

## Program notes and results: Ray synthetic seismogram programs for P-S<sub>V</sub> wave propagation in isotropic, anisotropic and viscoelastic plane layered media

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### INTRODUCTION

When processing seismic data modelling programs can be useful tools provided that they are fast, reasonably accurate, and require minimal user interaction in the form of data preparation. Also, the results should be in a form that makes their display as uncomplicated as possible and in a form that allows for a straightforward interpretation of events of interest. The general program type that is often used for this purpose is a plane parallel layered model for the coupled  $PS_V$  wave propagation amplitude versus offset (AVO) analysis of a geological structure. Although far from realistic, this class of programs provides a basic means of obtaining at least a general trend of the seismic response of a specified subsurface geology and as a consequence an indication of how accurately a data processor may be interpreting field data.

A determining factor in the usefulness of these programs is that their run time be in the range of seconds or at the most minutes for models with a large number of layers and many offsets. This allows a user to compute synthetics for a series of models in an attempt to iterate to the optimal solution of the problem being considered in a time efficient manner. Past experience has shown that this and the ease of use determine whether or not the software is used as an interpretation or modelling tool.

Programs also exist for more complicated geological structures in both two and three dimensions. This, however, introduces the not insignificant process of model building, which is almost a science in itself. Efficient use of these program types requires one or more persons dedicated solely to the tasks of building models, model databases and computing the synthetics, as the complexity of the model translates into a comparable complexity in the synthetic program with respect to data input as well as run time and turn around time. This takes control of the modelling aspect away from the primary user who, again from past experience, will not be inclined to use this service.

The three programs described below are of the first type described above, each with a mild increase in the degree of difficulty related to the input. They all assume a plane parallel layered geometry with both source and receivers on the surface and progress from isotropic layers (*isoavo*), to transversely isotropic layers with the axis of anisotropy aligned with the plane interfaces (*tiavo*), to viscoelastic layers with frequency dependent  $P$  and  $S_V$  quality factors,  $Q(\omega)$ , and hence frequency dependent complex valued velocities (*visavo*). The principal output from these three programs consist of two files containing the synthetic traces for the vertical and horizontal, generally complex, amplitude components of particle displacement written in SEG-Y format.

A very select subset of rays is used in the computations allowing for a large number of layers to be considered. Models with large number of layers usually consist of  $P$ -wave,  $S$ -wave and density logs together with the depth indication of these parameters. A blocked

model is probably preferable but this task can be time consuming. The use of logs extends the range of applicability for the use of asymptotic ray theory (ART), used in all of the above programs, to its limits, (Hron and Kanasewich., 1971; Cervený and Ravindra, 1970; Cervený et al., 1976; Cervený, 2001).

The programs described here are a preliminary subset of a more complete set. Other members include the vertical seismic profile (VSP) versions of the abovementioned programs. These are all in some state of flux at present, the most tested being *tivsp*, a program to produce synthetic traces resulting from surface sources and down hole receivers for a plane layered transversely isotropic medium

## PROGRAM HISTORIES AND DESCRIPTIONS

The first two of these programs, *isoavo* and *tiavo*, are radically modified versions of previous programs designed to compute synthetic traces in media composed of plane, parallel, homogeneous layers. What remains of these original industry production source codes, which (to put things in perspective) were typed on computer card decks, are just the concepts and certain aspects of the program construction, as they have been completely rewritten to reflect the requirements of the present day industry user. In addition, these major modifications were required to accommodate not only the changes in, and the introduction of, new computer languages, but also to take advantage of certain attributes of UNIX based workstations and PC platforms. Also, improved derivations of quantities such as geometrical spreading and reflection and transmission coefficients have been introduced. (Early FORTRAN or equivalent compilers did not have intrinsic complex data types and functions.) Vectorization and parallel processing possibilities have been taken into consideration. As all computations for these two programs are done in the time domain they are quite fast. Time domain computation requires that the Hilbert transform of the source wavelet be known. For this reason, the Gabor wavelet<sup>1</sup> is the default; all others including user defined pulses having been disabled for the present.

The original version of *isoavo* was written by F. Hron in the 1960's. His version employed a ray generation scheme known as the *partial ray expansion*, which produced a ray set that very closely approximated the total wavefield (Hron, 1971; Hron, 1972, and Hron et al., 1974). However, for unconverted arrivals, the number of rays for a  $J$  layer model is the sum of the first  $J$  rows of Pascal's triangle. The number of total rays with up to 2 or 3 conversions in a  $J$  layer model increases at an incredible rate in comparison. As a consequence even if constraints based on an a priori numerical analysis are imposed the number of layers, which can reasonably be considered using this ray generation scheme is limited. However, a synthetic comparable to an *exact* one computed by numerical methods is obtained in a fraction of the time and other computer resources. As mentioned previously amplitudes are computed using zero order Asymptotic Ray Theory (ART) and the associated critically refracted (head) wave computations have been disabled.

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<sup>1</sup> Gabor wavelet:  $g(t) = \sin(\omega_0 t) \exp\left[-(\omega_0 t / \gamma)^2\right]$ , with  $\omega_0 = 2\pi f_0$ ,  $f_0$  being the predominant frequency of the wavelet and  $\gamma$  is a factor controlling the side lobes in the time domain and is related to the waist parameter in the frequency domain. The Hilbert transform of the source wavelet  $g(t)$  is defined as  $H\{g(t)\} = \cos(\omega_0 t) \exp\left[-(\omega_0 t / \gamma)^2\right]$ .

The program *tiavo* is similar in structure to *isoavo*, the difference being that in this case synthetic traces are produced for a model comprised of transversely isotropic plane parallel layers in which the axes of anisotropy are aligned, in all layers and the underlying halfspace, with the plane interfaces. Run time is increased marginally due to the more complex nature of the model and the additional computations involved. The original program for this type of medium was written circa 1975 and has probably undergone more modifications than any of the other programs discussed here. Zero order asymptotic ray theory is used and critically refracted (head wave) events have again been disabled. Even though the plane wave reflection and transmission and surface conversion coefficients and ray tracing and related equations are computed using exact equations, the program is valid only for what has been termed by Schoenberg and Helbig (1996) as *mild* anisotropy. This indicates that the ray surfaces cannot have triplication points on them which is not an unreasonable constraint given the degree of anisotropy observed in geological structures. This is equivalent to requiring that the  $P$  and  $S_V$  slowness surfaces are convex.

The program *visavo* is a frequency rather than time domain program in which attenuation is introduced into the media using a frequency dependent quality factor  $Q = Q(\omega)$ . It is the latest of the AVO series of programs, being written in the 1990s. It has essentially the same framework as the previous two programs but is possibly more efficient as the version discussed here is all new code designed specifically for the extensions inherent in SUN Fortran and C compilers. As indicated, it is a frequency domain program which means that the run time is highly dependent on the width, in the frequency domain, of the spectrum of a band limited source wavelet. Again the Gabor wavelet is used, but in other existences any number of defined or digitized wavelets could be incorporated. Frequency domain synthetic programs require that the ray amplitude be computed at each frequency point in that domain where the spectrum of the source wavelet is not numerically equal to zero. This is in contrast with the previous two frequency independent programs where the generally complex amplitude of a ray need be computed only once for each offset.

### RAY GENERATION

At present, all three programs include only  $P$ -primary ( $PP$ ) and once converted  $PS_V$  ( $P$  down to reflector,  $S_V$  up to the surface receiver) reflected rays. Several orders of unconverted multiples and primary type converted rays with one and two conversions, together with the same scenario assuming a point source of  $S$  waves may also be included in all three programs, but have been disabled for the present. Even the addition of an extremely small subset of the partial ray expansion mentioned in the previous sentence dramatically increases run time, even if the concepts of kinematic and dynamic groups (Hron, 1972) are employed to reduce the number of computations required. In addition to the ray types listed above, a user specified ray file may be added either alone or in conjunction with the rays generated above.

## PROGRAM INPUT

Input to all three programs is basically the same and is of the following pattern which upon program initiation prompts the user for the following information:

- Input a **rootfilename**. This name is used to create and/or open:
  1. **rootfilename.data** – a file which exists and contains the velocity/density – depth data. If the model consists of  $J$  layers over a halfspace there are  $J+1$  lines in this file. The  $J+1$  layer is indicated by a zero value in the thickness field. A check is made to see that the file exists and that the number of layers in the model does not exceed the maximum allocated number.
  2. **rootfilename.print** – an output file which contains data relevant to the current job being run.
  3. **rootfilename\_v.segy** – an output file containing the vertical component of the synthetic traces computed at the offsets and time window specified in SEG Y format.
  4. **rootfilename\_h.segy** – an output file containing the horizontal component of the synthetic traces computed at the offsets and time window specified in SEG Y format.
  5. Auxiliary files: These are files which may also be generated. They include **rootfilename.amps**, which contains a code identifying the specific ray together with its complex amplitudes and traveltime and **rootfilename.rays**, in which are written a unique integer ray code together with the number of ray segments ( $P$  and  $S_V$ ) in each layer in which the ray propagates. For models with a large number of layers and/or added ray groups these files are not created.

Input the predominant frequency of the source wavelet together with its damping factor,  $f_0$  and  $\gamma$ . In the program *visavo*, a third field is requested on this line. A value  $f_{MAX}$  is requested with  $\Delta t = 1/f_{MAX}$ . At present the number of points in this frequency domain program is 2048 so that the maximum trace,  $t_{MAX}$  is given by  $t_{MAX} = 2048 \Delta t = 2048/f_{MAX}$ .

- Input the maximum length of traces in seconds,  $t_{MAX}$ . (Not required in *visavo*.)
- Input the sampling interval in milliseconds. (Not required in *visavo*.)
- PP ray primaries to be computed? **t** or **f**
- PS<sub>V</sub> ray primaries to be computed? **t** or **f**

- Number of offsets? First offset? Offset spacing? These 3 values are to be in consistent units with those contained in the input model file **rootfilename.data**.

**Table 1.** Input for *isoavo*. Velocity are in  $km/s^2$ , thicknesses in  $m$ , and densities in  $g/cm^3$ .

Layer #	P Velocity	S <sub>v</sub> Velocity	Layer Thickness	Density
1	<b>4000</b>	<b>2000</b>	<b>1000</b>	<b>2.00</b>
2	<b>5000</b>	<b>2500</b>	<b>300</b>	<b>2.05</b>
3	<b>6000</b>	<b>3000</b>	<b>300</b>	<b>2.10</b>
4	<b>5000</b>	<b>2500</b>	<b>250</b>	<b>2.05</b>
5	<b>7000</b>	<b>3500</b>	<b>150</b>	<b>2.15</b>
H-space	<b>8000</b>	<b>4000</b>	<b>000</b>	<b>2.20</b>

### INPUT DATA FILES

In this section, three simple examples of the data files required to be specified in **rootfilename.data** are discussed. The first to be considered is the simplest, being the input to the plane parallel layered isotropic program *isoavo*. The *data* for this program consists of one line for each layer in the model plus an additional line for the underlying halfspace. The four parameters required for each layer (line) are the *P*-wave velocity, *S<sub>v</sub>*-wave velocity, layer thickness and density. The parameters must be dimensionally consistent both in this file and the data prompted at the terminal. A simple example of a *data* file for this program may be found in Table 1.

Input for the transversely isotropic program *tiavo* requires essentially the same parameters as *isoavo* with the exception of velocity information. A line in the *data* file for this program requires that the velocities be given in  $A_{ij}$  notation, which have the dimensions of  $(\text{velocity})^2$ , rather than the commonly accepted  $(\alpha, \beta, \varepsilon, \delta)$  notation. The reason for the  $A_{ij}$  specification is to attempt to remove the possibility of the introduction of improperly computed values. The  $A_{ij}$  required to be input on each line corresponding to a layer are, in the following order,  $A_{11}$ ,  $A_{33}$ ,  $A_{44}$ , and  $A_{13}$ . In addition the layer thickness and density are required to fully define the layer. As in the previous case a zero entry in the thickness field indicates the underlying halfspace, layer  $J+1$  in a  $J$ -layered medium. A sample *data* input file for *tiavo* is presented in Table 2.

**Table 2.** Input for tiavo. The  $A_{ij}$  are in  $m^2/s^2$ , thicknesses in m, and densities in  $g/cm^3$ .

Layer #	$A_{11}$	$A_{33}$	$A_{44}$	$A_{13}$	Layer Thickness	Density
1	25000000	16000000	4000000	5000000	1000	2.00
2	36000000	25000000	6250000	9000000	300	2.05
3	49000000	36000000	9000000	11000000	300	2.10
4	36000000	25000000	6250000	9000000	250	2.05
5	64000000	49000000	12250000	16000000	150	2.15
H-space	81000000	64000000	16000000	20000000	000	2.20

The program *visavo* requires a similar but more complicated system of introducing variables into the program. This is due to the fact that in each layer, in addition to reference  $P$  and  $S_V$  velocities, the quality factors corresponding to the  $P$  and  $S_V$  ray propagation,  $Q_P(\omega)$  and  $Q_{S_V}(\omega)$ , must be specified together with a reference frequency,  $f_R = \omega_R/2\pi$ , at which the quality factors are assumed to be known. This manner of introducing attenuation into a layer is consistent with the method given in Futterman (1962) and Azimi et al. (1968). It is not the intent of this report to present the theory related to wave propagation in viscoelastic medium. Consequently a potential user of this program should have some basic knowledge of this topic. The other fields on an input line defining a layer contain the layer thickness and density. Table 3 contains a sample input *data* file.

## NUMERICAL RESULTS

A simple 5 layer isotropic homogeneous model has been constructed for computing results for the program module isoavo and the models used to obtain results using the programs tiavo and visavo are just variations on the five layered model. As previously mentioned, a Gabor wavelet is the default at present. The time sampling rate in all programs is 1ms. A wavelet of this type with a predominant frequency of 30Hz and a damping factor of  $\gamma = 5$  is used for computing synthetic traces when using isoavo and visavo. The wavelet used in tiavo is a 30Hz Ricker wavelet. As a 30Hz Gabor wavelet with a damping factor of  $\gamma = 5$  does not differ significantly with a 30Hz Ricker wavelet the use of the two wavelets was done as a check on the introduction of the wavelets into the synthetics in the time domain. Specifically, a check for the inclusion of the proper

**Table 3.** Input for *visavo*. Velocities are in  $m/s$ , thicknesses in  $m$ . reference frequencies ( $F_{REF}$ ) in  $Hz$ , densities in  $g/cm^3$  and the quality factors,  $Q_P$  and  $Q_{SV}$  are dimensionless. See text for more details.

LAYER #	P-VEL.	S <sub>V</sub> -VEL.	LAYER THICK	DENSITY	Q <sub>P</sub>	Q <sub>SV</sub>	F <sub>REF</sub>
1	<b>4000</b>	<b>2000</b>	<b>1000</b>	<b>2.00</b>	∞	∞	<b>40</b>
2	<b>5000</b>	<b>2500</b>	<b>300</b>	<b>2.05</b>	∞	∞	<b>40</b>
3	<b>6000</b>	<b>3000</b>	<b>300</b>	<b>2.10</b>	∞	∞	<b>40</b>
4	<b>5000</b>	<b>2500</b>	<b>250</b>	<b>2.05</b>	<b>30</b>	<b>25</b>	<b>40</b>
5	<b>7000</b>	<b>3500</b>	<b>150</b>	<b>2.15</b>	∞	∞	<b>40</b>
H-Space	<b>8000</b>	<b>4000</b>	<b>000</b>	<b>2.20</b>	∞	∞	<b>40</b>

number of time domain points of each wavelet; i.e., the location of numerical zero for each pulse was determined and checked.

Only one of the five layers in the viscoelastic modelling program, *visavo*, was given finite values of the quality factors,  $Q_P$  and  $Q_{SV}$ . As the models used for *isoavo* and *visavo* are identical except for the quality factors, some comparisons could be made between the elastic and viscoelastic programs by making a single layer viscoelastic. Making all layers viscoelastic would defeat this. Further, *visavo* is the latest of these three programs and was designed to run most efficiently when all model layers had some degree of viscoelasticity. The ability of the program to handle isotropic layers was checked.

The ray generation in each of the three programs was the same. At each of the 101 offsets for each synthetic section 10 rays were included, these being the primary  $PP$  and  $PS_V$  from the 5 interfaces in the model. The offset range was 0 to 5000m at steps of 50m. A very thick upper layer of 1000m was used so that the  $PP$  and  $PS_V$  arrivals were well separated. The total depth to the underlying halfspace was 1600m giving about a 3 to 1 offset to depth ratio. This is somewhat misleading as a consequence of the thickness of the first layer. A type of global amplitude scaling was applied to all of the  $PS_V$  to put them in a perspective that allows for the identification of individual arrivals. This was done on both the vertical and horizontal components of displacement recorded at the surface and the scaling factors used are indicated on the panels and in the figure captions. Other types of scaling routines in *PROMAX* and *SU* were tested but did not satisfactorily produce the desired results, which prompted the writing of a one time routine. In all three

programs both pre- and post-critical reflections for all arrivals are present in the synthetic sections. This was done to check for a smooth transition in the wavelets between the two regions. As mentioned, critically refracted arrivals were disabled in both *isoavo* and *tiavo*. The computation of an equivalent of these arrival types for *visavo* is ongoing.

The computed results are shown in Figures (1) through (6), with annotations on the panels and in the figure captions. Both vertical and horizontal components of particle displacement are shown for each program type for the models specified in Tables (1) – (3). Surface conversion coefficients for each of the three media types were derived and employed to properly account for energy partitioning for receivers located at a vacuum – solid interface. Some numerical experimentation has been done treating the earth’s surface as a fluid – solid interface (which it is, in fact) and mild changes were seen in the recorded  $PS_V$  particle displacements. These were not enough to rewrite the programs, at least not for the results obtained for the models tested.

### CONCLUSIONS

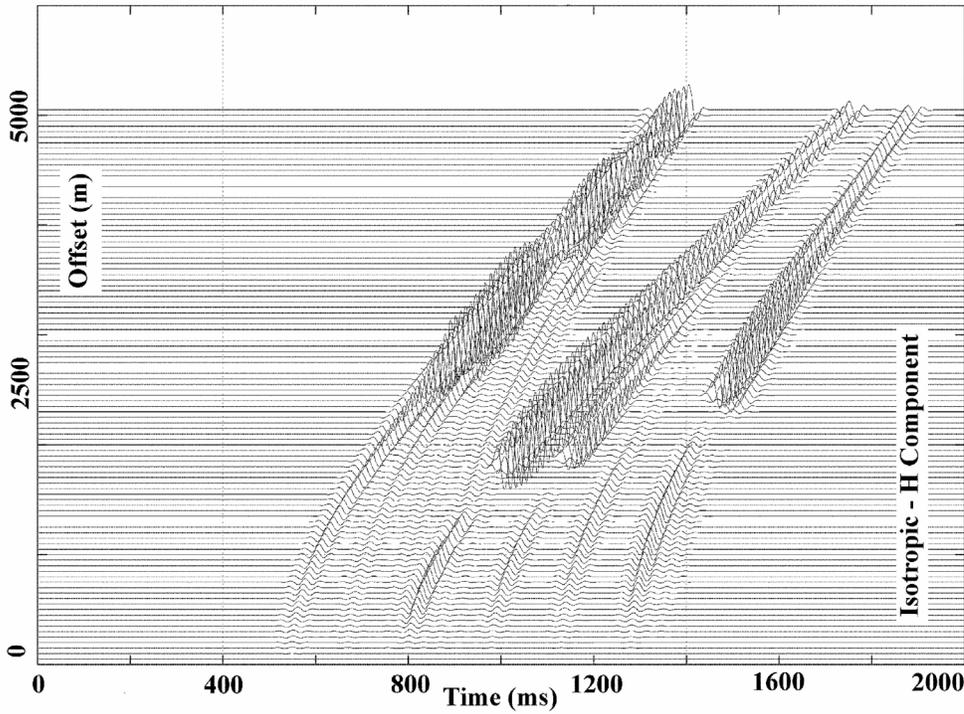
A set of three synthetic seismogram programs for plane layered media of different types have been described and the input and output outlined. These program modules were written in a very simplistic manner in that only a fundamental set of rays comprising a partial ray expansion are considered in the production of the synthetic traces. On a more positive note these programs require a minimum of interaction by the user and can produce at least a gross estimate of the total seismic response. Also, these programs may be used in an iterative scheme for a linearized inversion problem as a result of their reasonably fast execution time even for models with numerous layers and offsets.

This pared down type of program is often more useful than a “one size fits all” that may overwhelm a user with the input required. If users require some specialization of the basic program they may be accommodated with minimal effort by enabling some code that is currently not available, adding plug and play procedures from a database of existing code or by having a customized procedure written.

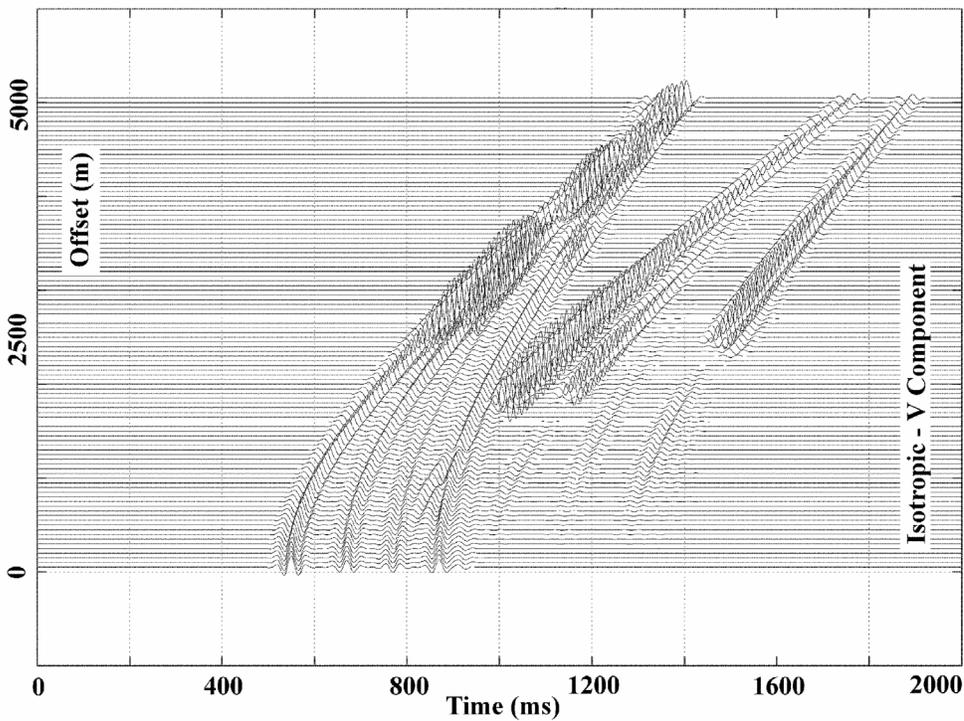
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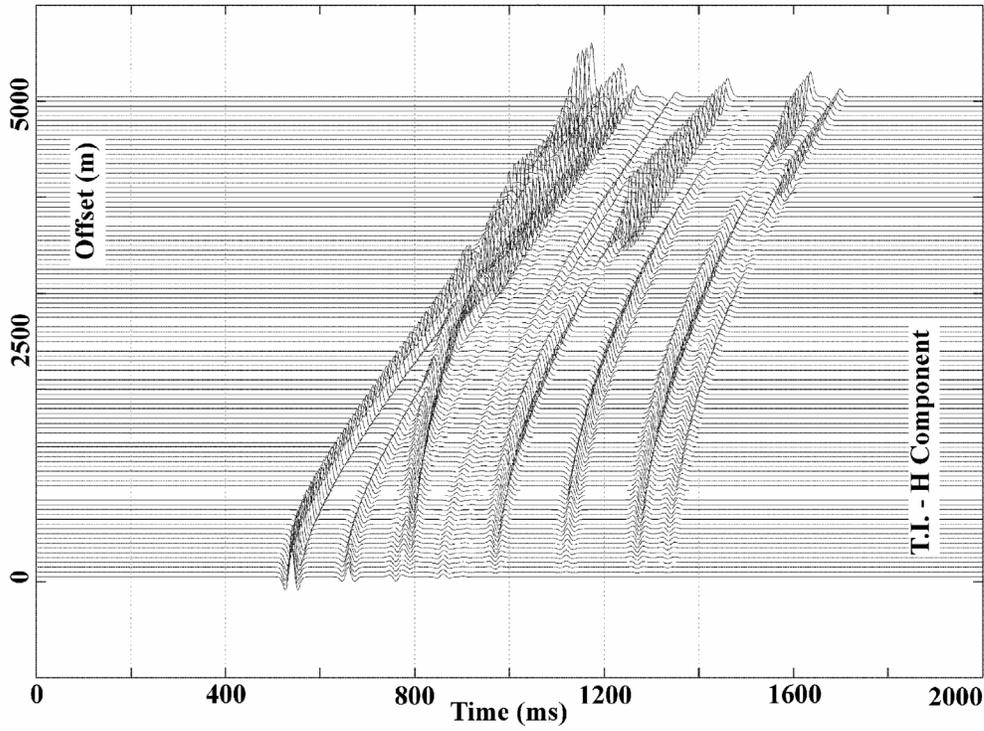
Shoenberg, M. and Helbig, K., 1996, Orthorhombic media: Modeling elastic wave behavior in a vertically fractured earth: *Geophysics*, **62**, 1954-1974.



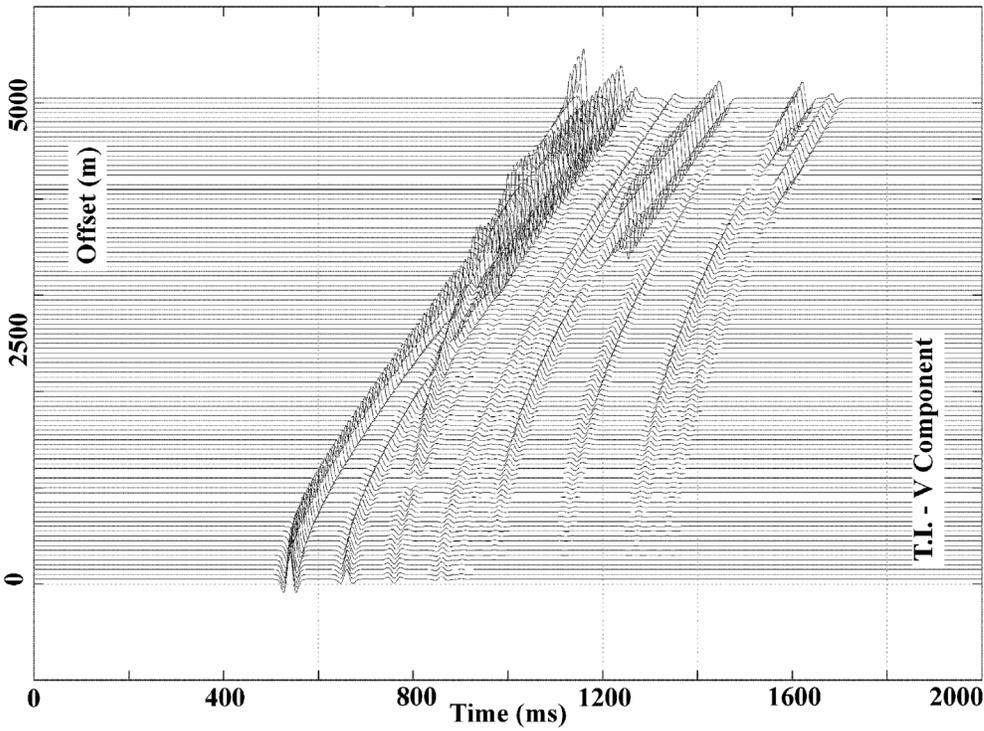
**FIG. 2.** Horizontal component of particle displacement for the isotropic model described in Table 1. The global scaling ratio for the arrivals is 2.18 with respect to the maximum arrival amplitude.



**FIG. 1.** Vertical component of particle displacement for the isotropic model described in Table 1. The global scaling ratio for the arrivals is 4.47 with respect to the maximum arrival amplitude.



**FIG. 4.** Horizontal component of particle displacement for the transversely isotropic model described in Table 2. The global scaling ratio for the PS arrivals is 1.49 with respect to the maximum PP arrival amplitude.



**FIG. 3.** Vertical component of particle displacement for the transversely isotropic model described in Table 2. The global scaling ratio for the PS arrivals is 3.49 with respect to the maximum PP arrival amplitude.

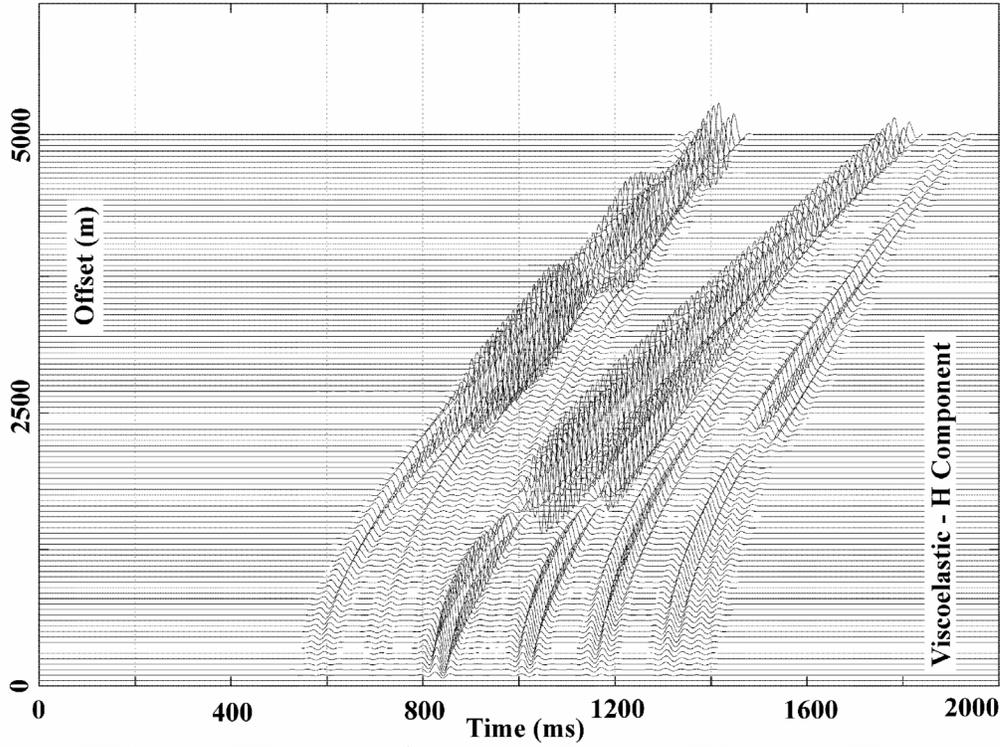


FIG. 6. Horizontal component of particle displacement for the viscoelastic model described in Table 3. The global scaling ratio for the PS arrivals is 2.37 with respect to the maximum PP arrival amplitude.

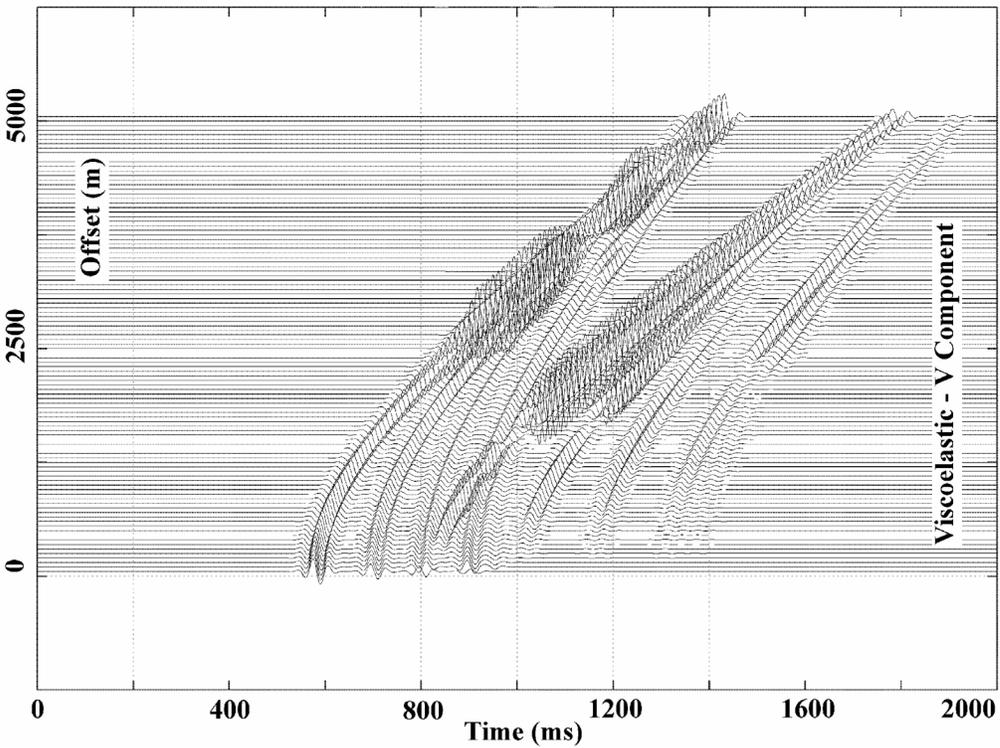


FIG. 5. Vertical component of particle displacement for the viscoelastic model described in Table 3. The global scaling ratio for the PS arrivals is 5.93 with respect to the maximum PP arrival amplitude.