AVO and AVA inversion challenges: a conceptual overview

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ABSTRACT

Inversion of seismic data for earth parameters involves two main steps: (1) estimate the reflectivity as a function of incidence angle for each point in the subsurface, and (2) in accordance with some mathematical model, invert the reflectivity to estimate the corresponding earth parameters. As simple as this observation may seem at first sight, there are many challenges associated with both of these steps. In conjunction with the extensive, up to date bibliography compiled here, it is my hope that my humble perspective on these issues will benefit the active researcher of seismic inversion theory.

OVERVIEW

Conventional AVO (amplitude variation with offset) analysis is based on the well-known Knott-Zoeppritz equations (Knott, 1899, and Zoeppritz, 1919). For a planar interface between two homogeneous isotropic elastic halfspaces in welded contact, these equations describe the various reflection and transmission coefficients for plane waves as a function of angle of incidence, the elastic constants and the densities of the two halfspaces (see e.g., Aki and Richards, 1980). Although these assumptions may appear rather restrictive, they nevertheless play a central role in the so-called forward problem of reflection seismology. This forward problem "consists of the determination of the data that would be recorded for a given subsurface configuration ... under the assumption that given laws of physics hold." (Treitel and Lines, 2000).

The corresponding inverse problem of reflection seismology, then, is nothing but the determination of the subsurface configuration from the observed data. What exactly is meant by subsurface configuration is open to debate. Practically speaking, I take it to mean the spatially varying elastic parameters -- this of course assumes that the data can be reasonably forward modelled by elasticity theory. Thus, for example, this assumes that any Q effects can be handled separately.

Reflections arise from discontinuities in earth parameters, i.e., reflectors. They can also arise from continuously varying parameters, e.g., turning rays, but we will ignore these and focus our attention on the reflectors. Since reflection data are recorded in time, the data must be migrated in order to estimate the locations of reflectors in depth. Thus, at least in principle, migration plays a central role in our inverse problem. Migration can be viewed in itself as a type of inversion, being part of the more general inverse problem known as imaging.

The sharpness of the image and the separation of nearby events are limited by our ability to remove the blurring effect of the seismic wavelet. Removal of this ambiguity is commonly known as deconvolution, yet another part of the inverse problem of seismic imaging. Ideally, each reflection event on each trace appears as a spike, located at the correct position (in time or depth), and having a distribution of amplitudes representing the reflection coefficients (Rpp, Rps, etc.) as a function of the unknown angle of incidence.

Once we are given a reasonably clean, sharp image, and corresponding functions describing the amplitude variations with incidence angle at each point in the image, we are in a position to consider the problem of inverting the AVA (amplitude variation with angle) information for the earth parameters.

We have neglected to mention some of the many stumbling blocks along the way: e.g., multiple reflections, multipathing, attenuation, anisotropy, acquisition geometry, data quality, ground roll, head waves, dispersion, "noise" such as power line hum, wind, traffic, or any other disturbance that fails to fit our simplified model, and the fact that seismic data are bandlimited and acquired with finite aperture. Thankfully, there are some good processing algorithms available to remove -- or at least reduce -- some of these problems.

A major obstruction to solving the seismic inversion problem is nonuniqueness. Given any data set, infinitely many distinct earth models will fit the data to within any given measure of error. A priori information such as well logs or experience with regional geology must be incorporated to constrain the set of allowable models.

The Zoeppritz equations are no longer sufficient if we are to include anisotropy in our model. In particular, Snell's law in its familiar form still applies to phase angles and phase velocities; but for rays, which are connected with group velocities and group angles, a generalized Snell's law is required (e.g., Slawinski et al., 2000). Daley and Hron (1977) (errata for that paper appear in Daley, 2002) develop reflection and transmission coefficients for the case of transversely isotropic media. Thomsen (1993) developed linear approximations to reflection coefficients for weak VTI media, incorporating his well-known anisotropy parameters, delta and epsilon, which he introduced earlier (Thomsen, 1986). However, some errors appeared in Thomsen's 1993 paper, which were later corrected by Rüger (1996).

Considering the complexity involved, it is quite remarkable that exploration seismology has enjoyed the level success that it has. Jackson (1972) elaborated on the complications associated with the seismic exploration problem in his paper: "Interpretation of inaccurate, insufficient and inconsistent data".

AVO/AVA INVERSION FOR ELASTIC PARAMETERS

There are two main steps to inverting seismic data for the elastic parameters: (1) estimate the reflectivity as a function of incidence angle for each point in the subsurface, and (2) in accordance with some mathematical model, invert the reflectivity to estimate the corresponding elastic earth parameters.

The first step can be approached in one of at least two ways: (1) using a time-dependent mapping from offset to incidence angle (Bale, et al., 2001, Ostrander, 1984), or (2) via prestack depth migration (de Bruin, et al., 1990, Hanitzsch, 1995, Bleistein, et al., 2001, Zhang, et al., 2001, Kühl and Sacchi, 2003). There is little doubt that the second alternative has greater potential than the first, although the additional cost might only be justified in regions of significant complexity. As Christian Hanitzsch (1995) stated in his Ph.D. Thesis:

> **Amplitude preserving prestack [depth] migration is the most sophisticated method to obtain [angle-dependent] reflectivity and replaces the techniques of binning, geometrical spreading correction, normal moveout (NMO) correction, dip moveout (DMO) correction, reflection angle estimation and zero offset (post-stack) migration in traditional amplitude versus offset (AVO) processing.**

The second step -- the inversion itself -- can be attempted in several ways. One way is to seek linearized approximations to the Zoeppritz equations, and then analytically solve these linear equations for the earth parameters. A second method is to determine a set of earth parameters that fit the data: that is, according to some error measure, minimize the difference between the angle-dependent reflectivity data and the forward-modelled synthetics obtained from trial earth parameters.

Often, sands and shales cannot be distinguished based on velocities alone; and in such cases, density can be a better discriminator. However, it is well known that the linearized approximations to the Zoeppritz equation for Rpp (e.g. Aki and Richards, 1980, Shuey, 1985, Fatti, 1994, Ramos and Castagna, 2001, Ursenbach, 2002a, 2002b, 2003a, 2003b) are relatively insensitive to changes in density, especially for limited-aperture experiments (Lines, 1998, and Ursenbach, 2002). For seismic exploration, this often means that density contrast cannot be satisfactorily estimated from P-wave data alone.

There is much demand for a robust method of density estimation, and since p-wave data alone has been shown to be generally insufficient, we naturally look toward multicomponent data to extract more information. Specifically, we are interested in PS converted waves, i.e., seismic energy originating from a compressional source, which partially converts upon reflection to shear wave energy. Although SS waves and SP converted waves are of theoretical interest, shear waves emanating from a point source (e.g., dynamite) tend to have much lower energies than compressional waves (although this is not universally agreed upon).

Having opted to exploit converted wave data, the question arises as to whether the PP and PS data should be inverted jointly or sequentially (Treitel and Lines, 2000). Joint, or simultaneous, inversion (Stewart, 1990, Larsen, 1999, Larsen, et al., 2000, Ronen, et al., 2000, Henley, et al., 2002) presupposes that interpreted events on the PP and PS sections can be registered (to put in exact alignment, as in printing or colour photography). This registration process is only necessary if the analysis is taking place in the time domain, since the misalignment of events there is purely a consequence of the different velocities for the two propagation modes. In depth, assuming an accurate migration output, PP and PS events are of course registered automatically. This suggests a nice quality-control check, i.e., how well the PP and PS migration outputs register in depth. However, in practice this can be difficult to judge, since the two reflectivities can be quite different, and can even have opposite polarities at the same depth (G. Margrave, personal communication, 2003).

There is also the problem of distinguishing events in the data among the various types of mode conversion. P-to-S converted reflections tend to arrive at near vertical trajectories, so they tend to be well represented by the radial component of the data. When the near-surface compressional velocities are very low compared to the deeper ones, PP reflections also arrive vertically; thus, PP events appear mostly on the vertical component.

In these typical situations, the data effectively separates itself into PP and PS events. However, an interesting exception to this rule is exploration over a very fast isotropic permafrost layer. Such a layer can seriously complicate the separation process since it deflects incoming rays away from the vertical.

OPTIMAL ZOEPPRITZ APPROXIMATIONS

Charles Ursenbach (2002a, 2003b) developed optimal pseudo-linear approximations to the Zoeppritz equations for Rpp and Rps. He calls these approximations pseudo-linear since they are expressed in a form similar to the approximations of Aki and Richards (1980) (in fact, their linear approximations are not, formally speaking, linear). Ursenbach's expressions are optimal in the sense that they minimize error while preserving the familiar format of the Aki-Richards approximations. They have the advantage over other approximations of maintaining good accuracy even at post-critical angles of incidence. They also bridge the gap between the Aki-Richards approximations and the full Zoeppritz equations, in that they are accurate yet amenable to standard AVO methods (Ursenbach, 2003b).

PARAMETER SELECTION

For the mathematical inversion step, not all parameterizations of the Zoeppritz equations are equal. Debski and Tarantola, (1995) looked at certain probabilities associated with AVA inversion for different choices of parameter sets. They found that choosing the parameter set {density, P-velocity, and S-velocity} to invert AVA information is a mistake. Unfortunately, this happens to be a common choice. They offer several better alternatives: {density, P-impedance, Poisson's ratio}, {density, P-impedance, S-impedance}, or {density, P-impedance, Jussieu's ratio}, where Jussieu's ratio is defined as the ratio of bulk modulus over shear modulus. It is conceivable that improved density estimates could be obtained by adopting one of these recommended parameterizations.

WHY PS AVO/AVA INVERSION?

Large incidence angles for PS converted waves are typically achieved at shorter offsets than for PP reflections. This is because rays follow the path of least traveltime, and thus travel longer distances in the fastest direction or mode of propagation. This means that for a given aperture, more complete AVA information is available for PS data than for PP data; thus allowing for a more reliable parameter inversion. Moreover, at least in the absence of noise, inversion of converted wave data is more accurate than other sources (Ursenbach, 2003a). In the presence of noise, Joint PP and PS inversion is better than either alone given that events can be aligned (G. Margrave, personal communication, 2003; demonstrated by Larsen, 1999).

Finally, converted wave data offers an extra, independent source of information. This leads to higher resolution elastic parameter estimates, and ultimately betters knowledge of subsurface rock properties.

AVO IN TI MEDIA

PP incidence angles increase more rapidly with offset in TI media, when the fast direction is horizontal, than they do in isotropic media. Therefore, PP AVO may be sufficient in many cases. However, as mentioned above, neither the Zoeppritz equations nor Snell's law apply any longer: the TI reflection coefficients are required (Daley and Hron, 1977, errata in Daley, 2002).

CONCLUSIONS

Inversion of seismic data for earth parameters involves two main steps: (1) estimate the reflectivity as a function of incidence angle for each point in the subsurface, and (2) in accordance with some mathematical model, invert the reflectivity to estimate the corresponding earth parameters. Some of the many challenges associated with both of these steps have been reviewed. An extensive, up to date bibliography has been provided. The active researcher of seismic inversion theory should find use for this work either as a starting point, or as a refresher on the subject.

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