Multiple attenuation by semblance weighted Radon transform

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ABSTRACT

The Radon transform is defined as summation along some specified family of trajectories over a set of time domain seismic data. The hyperbola is the most interesting summation curve for NMO-corrected seismic CMP-gathers since primaries become flat, but multiples remain hyperbolic. The primaries and multiples are hence able to be separated in the hyperbolic Radon domain. However the Radon transform is not a one-to-one transform, which means that near offset energy is transformed to the Radon domain many times. When we apply the inverse transform, the amplitudes of the reconstructed data are actually different from those of the original input data. In this paper, the strongest events on a record are first estimated by the semblance weighted Radon transform, and the corresponding energy is removed from the input data, forcing a forward Radon transform to be one-to-one. Removal is actually accomplished by a nonlinear filtering of multiples from the original input data rather than by subtraction.

INTRODUCTION

The Radon transform is defined by Johan Radon (1917) as an integral of some physical property of a medium along a particular path, which is given by

$$u(\tau,q)=\int_{-\infty}^{\infty} d\big(h,t=\tau+q\phi(h)\big)dh\,. \tag{1}$$

In exploration geophysics $d(h, t)$ is the original seismogram, $u(\tau, q)$ is the modeling data in the Radon transform domain, t is the two way time, h is a spatial variable such as offset, τ is the intercept two way time, q is the slope of the curvature based on which the transform trajectory is defined, and $\varphi(h)$ defines the curvature.

The conventional inverse Radon transform is given by

$$d^{'}(h,t)=\int_{-\infty}^{\infty} u\big(\tau=t-q\phi(h),q\big)dq\,. \tag{2}$$

Removing reverberations from reflection seismograms has been a long standing problem of exploration geophysics since multiple reflections often destructively interfere with the primary reflections of interest. In recent years, Radon transform approaches have attracted attention. For example, consider the hyperbolic Radon transform. After NMO correction, the primaries on a CMP gather would be flat events; but the multiples would still be hyperbolic events. Therefore the primaries and multiples could be separated in the Radon transform space.

Let us consider the main problem with the Radon transform. Because of energy sharing at near offsets on CMP gathers, a smearing problem occurs in the Radon domain, which decreases the resolution of the Radon transform. In this case, primaries and multiples are not easily separated in the model space. Subsequently, the reconstructed seismic data are different from the input data. This problem occurs because the equations (1) and (2) are not a transform pair, which is due to the form of the forward Radon transform.

In order to minimize this smearing problem, investigators have tried different methods for almost twenty years. Thorson and Claerbout (1985) worked on a least-squares method and a stochastic inverse method. The latter one has a relative high resolution but it is time consuming. To quicken the computation speed, Hampson (1986) moved the problem into the frequency domain and used the parabolic Radon transform. Beylkin (1987) applied a *rho* filter prior to integration over the q parameter in the inverse Radon transform. Zhou and Greenhalgh (1994b) showed that applying the *rho* filter in the forward Radon transform rather than the inverse transform could give better resolution in the modeling space. Sacchi and Tadeusz (1995) propose an improved algorithm for the parabolic Radon transform to improve the resolution.

We identify here an important paper by Ng and Bradshaw (1987), in which the time domain semblance weighted Gauss-Seidel forward Radon transform was introduced. Energy lying on a trajectory of h-t space (hyperbola) is mapped to a point in Radon τ-q space, then the amplitudes along the trajectory are subtracted from the input h-t space. Subtracting these values prevents their energy being incorporated into other trajectories. The modeling space could be enhanced by weighting it with the semblance. In this method, a threshold is set for each iteration of the Gauss-Seidel method. The selection of threshold would affect the result of the modeling space. A few iterations are needed to complete the forward Radon transform. Ng (2004) introduced the high resolution semblance weighted Gauss-Seidel Radon transform by first transforming the stronger energy and removing it from the input seismic data. After being Radon transformed, the energy of each q trace is measured in the modeling space and then the forward Radon transform is re-compute based on the strength of energy. Stronger events are computed first and removed from the input data, which further enhances the resolution of the modeling space. Since the energy measurement of each q is performed in the modeling space, it is still affected by the selection of threshold in the Gauss-Seidel method.

SEMBLANCE WEIGHTED RADON TRANSFORM

Semblance weighted Radon transform

Note that we don't mention Gauss-Seidel further because we don't use Gauss-Seidel method in this approach. We mentioned that the selection of threshold would affect the result of the modeling data. Before we examine our data, we can not know what thresholds would work. Experience can be helpful, but different data always have different requirements. Inappropriate thresholds would certainly dramatically affect our result. We can, however, estimate the energy of events relative to q in the modeling space in an objective way. Semblance is a very good estimation of the energy in the data space. We can therefore use semblance for objective selections, which also reduces computation time, since we only need a single computation of the forward Radon transform.

The semblance weighted hyperbolic Radon transform is given by Ng and Bradshaw (1987):

$$u(\tau,q)=S(\tau,q)\sum_{h} d\left(t^2=\tau^2+q*h^2,h\right). \tag{3}$$

Here $S(\tau, q)$ is the semblance of the input seismic data given by

$$S(\tau,p)=\frac{\sum_{l}\left(\sum_{h} d\left(t^2=\tau^2+ph^2,h\right)\right)^2}{N_h\sum_{l}\sum_{h} d^2\left(t^2=\tau^2+ph^2,h\right)}. \tag{4}$$

where N_h is the number of traces involved in the semblance, and l is a window size, usually a wavelet length.

In order to suppress multiples in the CMP gather, we need to mute the primaries in the modeling space, inverse Radon transform the multiples back to the original data space, and subtract these multiples from the data space. Then we have primaries and some other non-hyperbolic events left in the data space.

We now summarize the method:

1) Start with a CMP gather, $d(h, t)$;

2) Calculate the semblance of the input CMP data, $S(\tau, q)$;

3) Estimate the energy of the input CMP data relative to q based on the semblance;

4) Perform forward semblance weighted Radon transform in order of the descending power of q trace to get $u(\tau, q)$, i.e. first perform the Radon transform on the best fit trajectory of events, remove the amplitudes along this trajectory from the original input data, then move to the second best fit trajectory of events, and so on;

5) Mute the primaries in the Radon modeling space;

6) Perform inverse Radon transform on the remaining energy in the modeling space to reconstruct the multiples in the data space;

7) Suppress the reconstructed multiples from the original CMP gather $d(h, t)$;. The remaining energy in the data space is the demultipled seismic data.

Nonlinear multiple filter

Aside from the Radon transform itself, the multiple removal process is another critical issue. Zhou and Greenhalgh (1994a) introduced an elegant filter for multiple suppression with information about the multiples which can be derived by the semblance weighted Radon transform. The multiple data conveys to us just the location of multiples. By comparing the energy between the multiple data and the input data at each *h-t* point, an automatic nonlinear multiple rejection filter can be designed (Equation 5).

A Butterworth-type function was advised by Zhou and Greenhalgh (1994a) as a gain function, which in *h-t* domain is given by

$$g(h,t) = \frac{1}{\sqrt{1 + \left(\frac{B(h,t)}{\varepsilon A(h,t)} \right)^{n}}} . \tag{5}$$

Here $B(h, t)$ is the 2-D windowed sum over time and offset of the absolute value of the amplitude of the pixel centered at (h, t) on the multiple model traces; $A(h, t)$ is the same windowed sum over the absolute value of the amplitude of the original input section; n is the parameter used to control the smoothness of the filter; and ε is a multiple rejection parameter, which is related to the reflection coefficients of the sea bottom and sea surface, and ε should not be less than the absolute product of the reflection coefficients. This filter is nonlinear in the sense that the gain function is dependent on the input data.

NUMERICAL DATA EXAMPLES

Figure 1 is the synthetic NMO-corrected CMP gather of a simple layered model, where the NMO-stretch effect can be seen. Figure 2 is the semblance plot, which determines where the strongest events of Figure 1 are located. Figure 3 is the Radon model of Figure 1. There is some noise, but reasonably high resolution is evident. Even deep primaries and multiples, which don't have long period separation, can be easily separated in the model space. Figure 4 is the full reconstruction of Figure 1 from Figure 3. Compared with Figure 1, the original input data, Figure 4 has some small differences in the far offset. In Figure 5, the primary energy in the Radon domain is muted and only multiples are left. The reconstructed multiples are shown in Figure 6. The primaries after the nonlinear filtering of reconstructed multiples from the original input data are shown in Figure 7. Comparing Figure 7 with Figure 8, the primaries after ordinary subtraction of the reconstructed multiples from the original input data, we can see that the nonlinear filter does a better job whenever the multiple model traces map the exact locations of the multiples, although there is still some residual energy, especially where the primary and the multiple energy interfere.

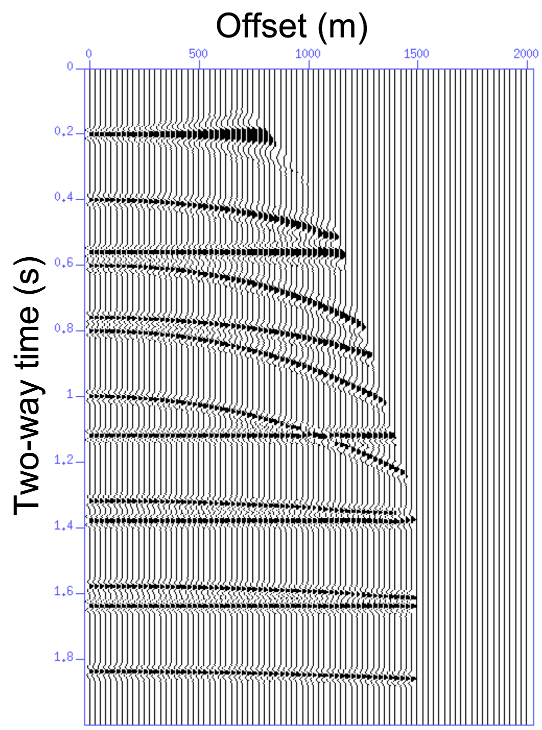

FIG. 1. An NMO-corrected muted synthetic CMP data. Trace interval: 25 m; sampling rate: 2 ms.

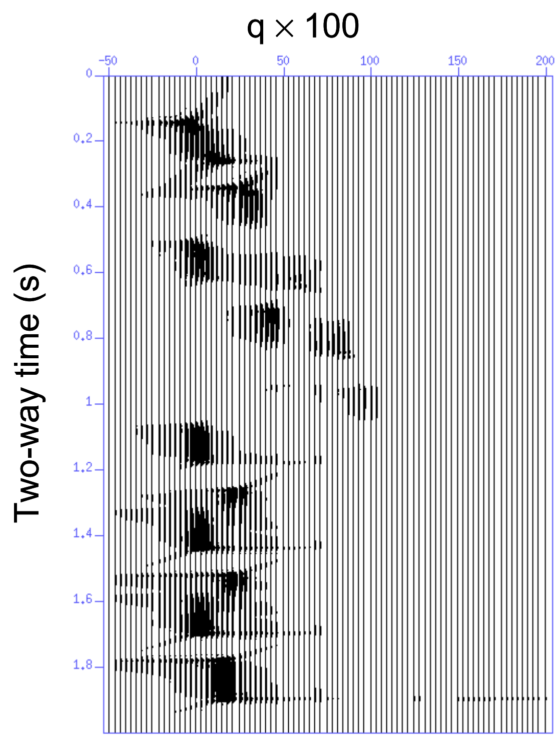

FIG. 2. The semblance plot of the data in FIG. 1

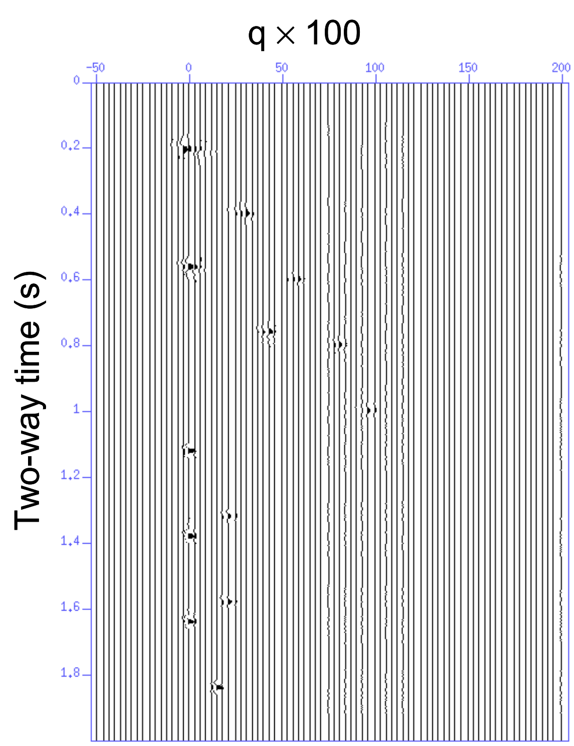

FIG. 3. The Radon domain data of FIG. 1. Primaries are aligned along q=0. The rest are multiples.

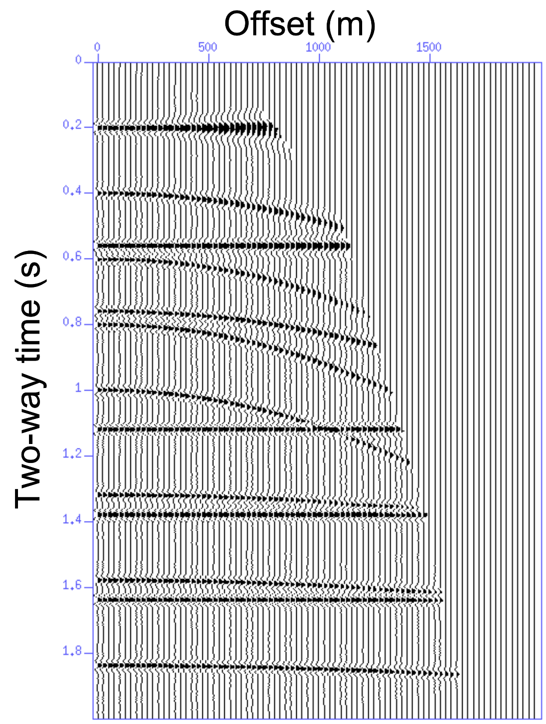

FIG. 4. The reconstructed primaries and multiples.

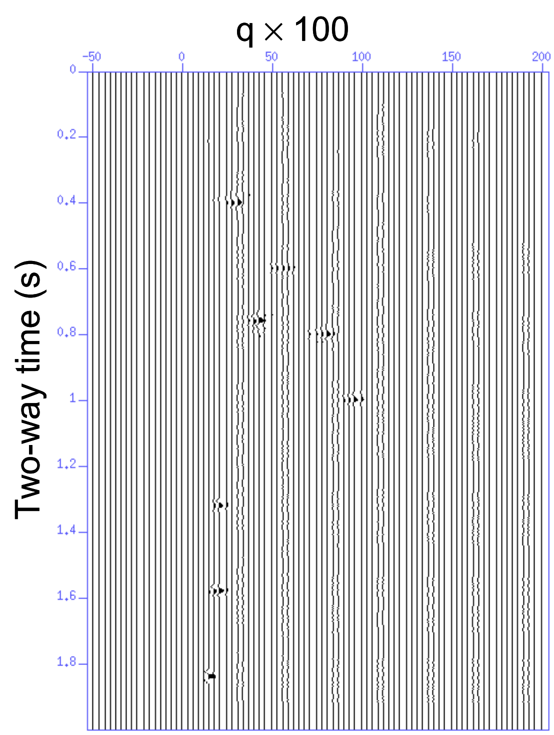

FIG. 5. Primaries are muted and multiples are left.

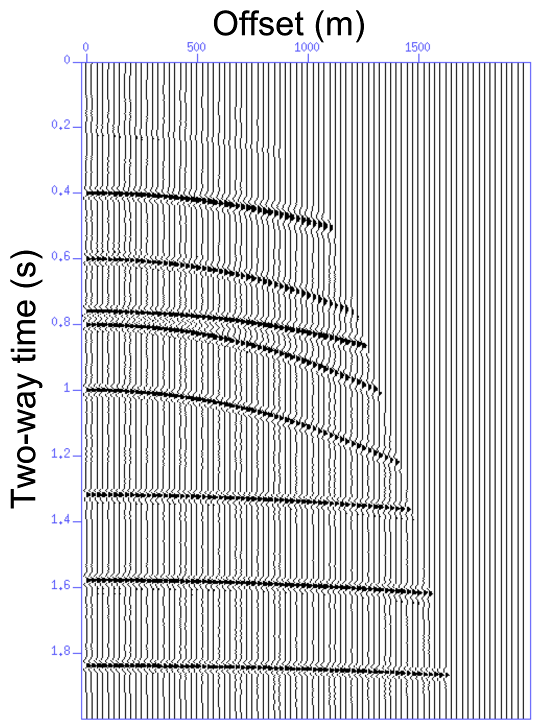

FIG. 6. The reconstructed multiples in time-space domain.

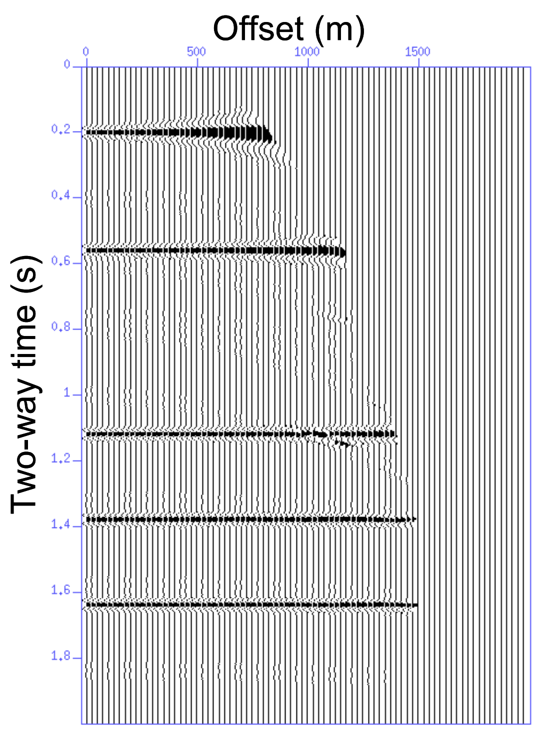

FIG. 7. The primaries after nonlinear filtering of multiples in time-space

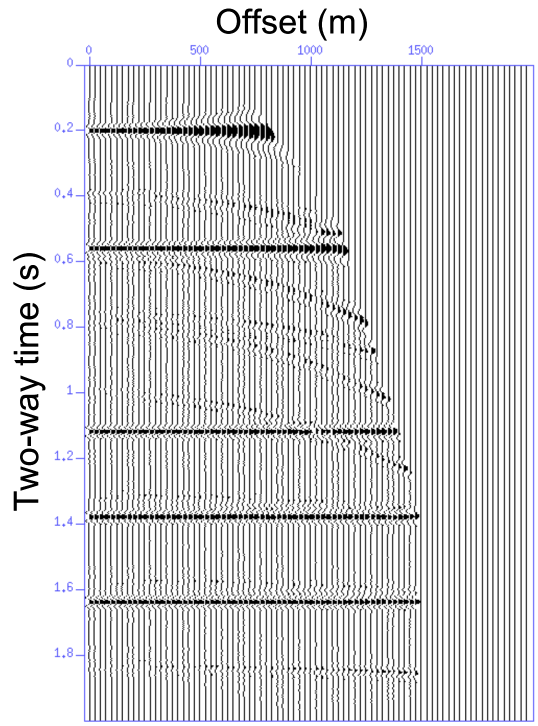

FIG. 8. The primaries after the subtraction of multiples from the original input data.

CONCLUSIONS

A new Radon transform approach based on Ng and Bradshaw (1987) and Ng (2004) is introduced. This approach can give high resolution in the Radon space. Combined with the nonlinear filter advised by Zhou and Greenhalgh (1994a), this method is effective in suppressing multiples.

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