Using the exact Zoeppritz equations in pseudo-linear form: Isolating the effects of input errors

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ABSTRACT

AVO inversions are carried out on synthetic data using a pseudo-linear form of the exact Zoeppritz equations in order to demonstrate the significance of various types of errors in the input data. Noise is represented by random Gaussian data added to the reflectivities, and this is shown to be one of the key sources of error. Error in the background parameters (β/α, $\Delta\alpha/\alpha$, $\Delta\beta/\beta$, $\Delta\rho/\rho$) is represented by adding deviations to their exact values. Error in the incidence angle may be either random or systematic. Random errors are modeled with noise added to the assumed values. Systematic errors are represented by linearly scaling the assumed values. Some input errors are seen to result in contrast errors which are correlated with the value of $\Delta\beta/\beta$. This method allows direct and quantitative comparison of the effect of input errors with the effect of approximations to the Zoeppritz equations. In this study comparisons are made with the effect of the Aki-Richards approximation.

INTRODUCTION

The Zoeppritz equations play a central role in AVO inversion. Errors in the inversion process can be grouped in at least four different categories:

1. Assumptions of the Zoeppritz equations (neglects anisotropy, anelasticity, tuning, etc.)

2. Approximations to the Zoeppritz Equations (e.g., linearization)

3. Limited input (limited offset range, data only at discrete points)

4. Errors in the input (noise in data, processing artifacts, errors in assumed prior information, velocity model errors, incidence angle errors, background parameter errors, etc.)

In previous work (Ursenbach 2003a,b,c, hereafter referred to as Papers I, II, and III) we have considered the effect of various approximations (category 2) on the accuracy of inverted parameters. In this work we will compare the influence of approximations to that of some errors in the input (category 4).

We will first describe the types of error to be considered, and outline our general methodology. This will be followed by the presentation of results and a discussion and conclusion.

MODELS OF ERROR

This study will consider the following types of input errors:

1. Data Noise: Synthetic reflectivities will be augmented with random Gaussian noise.

2. Background parameters: The quantities β/α and $\Delta\alpha/\alpha$ are normally provided to calculate the coefficients in the inversion process. Since we are employing the exact Zoeppritz relations, background values of $\Delta\beta/\beta$ and $\Delta\rho/\rho$ will be required as well. Errors will be represented by adding a small quantity to the exact value before substituting into the coefficients. In practice, errors in these quantities can result from imperfect estimation from velocity analysis, well-logs, empirical relations, etc.

3. Random angle-of-incidence errors: These will be assumed to be Gaussian in distribution. In practice such errors could result, for instance, from raytracing through a velocity model in which small lateral inhomogeneities are unaccounted for.

4. Systematic angle-of-incidence errors: It can be demonstrated that errors in either the velocities or thicknesses of a layered velocity model will result in raytraced angles of incidence whose error, to first order, is linear in offset and/or angle. We will model this in a given inversion by adding an error term to the angle of incidence which increases linearly with the angle to which it is added.

The last three items all represent several types of errors that can result from an imperfect velocity model. We do not consider in this study the important errors that can arise from processing of the data.

METHODOLOGY

We have previously described in detail (Papers I and II) the 3-parameter inversion of PP and PS data by three different theoretical approximations:

1. Aki-Richards approximation

2. Pseudo-linear approximation

3. Pseudo-quadratic approximation

It was noted that all three approximations are intrinsically non-linear in that their coefficients depend implicitly or explicitly on one or more of the contrast variables which are the object of the inversion. In Papers I and II, an iterative approach was used to supply these values. In this study, we will employ exact Zoeppritz expressions (given in I and II) which are structured to allow for convenient iterative inversion. However, since errors in the background contrasts are one quantity we wish to study, we will supply the coefficients with exact values of these quantities, except when we are explicitly considering the effect of one of them. Thus each calculation will be carried out with a single linear inversion.

In Paper III we presented various analytical representations of the contrast errors resulting from various approximations. We will employ the analytical expressions for the error of the $\Delta I_S/I_S$ estimate from Aki-Richards inversion of P-P and P-S data. These expressions will be used to compare errors resulting from use of approximations with errors inherent in input.

As in Papers I-III, we will employ data on 125 different interfaces in order to sample a broad range of potential behavior. As before, we will plot results as the error in a predicted contrast variable (i.e., $\Delta I_S/I_{S,predicted} - \Delta I_S/I_{S,exact}$). We found previously that most such quantities show a significant trend when plotted against the S-wave velocity contrast ($\Delta\beta/\beta_{exact}$) [or S-impedance contrast ($\Delta I_S/I_{S,\ exact}$)] and so this quantity will be used as the abscissa. The inversions will produce three parameters, and these may be combined in various ways, but we will focus on presenting predictions of $\Delta I_S/I_S$, as it predicted somewhat less accurately than $\Delta I_P/I_P$ by current methods. It is therefore an object of interest in current research.

RESULTS

In the sections below, we will consider various sources of error in turn. We will plot the error in the S-impedance contrast estimate for both conventional AVO and multicomponent AVO. In the plots these will be compared to the error resulting from Aki-Richards inversion of error-free data. A few points are helpful to keep in mind when looking at these results:

- Each source of error is controlled by some parameter. That parameter has been adjusted so that the resulting contrast errors are of a similar magnitude to the Aki-Richards contrast errors. This allows us to differentiate situations in which Aki-Richards is useful from those in which it may become the predominant source of error.

- Although the errors are shown only for a single value of each parameter, it was observed (not shown here) that the errors in predicted contrasts varied roughly linearly with the magnitude of the parameter. Changing the sign of the parameter changes the sign on the predicted contrast errors.

- Results are not shown for joint inversion. Such calculations have been carried out however and, in the present case, joint inversion results may be characterized as being similar to the P-S inversion results, but perturbed slightly toward the P-P inversion results.

Gaussian noise in the amplitudes

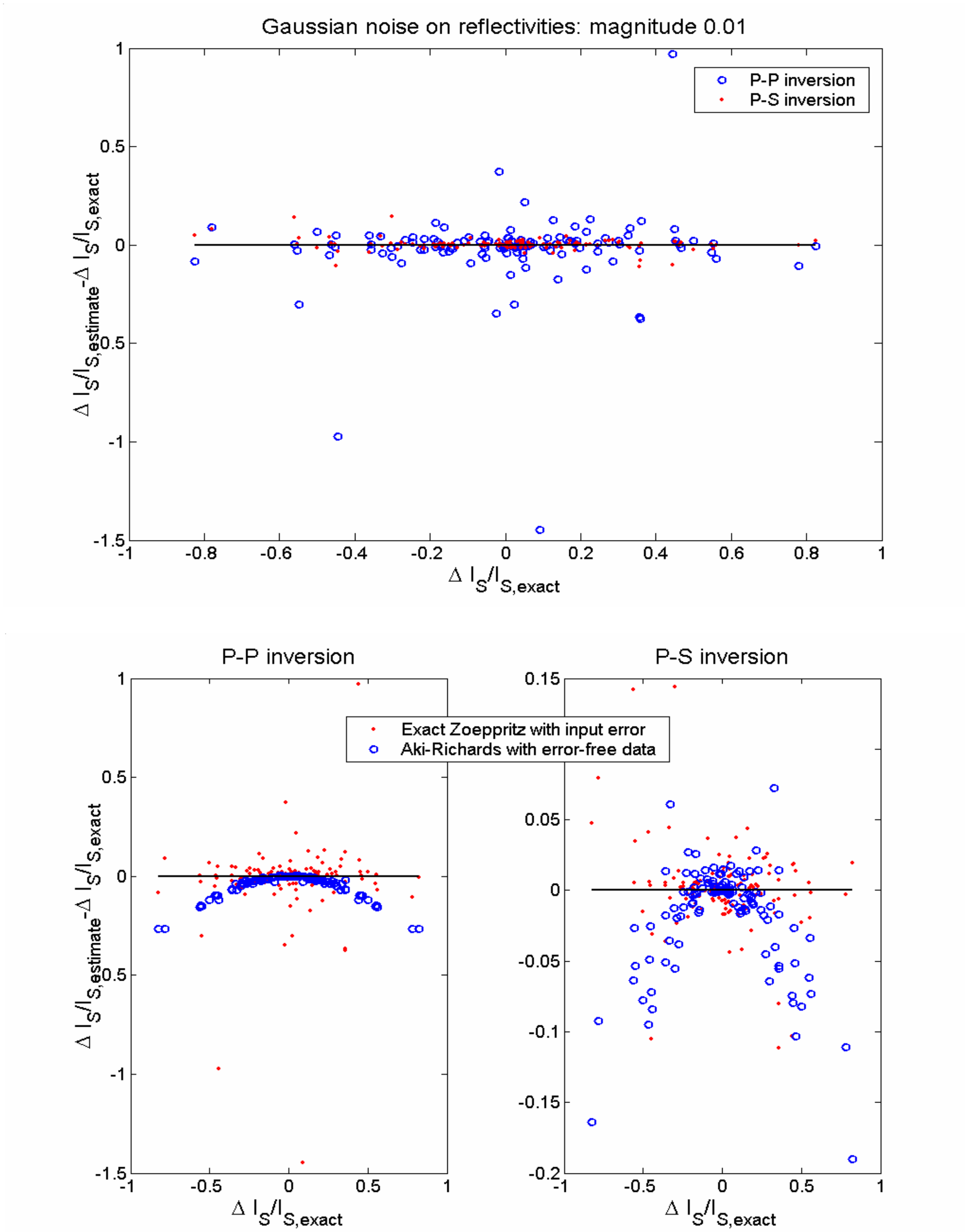

FIG. 1. The upper graph compares P-P and P-S inversions of data with random Gaussian noise (of order magnitude 0.01) added to the amplitude data points. The P-S inversion yields much smaller errors. The lower plots compare this same data to the corresponding Aki-Richards inversions of error-free data. The noise-induced errors are not correlated with $\Delta I_S/I_S$, and increase in magnitude linearly with the noise itself. It is clear that even modest amounts of noise can have a deleterious effect on AVO inversion.

Errors in background β/α

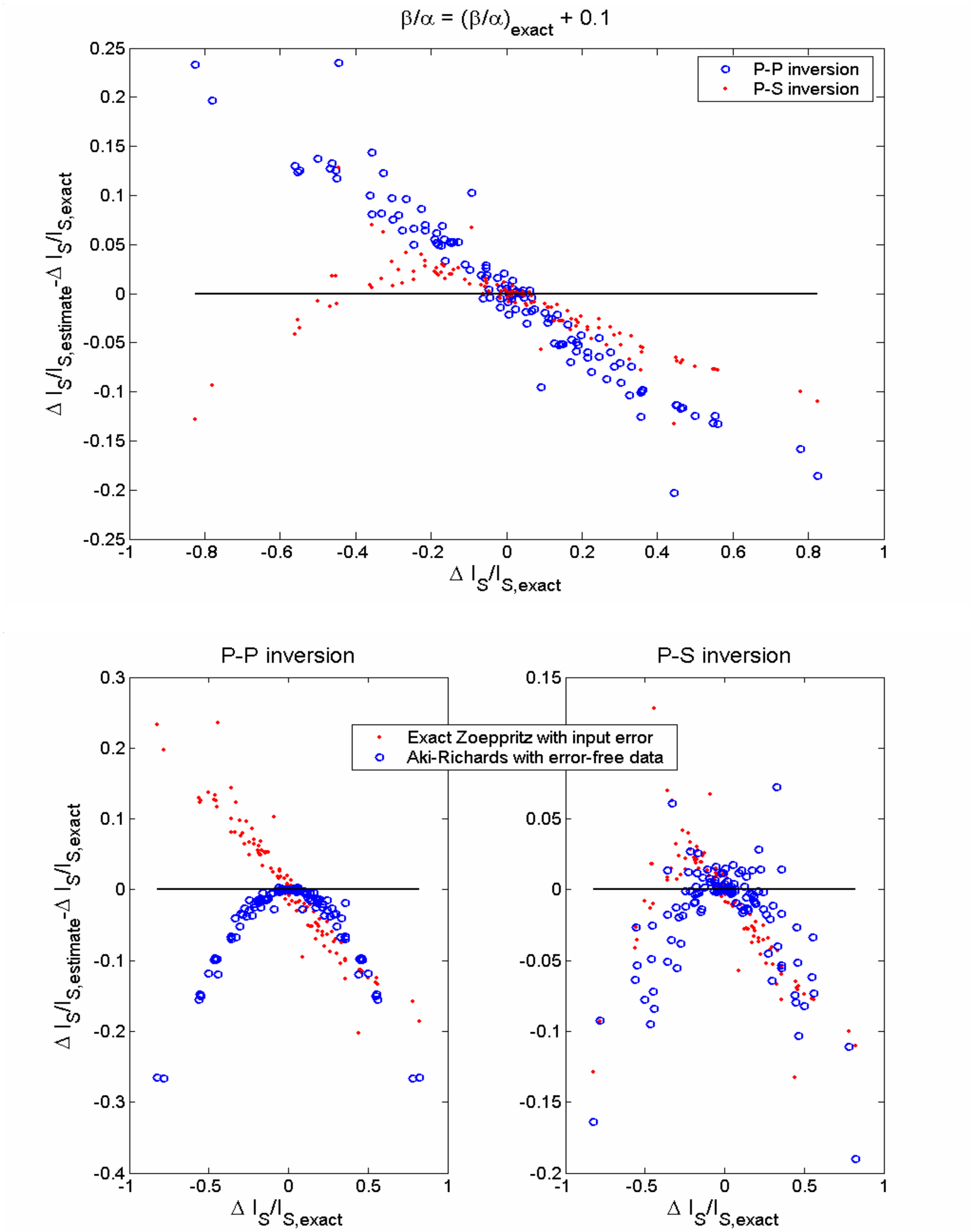

FIG. 2. The upper graph compares P-P and P-S inversions when the background value of β/α is overestimated by 0.1. The P-P inversion yields errors strongly correlated with $\Delta I_S/I_S$. The P-S inversion results correlate also, but in a more complex fashion. The lower plots compare this same data to the corresponding Aki-Richards inversions of error-free data. The latter differs in being quadratically rather than linearly correlated with $\Delta I_S/I_S$. However the errors are still comparable for this 0.1 deviation in β/α.

Errors in background $\Delta\alpha/\alpha$

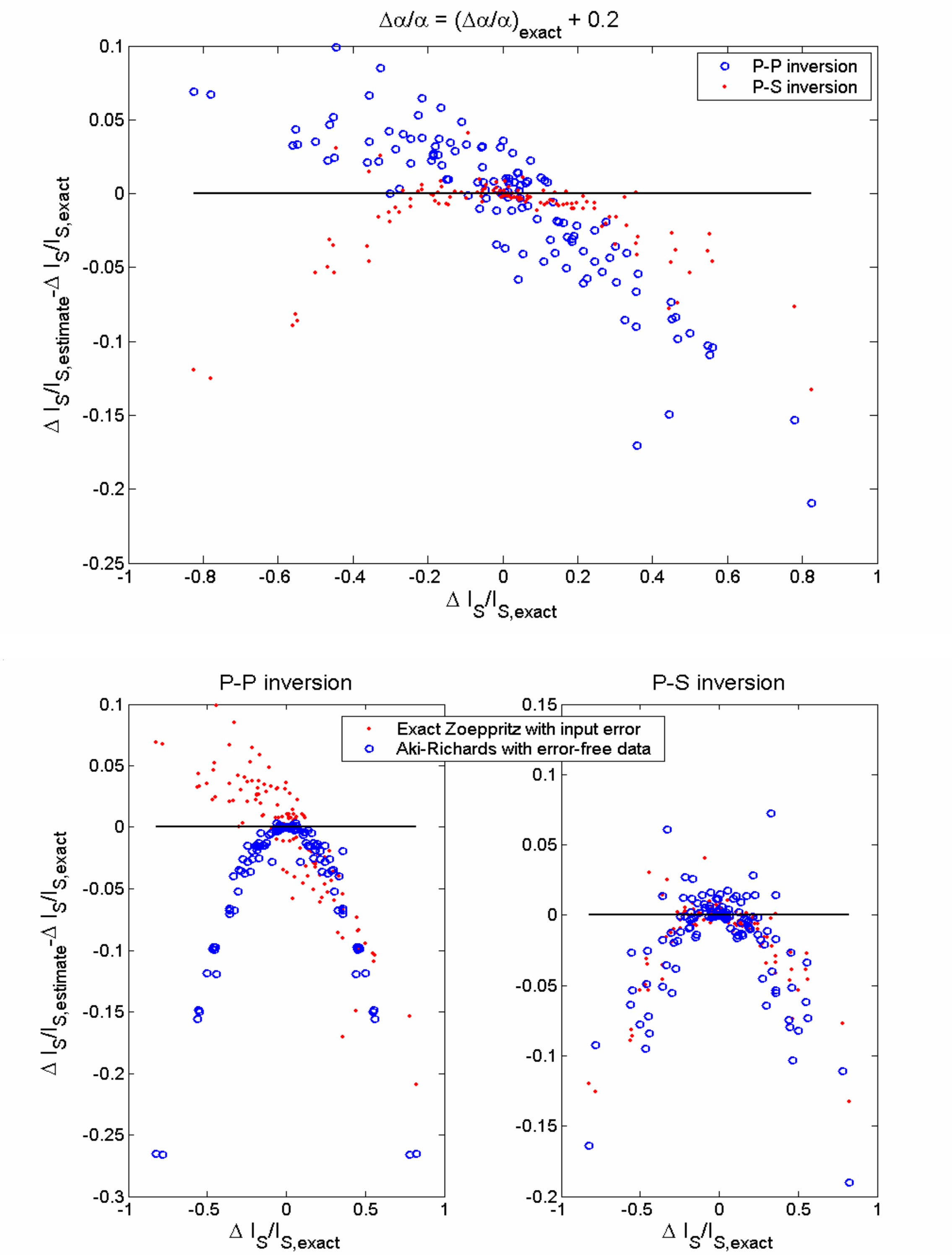

FIG. 3. The upper graph compares P-P and P-S inversions when the background value of $\Delta\alpha/\alpha$ is overestimated by 0.2. The P-P inversion results vary linearly with $\Delta I_S/I_S$, but with significant scatter. The P-S inversion results vary quadratically overall, but with a similar magnitude. The lower plots compare this same data to the corresponding Aki-Richards inversions of error-free data.

Errors in background $\Delta\beta/\beta$

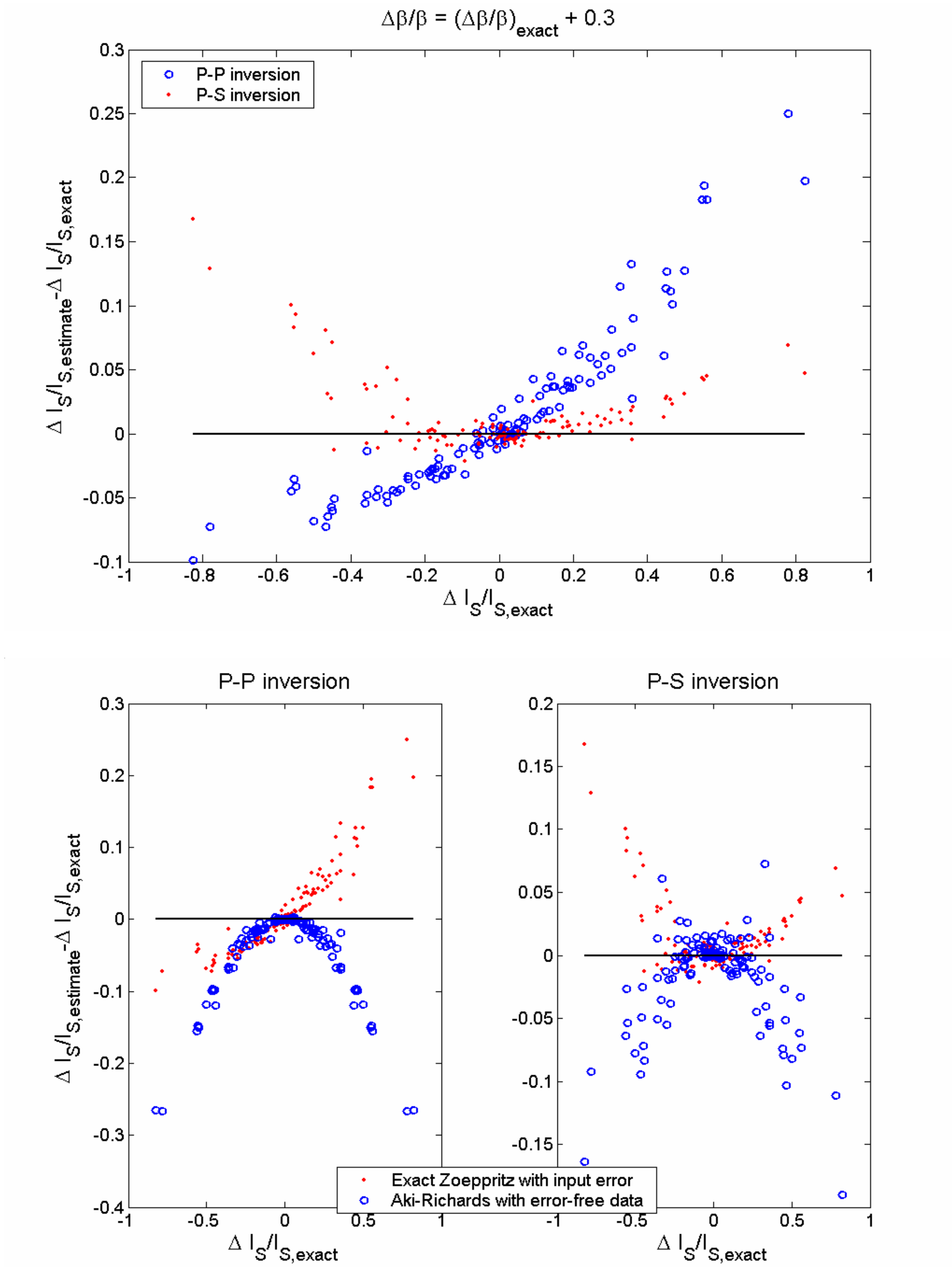

FIG. 4. The upper graph compares P-P and P-S inversions when the background value of $\Delta\beta/\beta$ is overestimated by 0.3. The general trends are reminiscent of Figure 3, but with the opposite sign. The lower plots compare this same data to the corresponding Aki-Richards inversions of error-free data.

Errors in background $\Delta\rho/\rho$

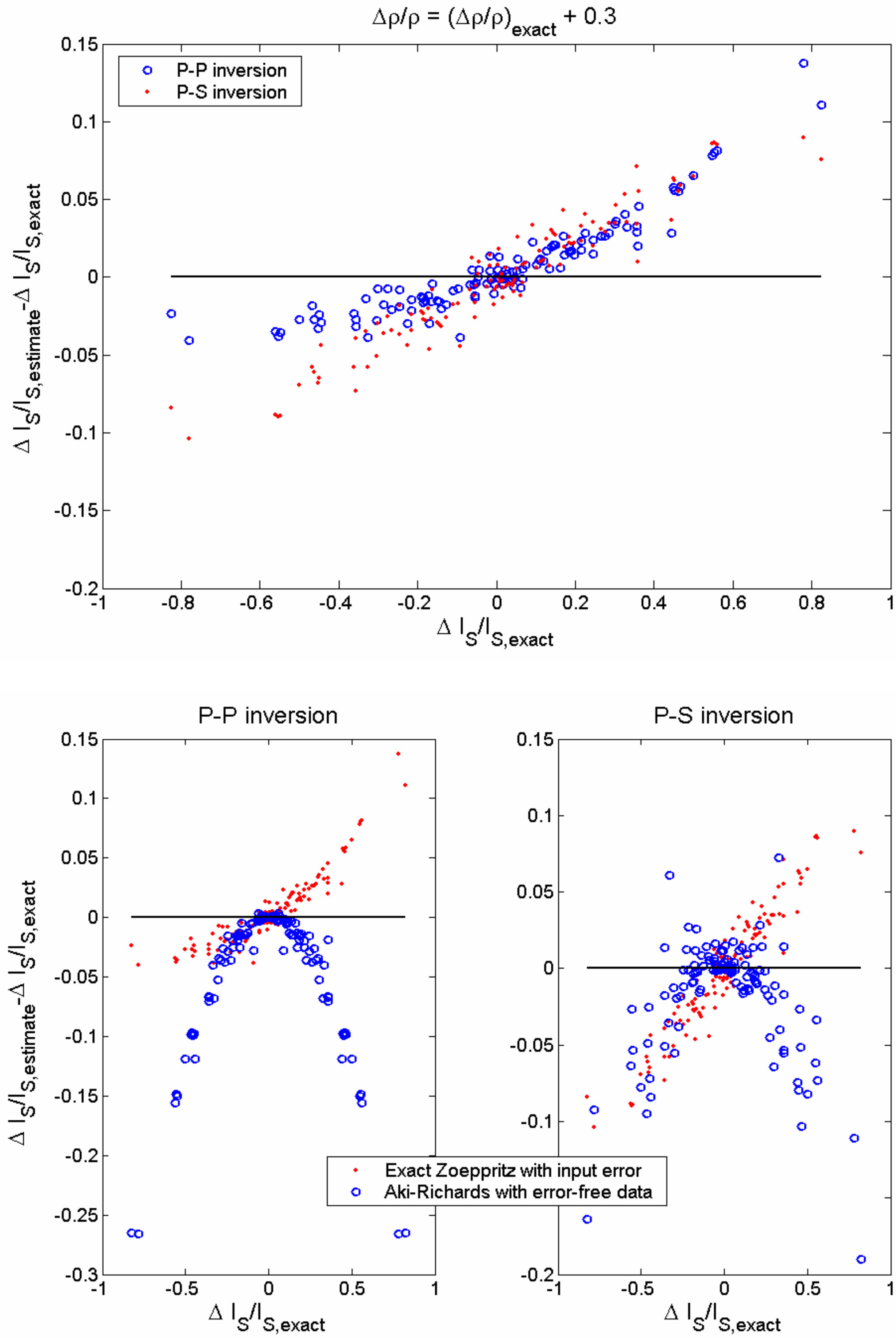

FIG. 5. The upper graph compares P-P and P-S inversions when the background value of $\Delta\rho/\rho$ is overestimated by 0.3. In this case both methods yield very similar errors, linearly correlated with $\Delta I_S/I_S$. The lower plots compare this same data to the corresponding Aki-Richards inversions of error-free data.

Gaussian noise on the angle of incidence

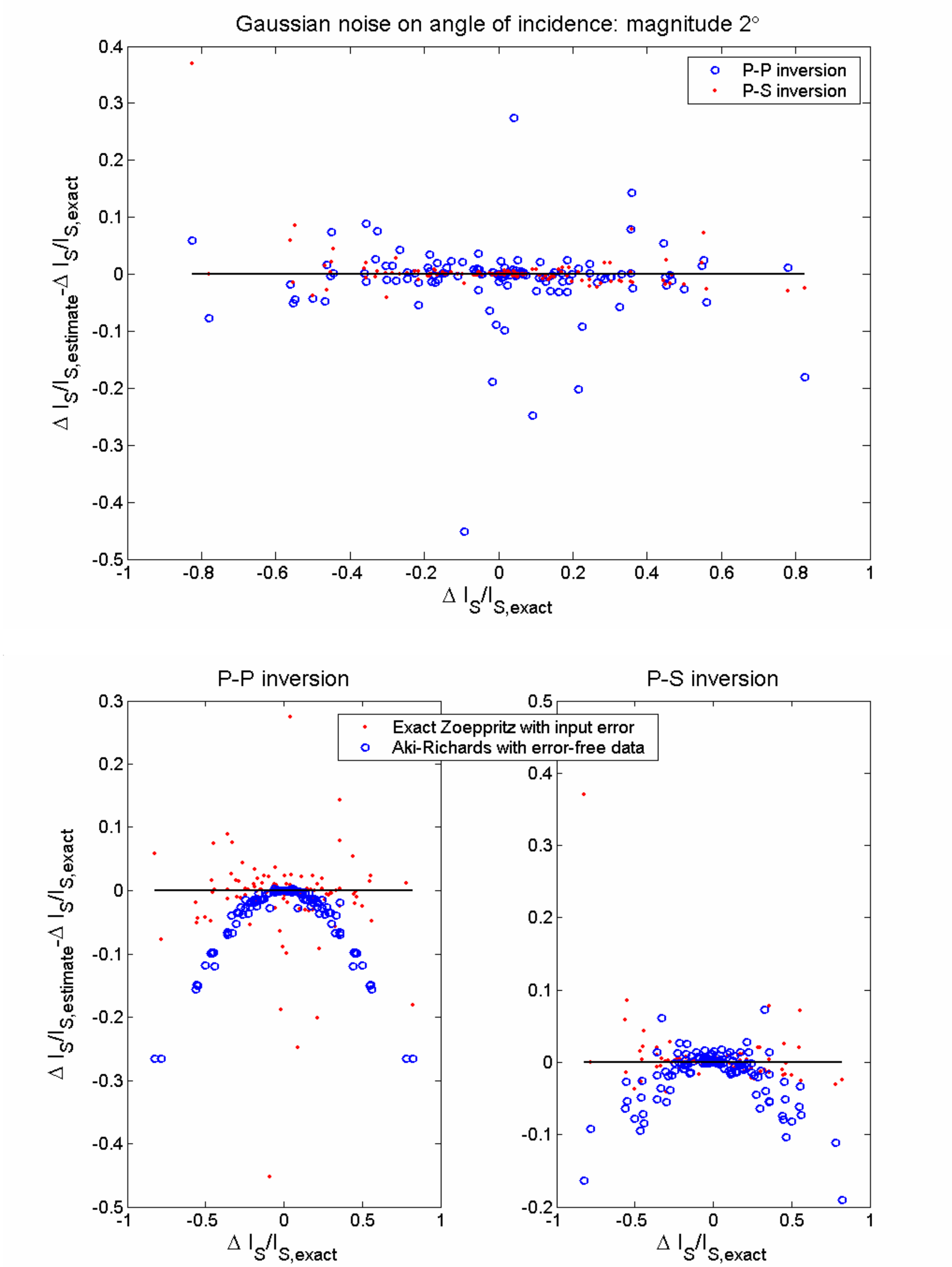

FIG. 6. The upper graph compares P-P and P-S inversions when Gaussian noise is added to the angle of incidence with an order of magnitude of 2°. The P-S errors are generally smaller for $\Delta I_S/I_S$ < 0.4, but both method are similar above that. The lower plots compare this same data to the corresponding Aki-Richards inversions of error-free data.

Systematic errors in the angle of incidence

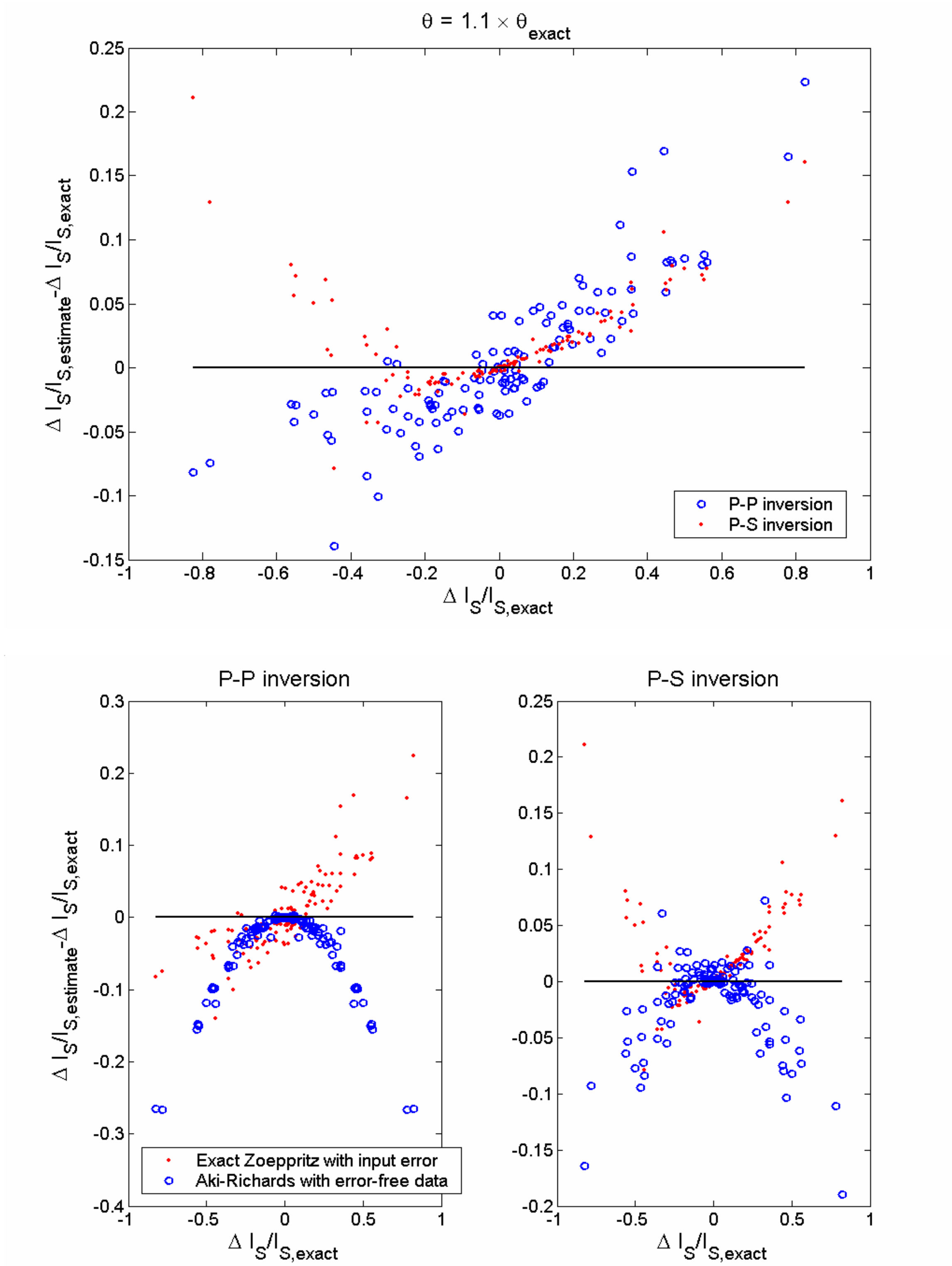

FIG. 7. The upper graph compares P-P and P-S inversions when the angles of incidence are linearly scaled up by 10%. The resulting contrast errors are correlated with $\Delta I_S/I_S$ but possess significant scatter. The lower plots compare this same data to the corresponding Aki-Richards inversions of error-free data.

DISCUSSION

The results above show that the input errors explored in this paper yield contrast errors of three types: 1) The random errors added to amplitudes and angles of incidence yield random errors in the predicted contrasts. The one partial exception to this is that in P-S inversion the errors propagated from random angle errors appear to be smaller if $\Delta I_S/I_S$ is small. 2) Adding error terms to background values of velocity ratios (β/α, $\Delta\alpha/\alpha$, $\Delta\beta/\beta$) and scaling the angle incidence by a linear factor all yield similar correlations of contrast errors with $\Delta I_S/I_S$. They differ to some degree in scale, scatter, and sign, but are all of a recognizably similar pattern. 3) Adding an error term to $\Delta\rho/\rho$ yields a slightly different pattern than for the other background ratios. The P-P contrast errors are similar, but the P-S differs in being much more linear and in matching the P-P result more closely.

A general observation can be made concerning the various patterns found. Those that are correlated with $\Delta I_S/I_S$ or $\Delta\beta/\beta$ have their smallest magnitudes when the shear-velocity/impedance contrast vanishes. This is the same as was found in Papers I-III for errors due to approximations. This suggests that AVO should be most accurate at fluid-fluid interfaces, where the rock matrix does not change. To a first approximation such interfaces are described by $\Delta\beta/\beta = 0$. The only errors which do not behave in this way are those due to random effects. Here though we note that there is a difference between P-P and P-S inversion. In the case of random noise added to reflectivities, the propagated errors were noticeably larger in P-P results than in P-S results, and in the case of random noise added to angles of incidence we have just noted above that in P-S results the propagated errors tend to become small with the magnitude of $\Delta\beta/\beta$. Thus we expect that multicomponent inversion would be particularly favorable at liquid-liquid interfaces. This is consistent with Jin et al. (2000) who found multicomponent AVO to be effective in detecting fluid contacts in a reservoir.

The results of this study suggest some practical applications. One use of the observations of this paper is that they can serve as a guide for translating uncertainties in various inputs into uncertainties in predicted contrasts. In this regard it is worth noting that if errors are introduced into, e.g., two background parameters, the resulting errors combine linearly, and thus add or cancel according to intuition.

These results also contain implications for the use of the Aki-Richards approximation in AVO inversion. If it is believed that background values have been estimated more accurately than to within ~0.2 of their correct value, for instance, then using Aki-Richards will nullify that accuracy. On the other hand, the errors of Aki-Richards may be quite small compared to those arising from simple noise. This underlines the need to constantly seek high quality data and to employ inversion techniques that account as capably as possible for the presence of noise. (See, for instance, Jin et al. (2000).)

CONCLUSIONS

The exact Zoeppritz equations, cast into the pseudo-linear form of Papers I and II, have been demonstrated to be useful as a means of carrying out simple AVO inversions. This convenient form has also been used to isolate the influence of various input errors, apart from any approximation errors. Such an approach could readily be applied to other sources of error as well.

Some of the errors in predicted contrasts have been shown to correlate with $\Delta I_S/I_S$ (or with $\Delta\beta/\beta$), just as was observed earlier with approximations arising from approximations to the Zoeppritz equations (see Papers I-III). The combined results suggest that AVO is particularly useful in detecting fluid-fluid interfaces within a homogeneous rock matrix, especially when multicomponent data is employed.

In this study the errors of various inputs were each adjusted to roughly match the errors obtained from error-free data using the Aki-Richards approximation. These results are useful in providing a guide to uncertainty estimation, and to indicate when Aki-Richards is and is not reasonable to apply.

REFERENCES

Jin, S., Cambois, G., and Vuillermoz, C., 2000, Shear-wave velocity and density estimation from PS-wave AVO analysis: Application to an OBS dataset from the North Sea, Geophys. **65**, 1446-1454.

Ursenbach, C.P., 2003a, Testing pseudo-linear Zoeppritz approximations: P-wave AVO inversion, CREWES 2003 Research Report.

Ursenbach, C.P., 2003b, Testing pseudo-linear Zoeppritz approximations: Multicomponent and joint AVO inversion, CREWES 2003 Research Report.

Ursenbach, C.P., 2003c, Testing pseudo-linear Zoeppritz approximations: Analytical error expressions, CREWES 2003 Research Report.