Finite-difference modelling corrections: application in a variable velocity medium

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ABSTRACT

The technique of correcting finite-difference wave-propagation models at each time step is applied in one and two dimensions. The accuracy of the method is shown for a one-dimensional case, and improved results are shown for two models in the two dimensional case.

INTRODUCTION

The theory of correction filters for one and two dimensional finite-difference modelling has been presented by the authors in earlier papers (Manning and Margrave 1999, 2000). In those papers we showed how a correction filter designed in the frequency domain could compensate for finite-difference sampling effects (with inherently stable conditions), and thereby obtained results that matched analytic modelling. These papers considered media with a single constant velocity.

The present paper shows how correction filters may be employed within a model with more than one velocity. We will illustrate our ideas with the one dimensional model because the points are easier to demonstrate and understand. The same considerations apply to two dimensional models, and two examples of these will be shown.

The technique depends on the use of time-domain filters which are applied to the wave field after each time step (Manning and Margrave 2003). These are optimized to approximate the desired frequency domain correction response given the specified limited filter length. Although these filters are designed to be used in a constant velocity medium, they work very well in a non-constant medium as well. This is because their small size limits the disadvantage of having some coefficients operate on data from a zone where the waves are shaped with the wrong velocity.

ONE DIMENSIONAL MODELLING EXAMPLE

An example of a correction filter designed for a one-dimensional model is shown in Figure 1. The relevant design factors for the frequency domain correction are: velocity 1000 m/sec, spatial sample rate 3 m., and time sample rate 0.0015 seconds. The frequency correction curve takes the parabolic shape (in blue). It shows, for example, that the Nyquist frequency amplitudes should be doubled for accurate propagation.

In the same figure is shown the frequency domain response of a designed limited size spatial filter (green). The filter was designed to match the frequency domain response from zero Hertz to one-half Nyquist, using a length of 5 points. The actual filter coefficients are shown in the top-left of the Figure. It is apparent that all frequencies within the design range will be propagated accurately.

A one-dimensional finite-difference model with and without the correction filter is shown in Figures 2 and 3. Figure 2 shows the initial Ormsby wavelet (frequencies 20/30 – 50/60) with a dashed line, about to propagate right, and the propagated wavelets, after reflection from and transmission through a velocity interface, with solid lines. The red vertical line marks a velocity boundary. To the left of the boundary the perfect sampling condition exists (dt = dx/v) because the velocity v is 2000 m/sec. (actually 1999 m/sec. to ensure stability). Under these conditions no correction filter is required, and the wavelet propagates without dispersion or instability. To the right of the boundary the velocity is 1000 m/sec., and the dispersion is apparent from the change in the waveform.

Figure 3 shows the same model, but with the 5 point correction filter from Figure 1 applied in the area to the right of the boundary. Near the boundary, when the correction filter extends across the boundary, we used the velocity at the center point of the filter. The preservation of the zero phase character of the wavelet in this zone is now consistent with analytic theory. .

Comparison of the transmitted and reflected peak event amplitudes is also very close to what analytic theory predicts. The reflected wavelet is within 8% of expected (as it is in the uncorrected case), and the transmitted wavelet is within 1% of expected (in the uncorrected case, the transmitted wavelet distorts so quickly it is very difficult to measure).

Figure 4 shows another set of parameters used on a model of the same physical dimensions. As with Figure 2, the processing is uncorrected, but with the sample rates in space and time halved. The character of the transmitted wavelet is much better than the previous uncorrected version, but deviates from the proper zero phase shape as it propagates further from the boundary. The relative amplitudes before moving far from the boundary are quite satisfactory, with the transmitted wave within 1% of expected, and the reflected wave within 3% of expected.

TWO DIMENSIONAL MODELLING EXAMPLES

The correction filters shown here may also be extended for use on two dimensional models. An example of an uncorrected and corrected model is shown in Figures (5) and (6). The filters used to correct the model in (6) consist of two sets of 5, each two dimensional of size 7 by 7. The Manning and Margrave (2001) paper defines these frequency domain corrections, and the 2003 paper defines the method used to construct the equivalent small spatial filters.

The High Velocity Wedge model is shown in Figures 7 and 8 as another uncorrected and corrected modelling pair. The model has 4000 m/sec (P-wave) zones in the wedge to the top left, and in the bottom layer. Between the two zones is a medium of 2000 m/sec. The S-wave velocities were set to one-half the P-wave velocities. The correction matrices here are of size 5 by 5, which may be seen to reduce, though not eliminate the obvious dispersion of Figure 7. The corrections may also be seen to have advanced the P wave at the bottom of Figure 8.

The same model pair is shown in Figures 9 and 10 in the interpreted P/T (Pressure/Torque) color format. This format tends to separate the pressure and shear wave energy. Figure 10 shows that the P wave dispersion has been almost eliminated, while the S wave dispersion has been reduced, but not eliminated.

CONCLUSIONS

We have shown the correction theory given in earlier papers may be used in the form of limited size spatial operators. These operators can then be used in the appropriate sections of a variable velocity medium to provide much more accurate wavelet propagation.

REFERENCES

Manning, P. M. and Margrave, G. F., 1999, Finite difference modeling, Fourier analysis, and stability: CREWES research report, v. **11**, p. 63-75.

Manning, P. M. and Margrave, G. F., 2000, Elastic finite difference modeling with stability and dispersion corrections: CREWES research report, v. **12**, p. 139-156.

Manning, P. M. and Margrave, G. F., 2003, Optimized corrections for finite-difference modeling in two dimensions: CREWES research report, v. **15**, Chapter 20.

FIGURES

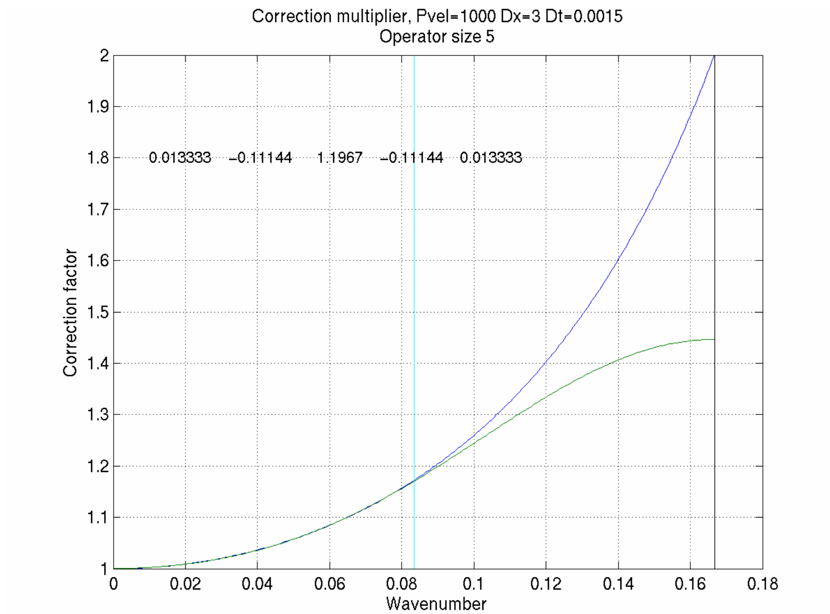

FIG. 1. A comparison of the ideal and the practical corrections, 1000 m/s medium.

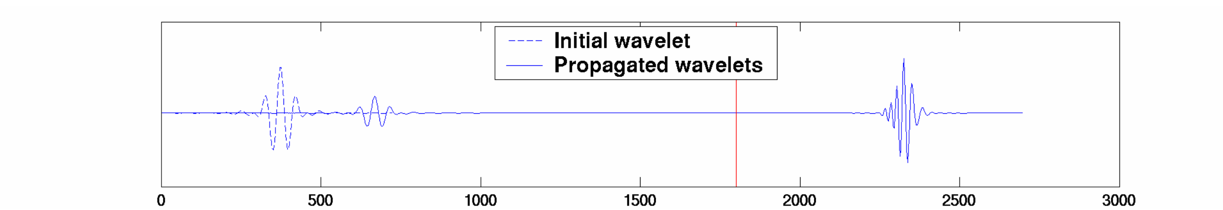

FIG. 2. Uncorrected wavelet propagation. Right side of model has half the velocity of the left side.

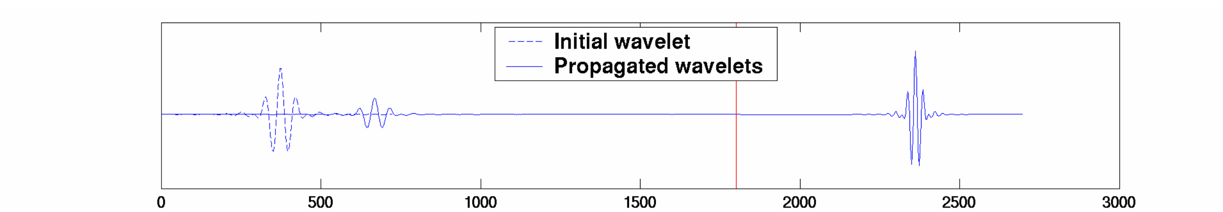

FIG. 3. Propagation with the right side of the model corrected at each step with a 5 point spatial operator.

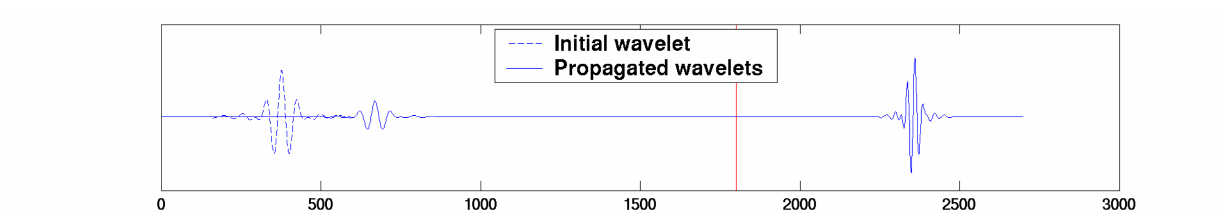

FIG. 4. Uncorrected wavelet propagation with half the sample rates of Figure 2.

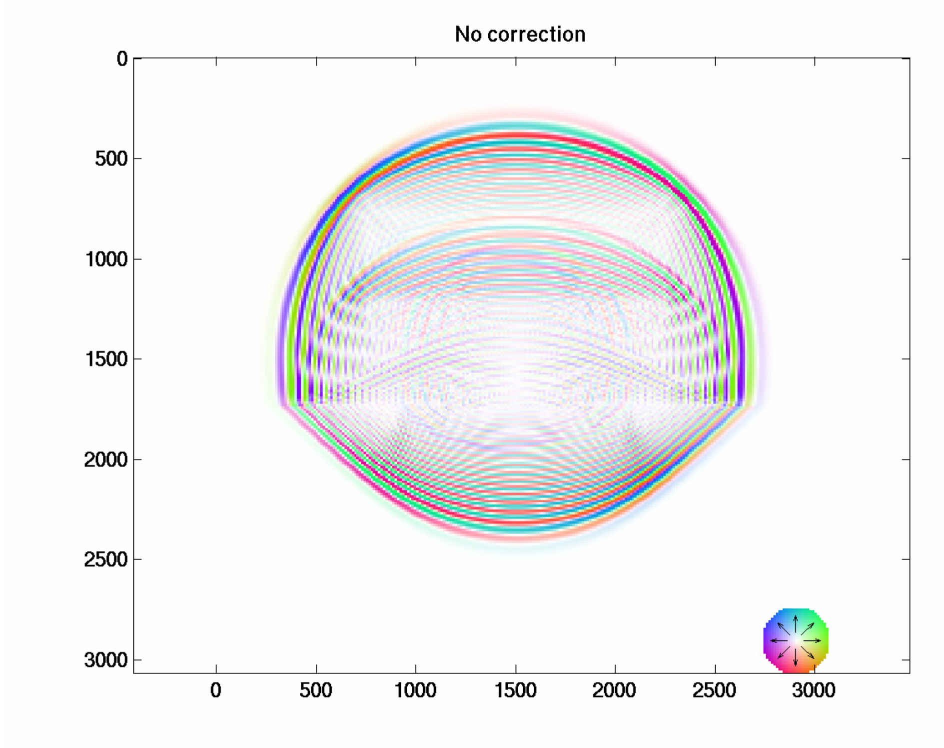

FIG. 5. Uncorrected two dimensional model. The source is centered at the (x,z) coordinates (1500,1500), just above the velocity contrast.

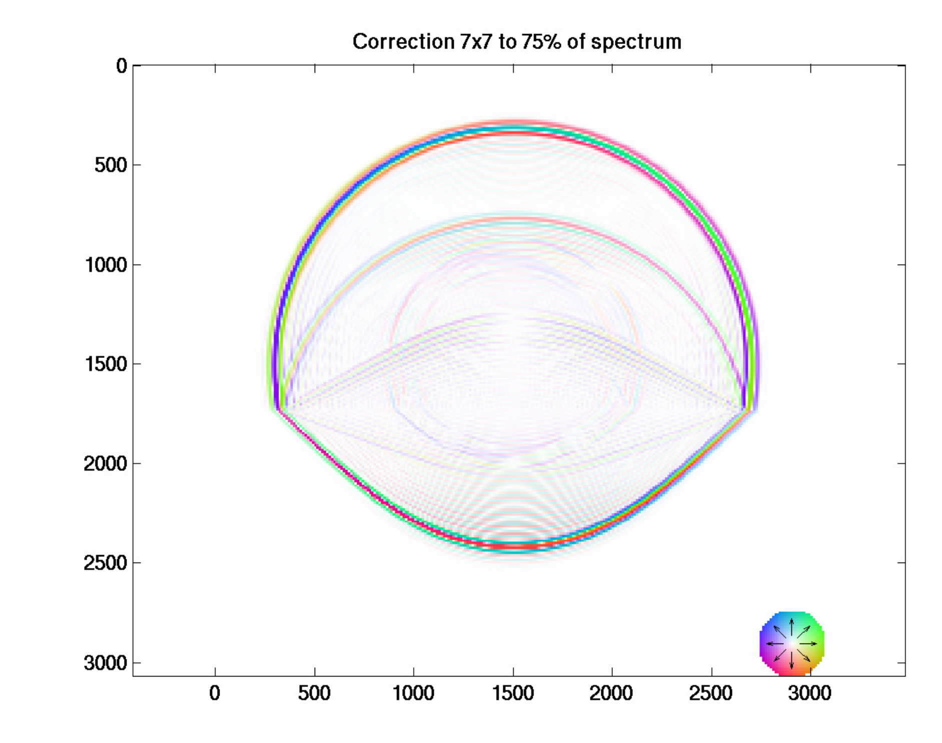

FIG. 6. Corrected model. Reflected P and S, and transmitted P and faint transmitted S waves can be seen.

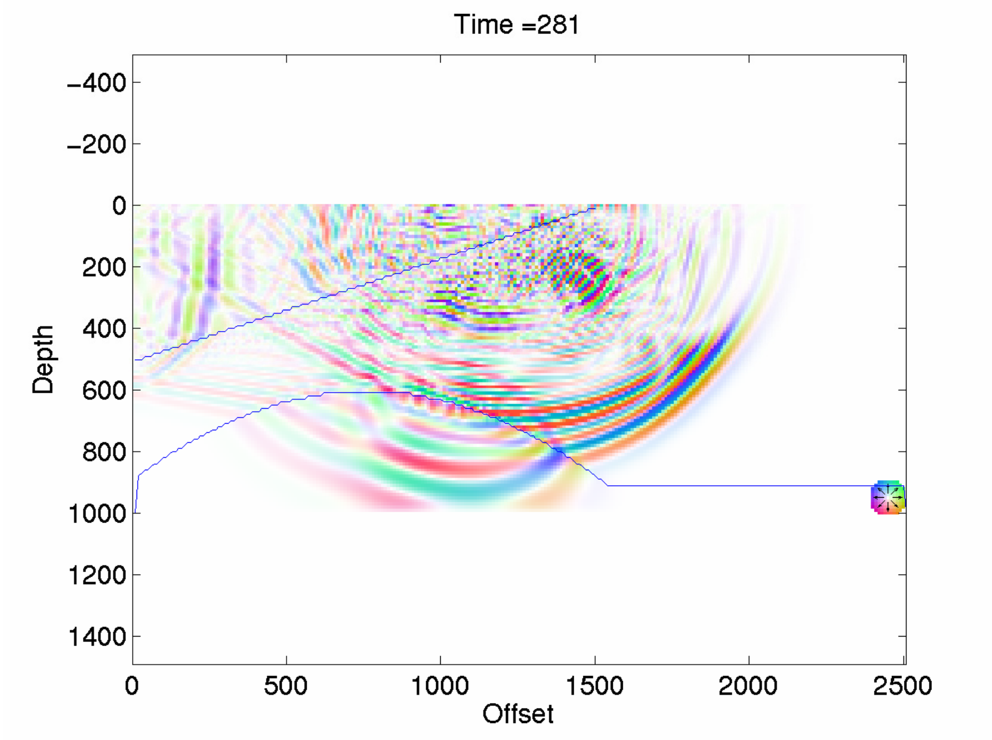

FIG. 7. Uncorrected high velocity wedge model. The source is in the centre of the model, just above the wedge boundary and below the surface.

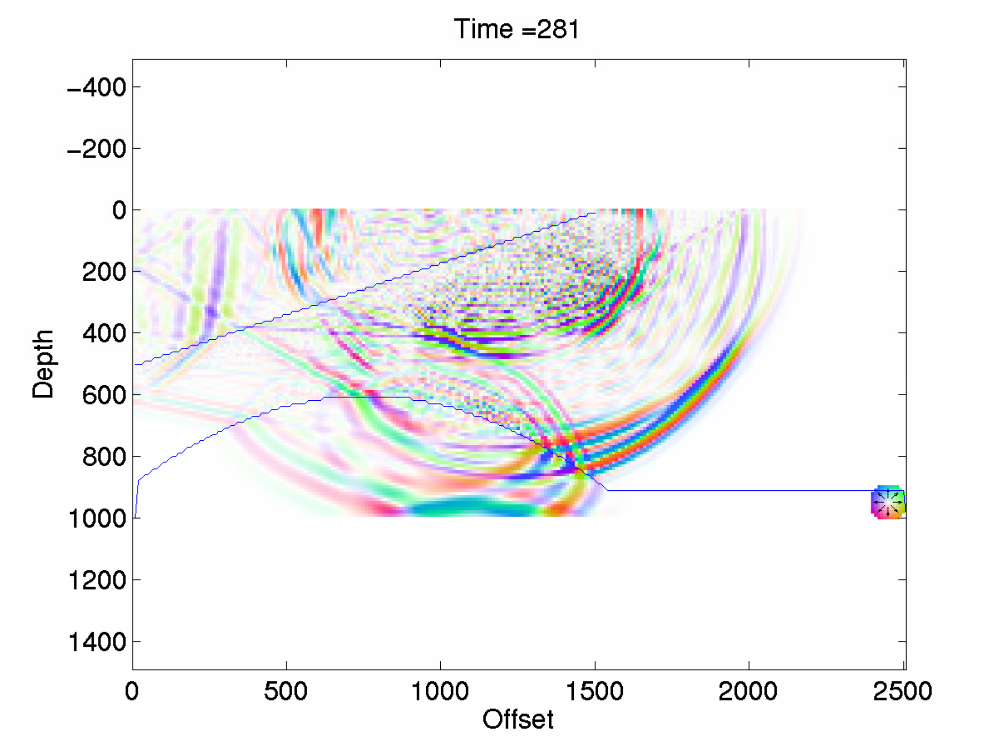

FIG. 8. Corrected model. The corrections may be seen to have concentrated the energy, although some events still show dispersion.

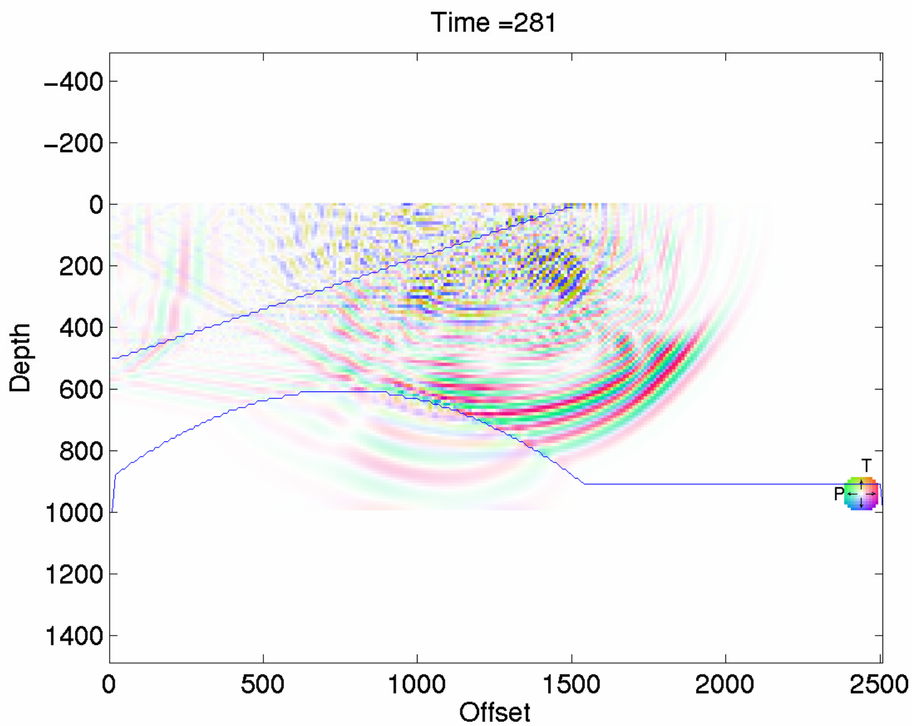

FIG. 9. Uncorrected model in the interpreted P/T (pressure/torque) format. There is obvious dispersion of both P and S waves.

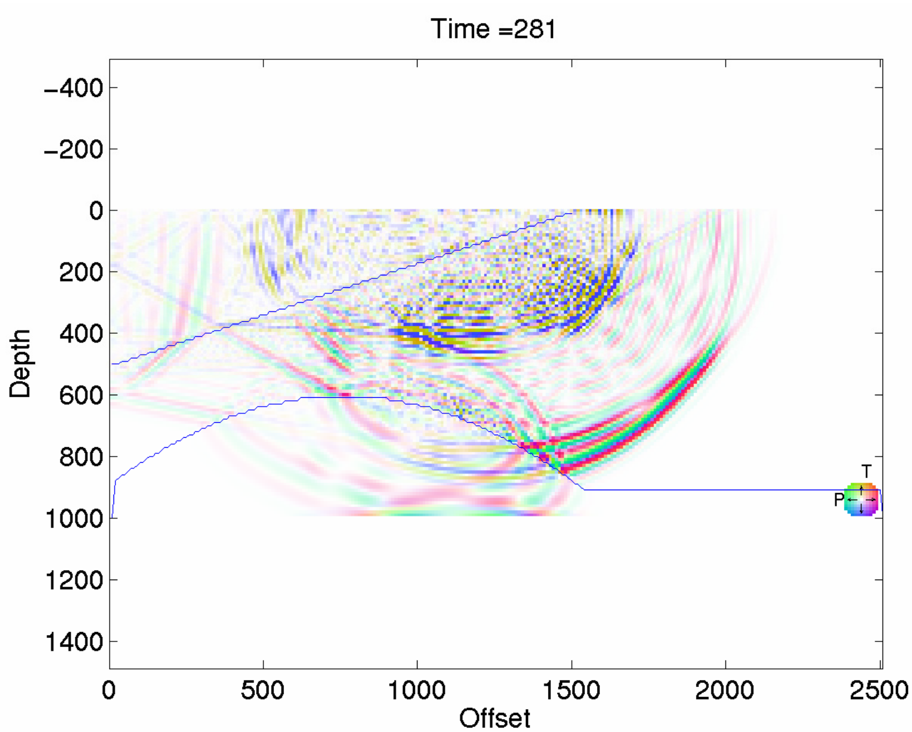

FIG. 10. Corrected model in P/T format. There is still some dispersion of the S waves, but most of the P wave energy has been concentrated.