).

Clay content has little effect on the distribution of the dry elastic moduli with total porosity (Figure 8), despite a large range of values. Figure 13 reflects this dissimilarity. The clay content is also poorly correlated with the dry elastic moduli in Figure 14. At a given total porosity, the pore geometrical parameter for each sample can be resolved into two parts, one from round pores and the other from cracks. As seen in Figure 15, the round pores have a consistent contribution to the strength of rocks because of their strong correlation with total porosity. The contribution from cracks is more highly variable. Disregarding the effect of the elastic moduli of rock solid, the cracks cause the variation in dry elastic moduli at a given total porosity. As seen in Figure 15, the upper bound in Figure 8 is the combined effect of the round pores and the upper bound of cracks.

KT modelling indicates that the decrease in the elastic moduli due to introduction of pores will be larger for rocks of high elastic moduli than for rocks of low elastic moduli. In other words, rocks of high elastic moduli are more sensitive to addition of pores. The elastic moduli solely due to round pores increase approximately linearly with decreasing porosity. Consequently, adding a fixed amount of cracks would decrease the elastic moduli more for low-porosity sandstones than for high-porosity sandstones. This is equivalent to saying low-porosity sandstones are more sensitive to cracks.

In summary, pore geometry is the major factor contributing the dry elastic moduli in sandstones. The variability in the elastic moduli at any given porosity is caused chiefly by the variability in cracks and the variability increases with decreasing porosity. The upper bound of the triangular distribution in Figure 8 is a result of both the linear trend of round pores and the upper bound of cracks.

Dvorkin et al. (1996) proposed a similar hypothesis to interpret the triangular distribution shown in Figure 11. They assume that the elastic moduli at any given porosity is a linear interpolation between zero at critical porosity and the elastic moduli of rock solid at zero porosity, i.e., $M=M_s(1-\phi/\phi_c)$. This assumption is equivalent to only considering round pores in our pore geometrical model. With cracks in the rock solid, M will change between $M_s (1-\phi/\phi_c)$ and $M_{s\text{-}crack} (1-\phi/\phi_c)$. The amplitude of the variation of elastic moduli due to introduction of cracks is zero at critical porosity and reaches the maximum at zero porosity, causing a triangular distribution as seen in Figure 8. However, they did not consider more variability in cracks at low porosity.

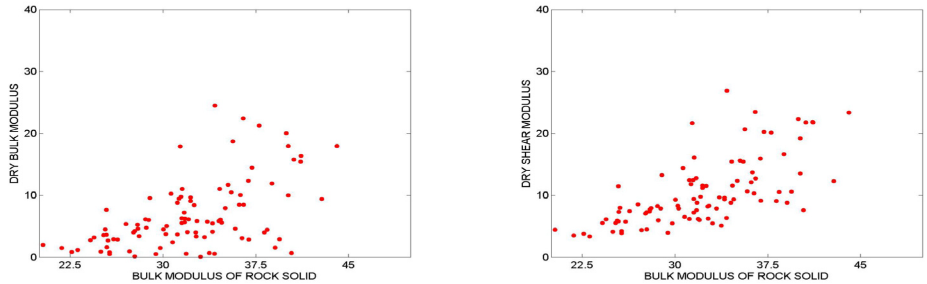


FIG. 12. Bulk modulus of rock solid versus dry bulk (left) and dry shear (right) moduli.

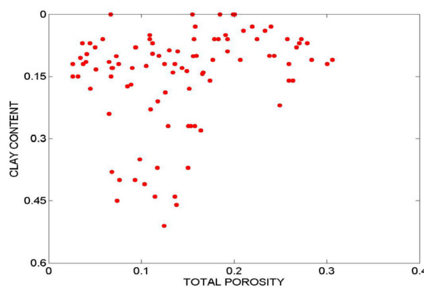


FIG. 13. Total porosity versus clay content.

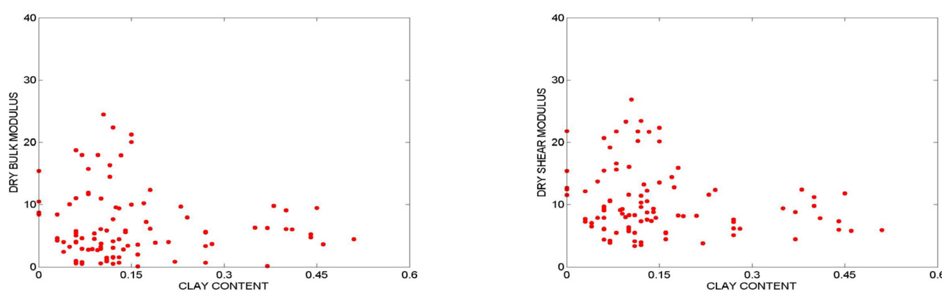


FIG. 14. Clay content versus dry bulk (left) and dry shear (right) moduli.

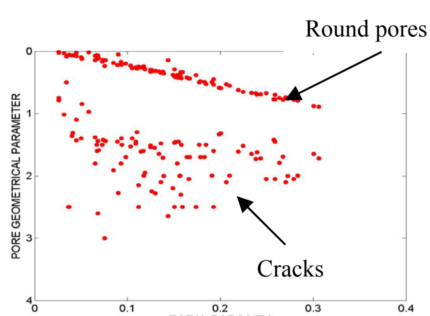


FIG. 15. Total porosity versus pore geometrical parameter.

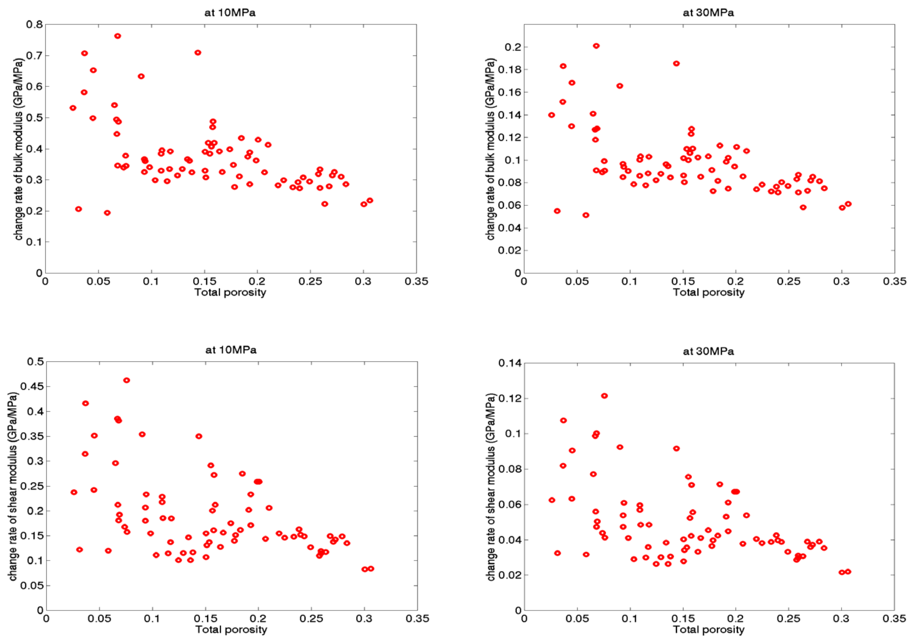


FIG. 16. Total porosity versus change rate of dry bulk modulus at 10 MPa (upper left) and 30 MPa (upper right) and dry shear modulus at 10 MPa (lower left) and 30 MPa (lower right).

Stress sensitivity of dry sandstones

The increase of the elastic moduli with increasing effective pressure is affected by the amount of cracks to be closed. As stated previously, the elastic moduli at any given porosity vary due to the variability in the amount of cracks. As a result, when effective pressure increases, sandstones with more cracks will have a greater increase in the elastic moduli than those with few cracks. This accounts for the variability in the dependence of the elastic moduli on effective pressure in Figure 16. Similarly, lab measurements by Eberhart-Phillips et al. (1989) showed that effective pressure dependence of velocities for samples Gulf124155 and IndianaDark 2 has distinctly different behaviour at low effective pressure despite porosities being similar (25.6% and 26.1% respectively).

The increase of the elastic moduli with effective pressure is also affected by the quantity of round pores or total porosity. Assuming the same amount of cracks, sandstones at low porosity will be more sensitive to closing of cracks at elevated effective pressure than those at high porosity. The trend is also strengthened by more variability in cracks at lower porosity. In Figure 16, the rate of change of the elastic moduli with effective pressure (dKd/dP and dUd/dP) increases on average with decreasing porosity. Dvorkin et al. (1996) arrived at the same conclusion that ‘In absolute terms, low-porosity sandstones may be more sensitive to effective stress than high-porosity sandstones.’

The above results suggest that the plot of porosity versus rate of change of the elastic moduli with effective pressure will form a triangular distribution similar to that in Figure 8. As indicated in Figure 16, at high porosity, closure of cracks incurs a small increase in elastic moduli, whereas, at low porosity, the amount of cracks plays an important role in determining the increase of elastic moduli with increasing effective pressure. The extreme cases are at critical porosity, where there is slight response to closure of rocks, and at zero porosity, where there is highest sensitivity to closure of cracks since cracked solid and pure solid differ considerably in elastic moduli (Dvorkin et al., 1996). Dvorkin et al. (1996) plotted the difference between elastic moduli at two values of effective pressure versus porosity and obtained similar triangular distributions.

In addition, the increase of the elastic moduli with effective pressure is affected by effective pressure. At low effective pressure, the closable cracks exist and sandstones are sensitive to effective pressure. At high effective pressure, however, most cracks are already closed and the rate of change of the elastic moduli with effective pressure becomes very small. The rate of change of elastic modul in Figure 16 dwindles with increasing effective pressure.

Linearity of the elastic moduli with total porosity

The linear equations of velocities and porosity developed by some authors (Castagna et al., 1985; Han et al., 1986; Kowallis et al., 1984; Tosaya and Nur, 1982) are often employed to predict porosity from seismic velocity or amplitude. Without the linearity assumption, inversion would be intractable. According to our geometrical model, linearity is possible in the following cases.

Linearity caused by the distribution of cracks. In a general sense, the distribution of cracks is independent of porosity because sandstones come from a wide range of geological settings. In a specific area or depth interval, however, the porosity of cracks may be related to total porosity. As discussed previously, porosity reduction results from compaction and cementation. For the small subset of sandstones sampled from sandstone reservoirs within the gas-producing fields in the southern Cooper basin, South Australia, quartz cementation plays an important role in controlling reservoir quality (Rezaee and Tingaate, 1997). As a result, in Figure 5 the porosity of cracks increases with increasing porosity and an approximately linear relationship between porosity and elastic moduli can be observed in Figure 17. The slope depends on the relationship of cracks and total porosity. If the amount of cracks varies inversely with porosity, the straight line tends to be gentle. The theoretical model of cementation (Dvorkin, 1991) assumes two types of cementation. In the first type, cements precipitate in grain contacts and the slope is steep. In the second type, cements fill space of large pores and a flat slope results. The variation in cracks may be the major reason why the slope of velocity versus porosity differs among different authors (Castagna et al., 1985; Han et al., 1986; Kowallis et al., 1984; Toyasa and Nur, 1982).

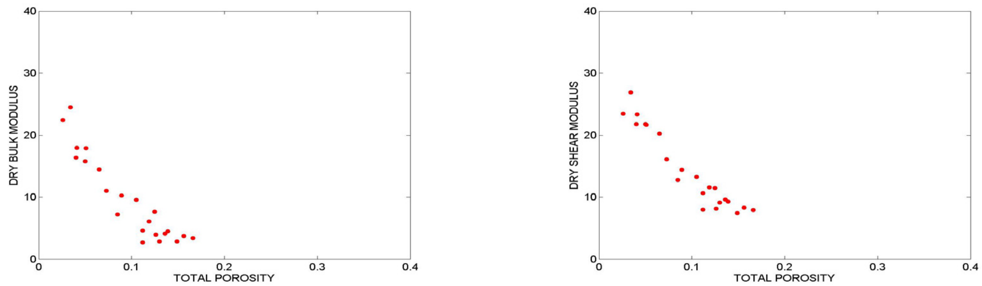


FIG. 17. Total porosity versus dry bulk (left) and dry shear moduli for a small set of samples.

Linearity caused by high effective pressure. At high effective pressure, most cracks are closed and the majority of remaining pores are round. The elastic moduli are determined chiefly by round pores, which are linearly related to total porosity in Figure 3. Linearity could then be predicted between total porosity and elastic moduli. The elastic modul at 100 MPa in Figure 18 are more linearly correlated with total porosity than those at atmospheric pressure in Figure 8. Similarly, Dvorkin et al. (1996) stated that elastic moduli and total porosity are poorly correlated at 5 MPa, but a good linear fit can be found at 40 MPa. Using a different pore geometrical model, Vernik (1997) predicted linearity at stress magnitudes beyond the closure of cracks introduced by natural or coring-related processes. Khaksar and Griffiths (2000) mentioned that at elevated effective stress, porosity is the rock property best correlated with velocity.

Linearity of the bolk modulus caused by saturation with a less compressible fluid When a less compressible fluid such as water saturates dry sandstones, the bulk modulus is not as sensitive to cracks, especially low-aspect-ratio cracks. In other words, the variation in dry elastic moduli due to the amount of cracks at given porosity decreases when pore space is filled with water. It is verified by numerous lab measurements that the gradient of compressional velocity with effective pressure is always smaller when sandstones are saturated with water than with air or vacuum. Figure 19 is the cross plot of porosity versus water-saturated bulk modulus calculated from Figure 8 (left) assuming the bulk modulus of saturating fluid of 2.3 GPa. Obviously it is more linear than in Figure 8 (left). At ultrasonic frequencies the linearity would be further strengthened.

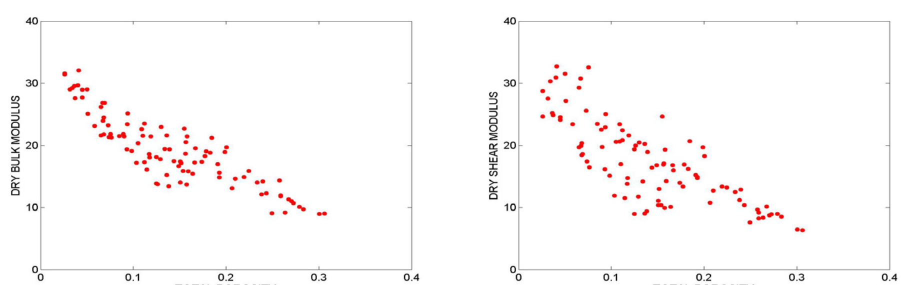


FIG. 18. Total porosity versus dry bulk (left) and dry shear moduli at 100 MPa.

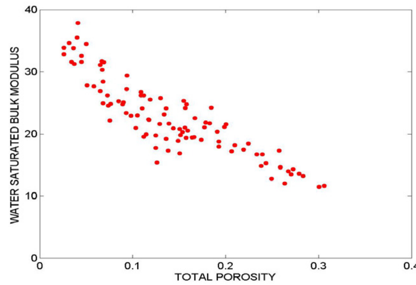


FIG. 19. Total porosity versus wet bulk modulus.

Vp/Vs ratio for wet sandstone

The elastic moduli depend directly on rock solid, pore geometry and pore fluids. At a given pore aspect ratio spectrum and pore fluid, rock solid or lithology becomes the only influencing factor. It is therefore theoretically reasonable to employ the elastic moduli in rocks to infer lithology. Among the elastic moduli, Vp/Vs ratio, or equivalently Poisson's ratio, is probably a better parameter than both the bulk and shear moduli because the ratio is less sensitive to pore geometry. In the past few decades, many people have observed the correlation of lithology with the Vp/Vs ratio (Benzing, 1983; Domenico, 1984; Kithas, 1976; Miller and Stewart, 1990; Pickett, 1963; Rafavich et al., 1983; Wilkens et al., 1984). These examples have a wide range of porosity (Pickett, 1963; Domenico, 1984; Tatham, 1982). The pore geometry is also expected to vary greatly. The observations suggest that the Vp/Vs ratio may depend chiefly on lithology. The Vp/Vs ratio of rock solid and water-saturated sandstones are cross-plotted in Figure 20. The Vp/Vs ratios for rock solid were calculated from the bulk and shear moduli of rock solid we found based on the method described earlier. The Vp/Vs ratios for water-saturated sandstones were obtained using KT modelling from the pore aspect ratio spectra and the elastic moduli of rock solid. A linear correlation coefficient of 0.86 exists between the Vp/Vs ratio of the rock solid and the saturated rock, implying that the Vp/Vs ratio in wet sandstones can predict the Vp/Vs ratio in rock solid. In other words, to the first-order approximation, the Vp/Vs ratio is a function of lithology. Figure 21 displays a cross plot of Vp/Vs ratio versus total porosity. The data points are poorly correlated (Miller and Stewart, 1990). It is impractical to establish a relationship between total porosity and Vp/Vs ratio. Figure 22 shows across plot of Vp/Vs ratio versus the porosity of all cracks. No correlation can be found between them. Lithology has the most influence on the Vp/Vs ratio in water-saturated sandstones and porosity and cracks are not well correlated to the Vp/Vs ratio. The result is consistent with many observations. Wilken's (1984) stated that variations in Vp/Vs ratio due to total porosity and pore geometry are around 0.1 whereas the change due to composition is 0.4. Domenico (1984) pointed out that separation of sandstone, limestone and dolomite appears to result from the difference in Poisson's ratio of the matrix materials (rock solid). Pickett (1963) found Vp/Vs values of 1.9 for limestone, 1.8 for dolomite and 1.6-1.75 for sandstones. Benzing (1983) indicated a shift from lower Vp/Vs (1.66-1.82) in clastic-dominated rocks to higher Vp/Vs (1.82-1.89) in carbonate-dominated rocks. Miller and Stewart (1990) obtained an average Vp/Vs ratio of 1.60 for sandstones and 1.89 for limestones

Despite the correlation of wet sandstones and lithology in Vp/Vs ratio, the overlap for different lithologies poses a real challenge for the application of this method. The empirical order of Vp/Vs increase from siliclastic to carbonate (Benzing, 1983; Miller and Stewart, 1990; Pickett, 1963) for lithological discrimination are valid only in certain cases. Due to variation in clay content, siliclastic rock solid can overlap the range of carbonate. For example, the Vp/Vs ratio of the rock solid in our sandstones ranges from 1.41 to 1.88. As a result, the Vp/Vs ratio for wet sandstones (at 5 MPa) spans the range of 1.58 to 2.02 (Han et al., 1986). These high values can not be misinterpreted as limestones.

Relation between Vp and Vs

The Vp and Vs relationship has been extensively studied (e.g, Castagna et al., 1985). Most authors believe that Vp and Vs are linearly related but the linear coefficients vary with location and with geological age. It seems from Figure 23 that Vp and Vs are well correlated even though the samples are from quite different geological settings. Vp and Vs for wet sandstones can be related as follows:

$$Vp=1.22\ Vs+1.29$$

For dry sandstones, we have:

$$Vp=1.66\ Vs-0.58$$

Two straight lines in Figure 23 are converging at high velocities. So the Vp and Vs relationship at high velocities may not be suitable for fluid identification.

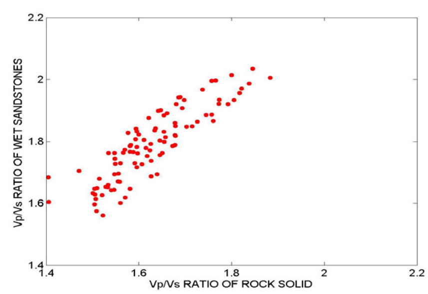


FIG. 20. Vp/Vs ratio of wet sandstones versus Vp/Vs ratio of rock solid.

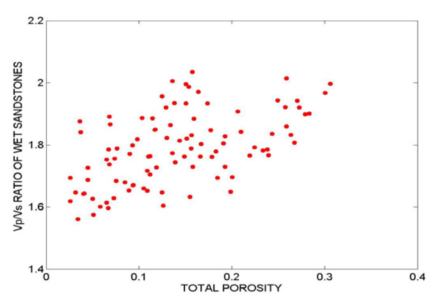


FIG. 21. Vp/Vs ratio of wet sandstones versus total porosity.

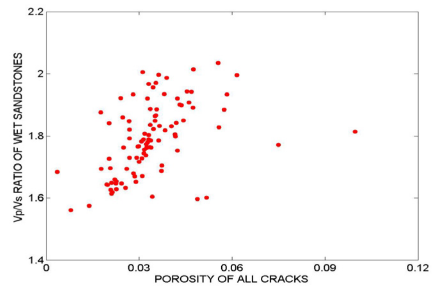


FIG. 22. Vp/Vs ratio of wet sandstones versus porosity of all cracks.

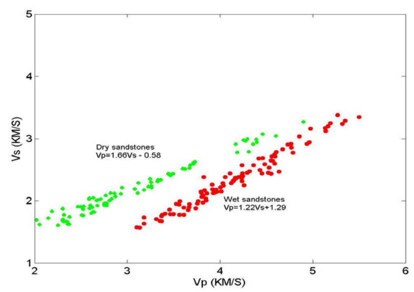


FIG. 23. Vp and Vs relationship for wet (red) and dry (green) sandstones.

CONCLUSIONS

Pore geometry represented by the pore aspect ratio spectrum is one of three direct factors to influence the elastic moduli. It is found from inverse KT modelling that the porosity of round pores is in direct proportion to total porosity, and that the porosity of cracks is small and varies randomly for a large set of samples from different geological backgrounds but may be linearly related to total porosity for a small subset from some specific area. This pore geometrical model is central to predicting the elastic properties in sandstones. The dry elastic moduli at a given total porosity vary chiefly due to cracks and the variability increases with decreasing porosity. This causes the cross-plot of dry elastic moduli versus total porosity to scatter in a triangular form, and is also responsible for high stress sensitivity of low-porosity sandstones. The linear correlation between total porosity and elastic moduli results from strong correlation between crack porosity and total porosity, and it becomes more pronounced at high effective pressure or at saturation with a less compressible fluid such as water. The Vp/Vs ratio in wet sandstones is affected by the Vp/Vs ratio of rock solid and pore geometry, but the former has far more influence, which may provide the rock physics basis for lithology identification using the Vp/Vs ratio. Vp and Vs are well correlated for both dry and wet sandstones. The two straight lines are separate at low velocities but converge at high velocities. In addition, clay content has been found to have little relationship to pore geometry but has some connection to the elastic moduli of rock solid.

ACKNOWLEDGEMENTS.

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APPENDIX: CALCULATION OF WEIGHT COEFFICIENTS

In order to use one parameter to represent the total combined effect of all pores of different aspect ratios on the dry elastic moduli, the fractional volume of pores for each aspect ratio needs to be weighted with a coefficient, which is related to the aspect ratio. The lower the aspect ratio, the higher the coefficient. The coefficients may be found by equating the dry elastic moduli bearing pores of single aspect ratios. In Figure A1, the bulk modulus versus the fractional volume of round pores is approximated by a straight line with the intercept equal to K_s. Likewise, the straight lines were obtained for other pores of low aspect ratios, as shown in Figures A2 to A4. Equating the dry bulk modulus at any point leads to:

$$-46.9C_{\alpha=1} = -167.2C_{\alpha=0.1} = -1598.8C_{\alpha=0.01} = -15952.8C_{\alpha=0.001}$$

i.e.,

$$2.9C_{\alpha=1} = 10.5C_{\alpha=0.1} = 100.2C_{\alpha=0.01} = 1000C_{\alpha=0.001}$$

i.e.,

$$2.9C_{\alpha=1} \cong (1/0.1)C_{\alpha=0.1} \cong (1/0.01)C_{\alpha=0.01} \cong (1/0.001)C_{\alpha=0.001}$$

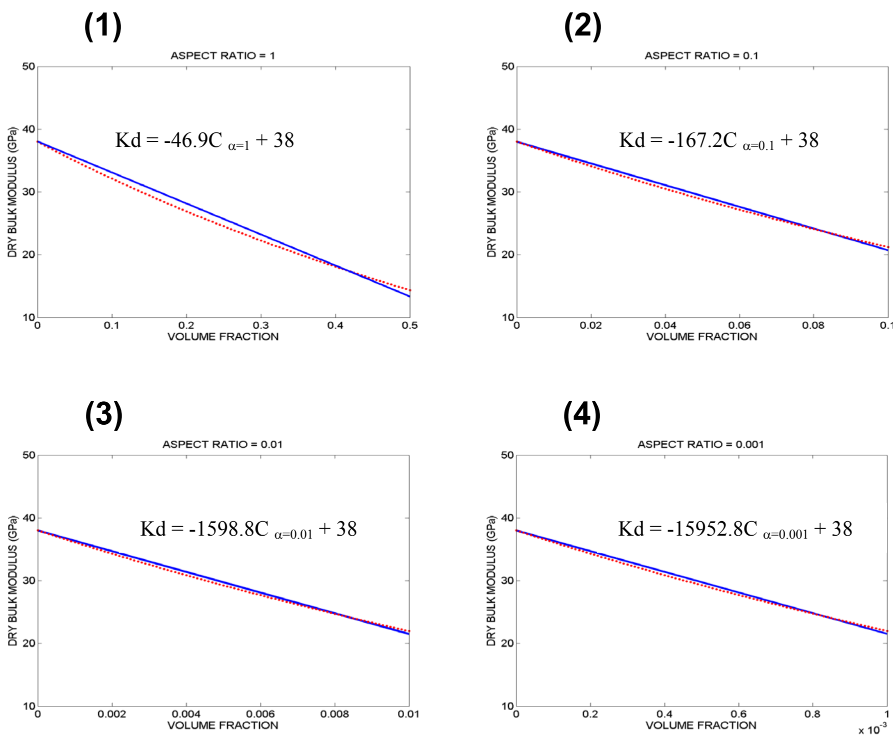

FIG. A. Dry bulk modulus as a function of the fractional volumes for aspect ratios of 1 (1), 0.1 (2), 0.01 (3) and 0.001 (4)

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