Prestack Depth Migration in a Viscoelastic Medium

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ABSTRACT

Theory and experiment have shown that the propagation of seismic waves in real media is in many respects different from propagation in ideal solids. In data processing, the attenuation and dispersion of the seismic wave has often been neglected. Presented here is a method for accommodating absorption and dispersion effects in prestack depth migration schemes. Extrapolation operators that compensate for absorption and dispersion have been designed. The algorithm is developed in the frequency-wavenumber domain, and is characterized by simplicity, speed, little dependence on stratum obliquity, and good stability. To test the validity of the method, a viscoelastic geology model was designed. Synthetic seismic data were generated based on the model. Viscoelastic and elastic prestack depth migration were performed on the synthetic data; the two results obtained are compared in this paper and show the influence of absorption and attenuation.

INTRODUCTION

Elasticity is a good model for mechanical wave propagation through the earth. No real materials, however, are perfectly elastic. Wave energy is gradually converted into heat. The propagation of seismic waves in real media is in many respects different from propagation in an ideal solid. The real medium will cause dissipation of seismic energy, thus decreasing the amplitude and modifying the frequency content of the propagating wavelet. This attenuation and dispersion of the seismic wave is strongly affected by the saturation state and physical condition of the rock (Jones, 1986). Therefore, these effects are important in exploration geophysics since they may allow us to extract more detailed information about the subsurface from seismic data or to construct images with better resolution, if the quality factor Q is satisfactorily approximated. There is, at the outset, no justification for neglecting the absorption and dispersion of seismic energy, and the effect has been incorporated into seismic modeling schemes (Emmerich and Korn, 1987; Carcione et al., 1988) and migration schemes (Mittet al., 1995). Attenuation of propagating waveforms is, in some cases, quite significant and could be a source of erroneous results in forward modeling, inversion, and imaging if neglected (e.g., Samec and Blangy, 1992). In recent years, the inclusion of second-order effects, such as absorption and anisotropy, into seismic processing schemes has become more important. Finite-difference modeling in a viscoacoustic medium has been developed (Carcione, 1993), and 3-D prestack migration in anisotropic media has been performed (Dong and McMechan, 1993).

The main purpose of this work is to compensate for the absorption of energy from the source location to the receiver location. This implies that both forward-propagated and backward-propagated waves should be compensated. Formally, this is equivalent to starting at the receiver location and repropagating the wave to the source location and at the same time adding the lost frequency components to the wavefield. The implementation performed in this paper is 2-D, but there is nothing in the formalism that prevents us from using the same method in the 3-D case.

The rest of this paper is organized as follows. In the next section, we present the basic equations for the depth extrapolation, and the formalism for absorption is then presented and introduced into the migration scheme. Next, we include a section with a numerical example of the scheme.

VISCOELASTIC WAVE EQUATION PRESTACK DEPTH MIGRATION

Since Stolt put forward the f-k migration method, much research has been done to improve it, and f-k migration arithmetic has been one of the most popular migration methods. The work in this paper is based on Stolt's migration (1978). In viscoelastic media, the viscoelastic wave equation can be expressed as:

$$\nabla^2 (P(x,z,t) + \frac{1}{\omega_0}\frac{\partial P(x,z,t)}{\partial t}) = \frac{1}{c^2}\frac{\partial^2 P(x,z,t)}{\partial t^2}. \quad (1)$$

In the 2-D case, equation (1) can be expressed as:

$$(1+\frac{1}{\omega_0}\frac{\partial}{\partial t})(\frac{\partial^2 P(x,z,t)}{\partial x^2}+\frac{\partial^2 P(x,z,t)}{\partial z^2}) = \frac{1}{c^2}\frac{\partial^2 P(x,z,t)}{\partial t^2}, \quad (2)$$

where *P(x,z,t)* is the pressure, ρ is the density, c is the velocity, and ω_0 is the transition frequency,

$$\frac{1}{\omega_0} = \frac{\eta_1 + \frac{4}{3}\eta_2}{\rho c^2},$$

where η_1 and η_2 are the viscoelastic coefficients. We perform a Fourier transform in the x-direction and t-direction of equation (2) to obtain

$$A\frac{\partial \overline{P}(k_x,z,\omega)^2}{\partial z^2} = B\overline{P}(k_x,z,\omega), \quad (3)$$

where k_x is the wave number responding to x, and ω is apparent frequency responding to t. $\overline{P}(k_x,z,\omega)$ is a 2-D Fourier transform of $P(x,z,t)$,

$$\overline{P}(k_x,z,\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(x,z,t)e^{-(i\omega t - ik_x x)}dxdt,$$

its inverse Fourier transform is

$$P(x,z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{P}(k_x, z, \omega) e^{i(\omega t - k_x x)} dk_x d\omega ,$$

$$A = \frac{i\omega}{\omega_0} + 1 ,$$

$$B = k_x + \frac{ik_x^2\omega}{\omega_0} + \frac{\omega^2}{c^2} ,$$

with the dispersion relation expressed as $\omega^2 = c^2\left(k_x^2 + k_z^2\right)$, then

$$B = k_z^2\left(\frac{ik_x^2\omega}{k_z^2\omega_0} - 1\right) ,$$

where k_z is the wave number corresponding to z.

The wave equation (3) has two independent solutions, corresponding to extrapolation of up-going waves and down-going waves, respectively.

$$\overline{P}(k_x, z, \omega) = \overline{P}(k_x, 0, \omega) e^{\pm r z} \tag{4}$$

where $r = \sqrt{\frac{B}{A}}$, $\overline{P}(k_x, 0, \omega)$ is the 2-D Fourier transform of $P(x,0,t)$. Thus, for a down-going wave, we have

$$\overline{P}(k_x, z, \omega) = \overline{P}(k_x, 0, \omega) e^{r z} , \tag{5}$$

the extrapolator e^{rz} can be used to compensate for the energy-loss that the wave has experienced during propagation from source to the reflecting interface (see Figure 1). For an up-going wave, the extrapolation will give

$$\overline{P}(k_x, z, \omega) = \overline{P}(k_x, 0, \omega) e^{-r z} . \tag{6}$$

up-going wave has been propagated upwards, losing amplitude on its way. The extrapolator e^{-rz} will boost the amplitude, thereby compensating for the lost amplitude on the way up from the reflection point (see Figure 1).

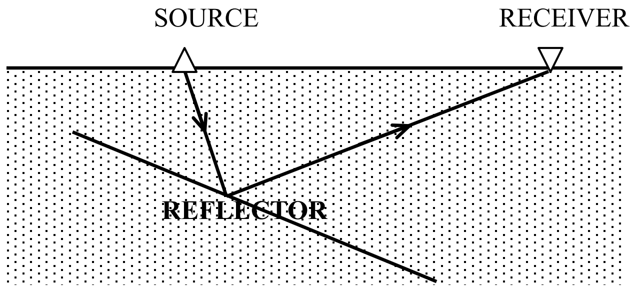

FIG. 1. Raypath of a reflected signal. The wave is compensated for absorption on the way from the source down to the reflector, and up to the receiver.

We perform a inverse Fourier transform in the k_x-direction and k_z-direction of equation (4) to obtain

$$P(x,z,t)=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{P}(k_x,0,\omega)e^{\pm rz}e^{i(k_x x+\omega t)}dk_x d\omega \quad (7)$$

According to the exploding reflector theory, the reflected point is located in the position when t=0. So, let t=0,

$$P(x,z)=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{P}(k_x,\omega)e^{\pm rz}e^{ik_x x}dk_x d\omega \quad , \quad (8)$$

then equation (8) is the basic migration formula in wave number domain.

There is a problem with equation (8) in that we can not apply FFT directly, and the computation speed is extremely slow. Such solutions are known to exist for a large class of linear partial differential equations with constant coefficients. Thus, migration with this algorithm is limited to homogeneous media with a constant-velocity function. In order to overcome this limitation, Gazdag (1978) developed solution methods for the migration of seismic records in inhomogeneous media. This called for the numerical solution of partial differential equations with variable coefficients. The numerical operations are defined in the frequency domain rather than in configuration space. The aim was to obtain the solution by operating on the Fourier coefficients of the seismic section. We will try to solve the problem this way.

Suppose that the geological model is composed of multiple horizontal layers, the coefficients are constant in the same layer ($z_i \le z \le z_i + \Delta z_i$), and they are variable in the vertical direction. We will only show the up-going wave extrapolation here, for the down-going wave extrapolation is the same. At different layers we can get a migration equation like equation (4)

$$\overline{P}(k_x, z_i + \Delta z_i, \omega) = \overline{P}(k_x, z_i, \omega) e^{r_{2i}\Delta z_i}, \qquad (9)$$

$$\overline{P}(k_x, z_i, \omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(x, z_i, t) e^{-i(\omega t - k_x x)} dxdt, \qquad (10)$$

where Δz_i is the distance from the layer $z = z_i$ to layer $z = z_{i+1}$; $\Delta z_i = z_{i+1} - z_i$;

$$r_{2i} = -\sqrt{\frac{|B_i|}{|A_i|}}[\cos(\frac{\beta_i - \alpha_i}{2} + \pi) + i\sin(\frac{\beta_i - \alpha_i}{2} + \pi)], \qquad (11)$$

$$A_i = \frac{i\omega}{\omega_{0i}} + 1,$$

$$B_i = k_{zi}^2 (\frac{ik_x^2 \omega}{k_{zi}^2 \omega_{i0}} - 1),$$

where ω_{0i} is the transition frequency of media in the Δz_i interval. c_{zi} is velocity in the Δz_i interval. α_i, β_i are arguments of A_i, B_i, respectively. k_{zi} is the wave number of z and is defined as

$$k_{zi}^2 = \frac{\omega^2}{c_{zi}^2} - k_x^2. \qquad (12)$$

From equation (9), follows that if $z_i = 0$, $P(x,0,t)$ and $\overline{P}(k_x,0,\omega)$ are the zero offset record and its Fourier transform.

Equation (10) is a migration algorithm corresponding to the vertical variable relative coefficient. But real media are complex, and relative coefficients vary in all directions. So we tried to develop an algorithm that fits complex situations. Let's consider the phase-shift operator $e^{r_{2i}\Delta z_i}$, where r_{2i} is determined by A_i, B_i, equation (11) can be expressed below

$$r_{2i} = -k_{zi}(m + in), \qquad (13)$$

where m, n corresponds to the real and imaginary part of equation (11) after k_{zi} is picked up.

Suppose that the coefficient varies in the x and z directions, then $c_{zi} = c(x,z)$, equation (12) can be expressed as

$$k_{zi} = \sqrt{\frac{\omega^2}{c_x^2} - k_x^2} \,. \tag{14}$$

Also, equation (14) can be expressed as

$$k_{zi} = \sqrt{\frac{\omega^2}{c_a^2} - k_x^2} + \sqrt{\frac{\omega^2}{c_x^2} - k_x^2} - \sqrt{\frac{\omega^2}{c_a^2} - k_x^2} \,, \tag{15}$$

where c_a is the average velocity, and c_x is the velocity along the x direction. Then the phase-shift operator is

$$e^{-dz(\sqrt{\frac{\omega^2}{c_a^2}-k_x^2})(m+in)} e^{-dz(\sqrt{\frac{\omega^2}{c_x^2}-k_x^2}-\sqrt{\frac{\omega^2}{c_a^2}-k_x^2})(m+in)} \,. \tag{16}$$

The seismic wavefield in vertical variable velocity situations can be obtained by migration, using c_a. The wavefield after migration is

$$\overline{P}_{c_a}(k_x, z+\Delta z, \omega) = \overline{P}(k_x, z, \omega) e^{-dz(\sqrt{\frac{\omega^2}{c_a^2}-k_x^2})(m+in)} \,. \tag{17}$$

The seismic wavefield at horizontal variable velocity situation is

$$\overline{P}\ (k_x, z+\Delta z, \omega) = \overline{P}_{c_a}(k_x, z+\Delta z, \omega) e^{-dz(\sqrt{\frac{\omega^2}{c_x^2}-k_x^2}-\sqrt{\frac{\omega^2}{c_a^2}-k_x^2})(m+in)} \,, \tag{18}$$

where $e^{-dz(\sqrt{\frac{\omega^2}{c_x^2}-k_x^2}-\sqrt{\frac{\omega^2}{c_a^2}-k_x^2})(m+in)}$ is the corrector. For the approximation formula $\sqrt{1-a^2} \approx 1 - \frac{a^2}{2}$ the corrector can be expressed as,

$$e^{-dz\omega(\frac{1}{c_x}-\frac{1}{c_a})(m+in)} e^{-dzk_x^2(c_a-c_x)(m+in)/(2\omega)} \,. \tag{19}$$

Then equation (18) becomes

$$\overline{P}\ (k_x, z+\Delta z, \omega) = \overline{P}_{c_a}(k_x, z+\Delta z, \omega) e^{-dz\omega(\frac{1}{c_x}-\frac{1}{c_a})(m+in)} \,. \tag{22}$$

Thus, the final algorithm that can fit a horizontal and a vertical variable velocity is

$$\overline{P}(k_x, z+\Delta z, \omega) = \overline{P}(k_x, z, \omega) e^{-dz(\sqrt{\frac{\omega^2}{c_a^2} - k_x^2})(m+in)} e^{-dz\omega(\frac{1}{c_x} - \frac{1}{c_a})(m+in)} \quad . \tag{23}$$

Prestack depth migrations are commonly obtained by using Claerbout's imaging principle (Claerbout, 1971). The upcoming wavefield is correlated with the down-going wavefield at each depth level. The details will be presented in this paper.

NUMERICAL EXAMPLE

Synthetic seismic data were generated to check the formalism developed in the previous section, and to demonstrate the compensation for absorption. A geological model was designed (see Figure 2), including a declining layer, two faults, and a flat seam. The velocity of each layer is labeled in the figure. The transition frequency ω_0 of this model is 20000. Figure 2 contains the geometry of the model used, together with the shot and receiver configuration. The dashed line defines the area to be imaged in this test. The test contains 30 shots. The streamer shown in the figure consists of 128 geophones with an interval of 20m; the offset is from 20m to 1280m, and shot in the middle. Data were recorded for 2,048ms with a simple rate of 1ms. The shot gather is shown in figure 3. The source signature is a Ricker wavelet. First, viscoelastic modeling with absorption was performed. The result of the modeling is shown in Figure 3. The synthetic data were generated without surface multiples. For field data, one should preprocess with surface multiple removal schemes, e.g., using methods such as those proposed in Fokkema and Vanden Berg (1990) and Wapenaar et al. (1990).

The data were processed with the prestack depth migration scheme outlined earlier. In Figure 4, we show the result using elastic prestack depth migration. Note the diffuse image of the reflectors due to the dispersion of the wavelet as it propagates down in the subsurface and up again. Also the locations of some of the reflectors are incorrect. This is a consequence of the fact that both an amplitudes and arrival times are changed in a viscoelastic medium with absorption (Carcione et al., 1988). In Figure 5 we show the result of our proposed viscoelastic prestack depth migration scheme. Note the improved quality of the image; in the points of the faults, and how reflectors are imaged clearly.

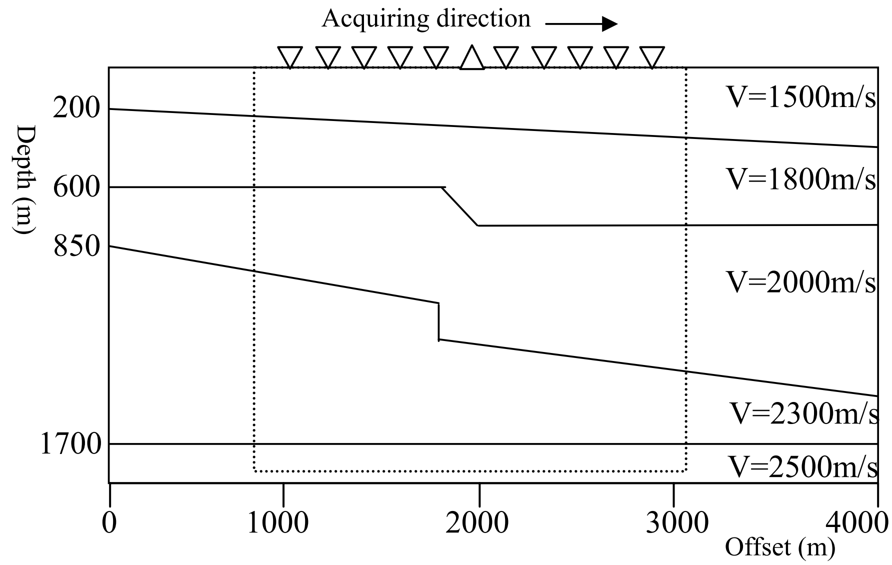

FIG.2. Subsurface model. The dashed line defines the image window. The velocity for each layer is labeled in the model. The transition frequency ω_0 of this model is 20000.

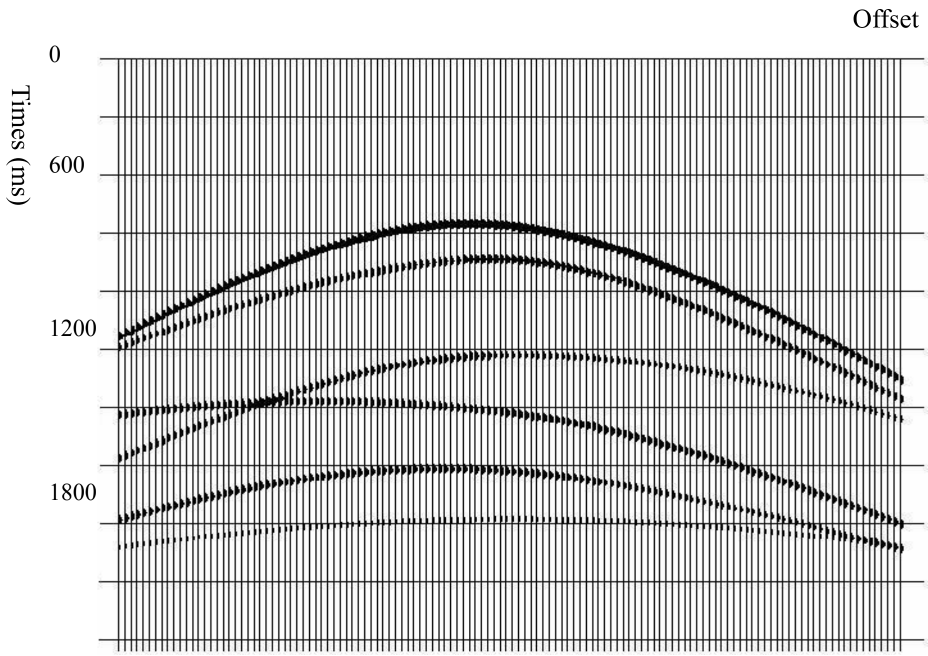

FIG. 3. Modeled data of a shot gather

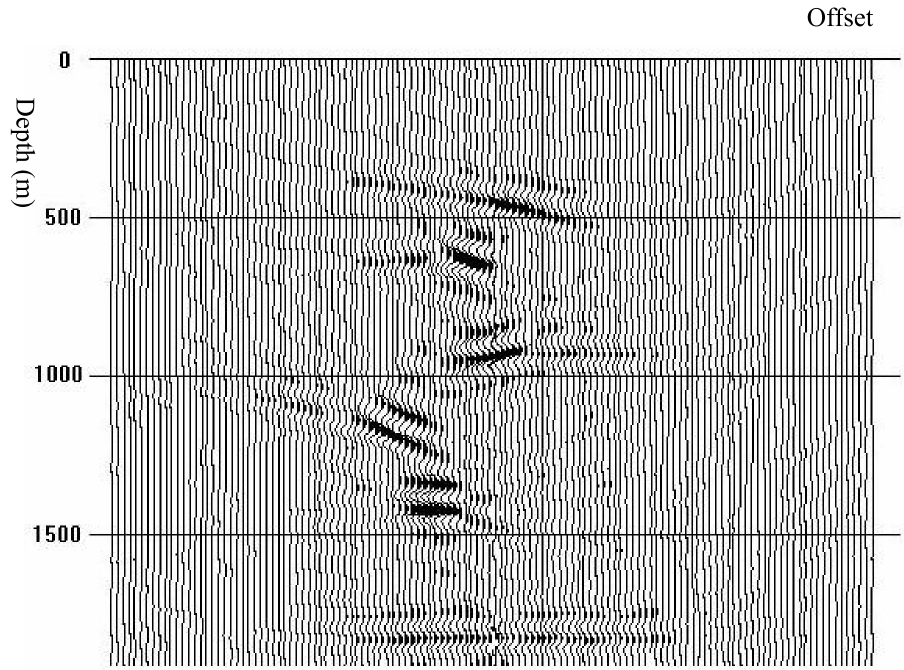

FIG. 4. The images resulting from elastic wave equation prestack depth migration

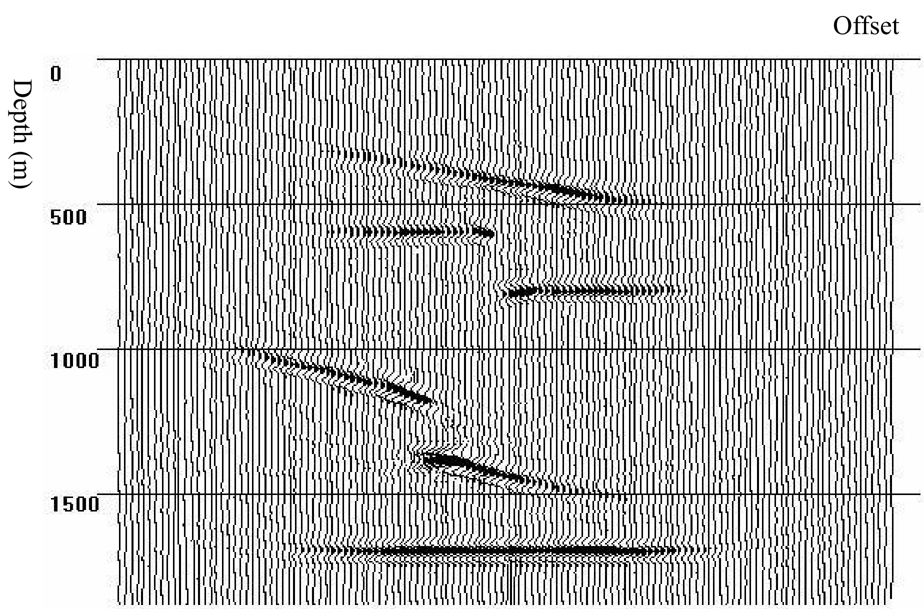

FIG. 5. The images resulting from viscoelastic wave equation prestack depth migration.

CONCLUSIONS

We have shown that the effect of absorption and dispersion of seismic energy in a viscoelastic medium can be compensated for in prestack depth migration schemes. New extrapolator coefficients that account for the attenuation and dispersion of a wavefield have been designed in this paper. From prior information about the variable velocity and the absorption coefficient of the medium, the correct extrapolator coefficient for a given point in space can be accessed and used in the depth extrapolation.

A viscoelastic model was designed, and synthetic data were obtained. The extrapolation operator can handle absorption laws. The results of the numerical test show a significant improvement of the images when migrating with compensation for absorption, as compared to images using elastic prestack migration extrapolation. The images are less diffuse, and the locations of the reflectors are improved.

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