Solving surface-consistent statics with multigrid

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ABSTRACT

By expressing the surface-consistent equations as a matrix operation, a multigrid method was adapted to separate source and geophone statics. The method is compared to an approximation allowing us to perform a direct inversion, and a Gauss-Seidel relaxation method. Multigrid shows a greater ability to resolve the long-wavelength components of the statics on a synthetic data set.

INTRODUCTION

The application of surface consistent residual statics are an important part of any seismic data processing flow. Static corrections are due to variations in the near-surface geology. The effect of the near surface on event traveltimes can often be approximated as a bulk shift on each recorded trace.

Most methods for estimating the bulk static shift needed to correct a prestack trace involve cross correlating a trace with a model trace. The time delay of the maximum of the crosscorrelation is the total required static shift.

The method assumes that each trace has a timeshift due to a combined delay from each geophone and source pair. The static shift comes from the effect of the source location on the downward-travelling wave, and the receiver location on the upward-travelling wave. We separate the two time delays into source and receiver components before applying them to the data. This is to help avoid arbitrarily changing the structure of the stack section (Bancroft et al., 2000).

The system of equations that results from this problem is over-determined (more equations than unknowns), requiring a least-squares solution. As well, the system is under-constrained (more unknowns than independent equations), restricting the available methods to obtain a solution (Marsden, 1993).

In this paper, the surface-consistent equations are studied. The problem of decoupling the source and receiver statics are expressed as a matrix operation of the form

$$\mathbf{A}\mathbf{s} = \mathbf{t}. \tag{1}$$

Here, $\mathbf{A}$ is a matrix of coefficients, $\mathbf{t}$ is a vector with all of the calculated time shifts for a trace, and $\mathbf{s}$ is an unknown vector of the separated source and receiver statics.

Several methods of solution are explored, including a multigrid method. Multigrid methods are a *decomposition of scale* for the problem. With the help of an antialias filter, the number of the unknowns in the problem is temporarily reduced, as per Figure 1. After this reduced system is solved, the solution is interpolated to a higher sampling rate. The interpolated points are corrected using an iterative method, such as Gauss-Seidel. This cycle of interpolation and correction is repeated until the desired grid spacing is achieved.

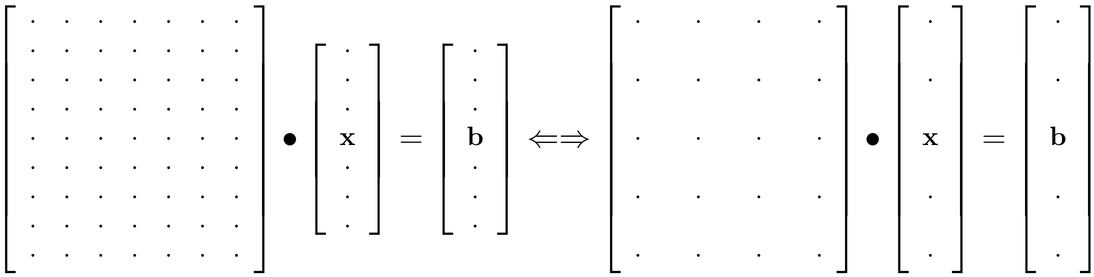

FIG. 1. Restriction (⇒) versus Interpolation (⇐).

THE SURFACE-CONSISTENT EQUATIONS

The total traveltime error for each trace T_{ijkl} can be expressed as

$$T_{ijkl} = S_i + R_j + M_k + O_l, \tag{2}$$

taking a contribution from the i^{th} source static, S_i, and the j^{th} receiver static, R_j. The M_k refers to the structural term at midpoint k, and an offset-dependent moveout error O_l. For simplicity, in this paper we assume that both $M = O = 0$,

$$T_{ij} = S_i + R_j, \tag{3}$$

which we justify in our conclusions.

To express T_{ij} as a vector, T_n, we employ the formula

$$n = (i-1) * N_{live} + i^*, \tag{4}$$

with N_{live} as the number of live geophones per shot gather, and i^* is the trace number within the shot record.

Consider a simple seismic survey, with 3 shots and 4 receivers. Writing down an equation for each of the calculated time shifts, and organizing the sources and receivers into columns,

$$\begin{bmatrix}
S_1 & & & + & R_1 & & & \\
S_1 & & & + & & R_2 & & \\
S_1 & & & + & & & R_3 & \\
S_1 & & & + & & & & R_4 \\
& S_2 & & + & R_1 & & & \\
& S_2 & & + & & R_2 & & \\
& S_2 & & + & & & R_3 & \\
& S_2 & & + & & & & R_4 \\
& & S_3 & + & R_1 & & & \\
& & S_3 & + & & R_2 & & \\
& & S_3 & + & & & R_3 & \\
& & S_3 & + & & & & R_4
\end{bmatrix}
=
\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \\ T_{10} \\ T_{11} \\ T_{12} \end{bmatrix}
=
\begin{bmatrix} T_{1,1} \\ T_{1,2} \\ T_{1,3} \\ T_{1,4} \\ T_{2,1} \\ T_{2,2} \\ T_{2,3} \\ T_{2,4} \\ T_{3,1} \\ T_{3,2} \\ T_{3,3} \\ T_{3,4} \end{bmatrix} \tag{5}$$

We can put (5) into the matrix equation

$$\begin{bmatrix} 1 & & & 1 & & & \\ 1 & & & & 1 & & \\ 1 & & & & & 1 & \\ 1 & & & & & & 1 \\ & 1 & & 1 & & & \\ & 1 & & & 1 & & \\ & 1 & & & & 1 & \\ & 1 & & & & & 1 \\ & & 1 & 1 & & & \\ & & 1 & & 1 & & \\ & & 1 & & & 1 & \\ & & 1 & & & & 1 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \\ T_{10} \\ T_{11} \\ T_{12} \end{bmatrix} = \begin{bmatrix} T_{1,1} \\ T_{1,2} \\ T_{1,3} \\ T_{1,4} \\ T_{2,1} \\ T_{2,2} \\ T_{2,3} \\ T_{2,4} \\ T_{3,1} \\ T_{3,2} \\ T_{3,3} \\ T_{3,4} \end{bmatrix} \tag{6}$$

To form this matrix, all 4 of the receivers were live for all 3 shots. For most seismic surveys, only a subset of all of the receivers laid out are live. For the same number of live receivers, where the spread advances 2 stations with each shot, we arrive at the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & & & 1 & & & & & & & \\ 1 & & & & 1 & & & & & & \\ 1 & & & & & 1 & & & & & \\ 1 & & & & & & 1 & & & & \\ & 1 & & & & 1 & & & & & \\ & 1 & & & & & 1 & & & & \\ & 1 & & & & & & 1 & & & \\ & 1 & & & & & & & 1 & & \\ & & 1 & & & & & 1 & & & \\ & & 1 & & & & & & 1 & & \\ & & 1 & & & & & & & 1 & \\ & & 1 & & & & & & & & 1 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \\ R_7 \\ R_8 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \\ T_{10} \\ T_{11} \\ T_{12} \end{bmatrix} = \begin{bmatrix} T_{1,1} \\ T_{1,2} \\ T_{1,3} \\ T_{1,4} \\ T_{2,3} \\ T_{2,4} \\ T_{2,5} \\ T_{2,6} \\ T_{3,5} \\ T_{3,6} \\ T_{3,7} \\ T_{3,8} \end{bmatrix}. \tag{7}$$

The number of unknowns in this system of equations is equal to the total number of source and receivers, $N_s + N_r$. Note that $N_{live} \neq N_r$. The number of calculated time shifts is $N_s \cdot N_{live}$ (length of $\mathbf{t}$). Therefore, $\mathbf{A}$ is of size $(N_s \cdot N_{live}) \times (N_s + N_r)$.

THE LEAST-SQUARES APPROACH

The system of equations represented by matrices in Equations 5 and 7 are over-determined. In order to solve it effectively, we use the normal equations to calculate the least-squares approximation,

$$\mathbf{A^T A s} = \mathbf{A^T t} \tag{8}$$

The matrix $\mathbf{A^T A}$ can be partitioned into 4 sub-matrices, as

$$\mathbf{A^T A} = \left[\begin{array}{c|c} \mathbf{S} & \mathbf{B} \\ \hline \mathbf{B^T} & \mathbf{R} \end{array}\right]. \tag{9}$$

The matrix $\mathbf{S}$ is $(N_s \times N_s)$, with entries only on the main diagonal, all equal to N_{live}. Similarly, $\mathbf{R}$ is an $(N_r \times N_r)$ matrix, with entries on the main diagonal equaling the receiver fold, or the number of shots each receiver is live for. To form $\mathbf{B}$, the i^{th} row has a 1 in the columns associated with all the receivers live for the i^{th} shot, and is $(N_s \times N_r)$.

The $\mathbf{A^T A}$ matrix corresponding to Equation 7 (and partitioned the same as Equation 9) is

$$\left[\begin{array}{ccc|cccccccc}
4 & & & 1 & 1 & 1 & 1 & & & & \\
 & 4 & & & & 1 & 1 & 1 & 1 & & \\
 & & 4 & & & & 1 & 1 & 1 & 1 & \\
\hline
1 & & & 1 & & & & & & & \\
1 & & & & 1 & & & & & & \\
1 & 1 & & & & 2 & & & & & \\
1 & 1 & & & & & 2 & & & & \\
 & 1 & 1 & & & & & 2 & & & \\
 & 1 & 1 & & & & & & 2 & & \\
 & & 1 & & & & & & & 1 & \\
 & & 1 & & & & & & & & 1
\end{array}\right] \tag{10}$$

Left-multiplying the vector of calculated time shifts $\mathbf{t}$ by $\mathbf{A^T}$ yields

$$\mathbf{A^T A s} = \mathbf{A^T t} = \left[\begin{array}{c}
\sum_j T_{1j} \\
\sum_j T_{2j} \\
\vdots \\
\sum_j T_{N_s j} \\
\\
\sum_i T_{i1} \\
\sum_i T_{i2} \\
\vdots \\
\sum_i T_{iN_r}
\end{array}\right] \tag{11}$$

The limits of the summation are not specified. However it is implied that the sum is across only the live receivers for a given source, and vice versa.

The vector $\mathbf{A^T t}$ is of size $(N_s + N_r) \times (1)$. The first entry in $\mathbf{t}$ is the sum of all the time shifts associated with all of the recordings in the first shot record. Likewise, the 2^{nd} entry corresponds to the sum of the statics for the 2^{nd} shot record, etc. Following the shot records, each static associated with a particular receiver is summed, and entered in its corresponding position in $\mathbf{t}$.

Using the example from Equation 7, and left-multiplying both sides by the transpose of $\mathbf{A}$,

$$\left[\begin{array}{ccc|cccccccc} 4 & & & 1 & 1 & 1 & 1 & & & & \\ & 4 & & & & 1 & 1 & 1 & 1 & & \\ & & 4 & & & & 1 & 1 & 1 & 1 & \\ \hline 1 & & & 1 & & & & & & & \\ 1 & & & & 1 & & & & & & \\ 1 & 1 & & & & 2 & & & & & \\ 1 & 1 & & & & & 2 & & & & \\ & 1 & 1 & & & & & 2 & & & \\ & 1 & 1 & & & & & & 2 & & \\ & & 1 & & & & & & & 1 & \\ & & 1 & & & & & & & & 1 \end{array}\right] \left[\begin{array}{c} S_1 \\ S_2 \\ S_3 \\ \hline R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \\ R_7 \\ R_8 \end{array}\right] = \left[\begin{array}{c} T_{1,1}+T_{1,2}+T_{1,3}+T_{1,4} \\ T_{2,3}+T_{2,4}+T_{2,5}+T_{2,6} \\ T_{3,5}+T_{3,6}+T_{3,7}+T_{3,8} \\ \hline T_{1,1} \\ T_{1,2} \\ T_{1,3}+T_{2,3} \\ T_{1,4}+T_{2,4} \\ T_{2,5}+T_{3,5} \\ T_{2,6}+T_{3,6} \\ T_{3,7} \\ T_{3,8} \end{array}\right] = \left[\begin{array}{c} T_1+T_2+T_3+T_4 \\ T_5+T_6+T_7+T_8 \\ T_9+T_{10}+T_{11}+T_{12} \\ \hline T_1 \\ T_2 \\ T_3+T_5 \\ T_4+T_6 \\ T_7+T_9 \\ T_8+T_{10} \\ T_{11} \\ T_{12} \end{array}\right] \tag{12}$$

By examining one line of the equation, we gain insight into the calculation. A line from the upper partition of Equation 9, in general, reads

$$N_{live} S_i + \sum_j R_j = \sum_j T_{ij}. \tag{13}$$

Solving for S_i,

$$S_i = \frac{\sum_j (T_{ij} - R_j)}{N_{live}}. \tag{14}$$

Likewise,

$$R_j = \frac{\sum_i (T_{ij} - S_i)}{N_{live}}, \tag{15}$$

from the lower partition of the matrix equation.

Physically, the significance is that the least-squares static solution for a particular source is the difference between the average calculated time shift and the average receiver static for that source,

$$S_i = T_{ave} - R_{ave}. \tag{16}$$

The converse is also true,

$$R_j = T_{ave} - S_{ave}. \tag{17}$$

INVERSION RESULTS

Direct inversion

As well as being over-determined, the normal equations are under-constrained by 1, (1 more unknown than independent equations). This makes some inversion methods difficult to use, as there is no true unique inverse. To perform a direct inversion, we need to add an additional condition or equation to make the method stable. By adding a single equation to the matrix, we can force all of the sources or receivers in each gather to have a zero average. This zeros all the off-diagonal coefficients in $\mathbf{A^T A}$. From there it is straightforward to calculate the inverse, as Equation 9 becomes

$$\mathbf{A^T A} = \left[\begin{array}{c|c} \mathbf{S} & 0 \\ \hline 0 & \mathbf{R} \end{array}\right]. \tag{18}$$

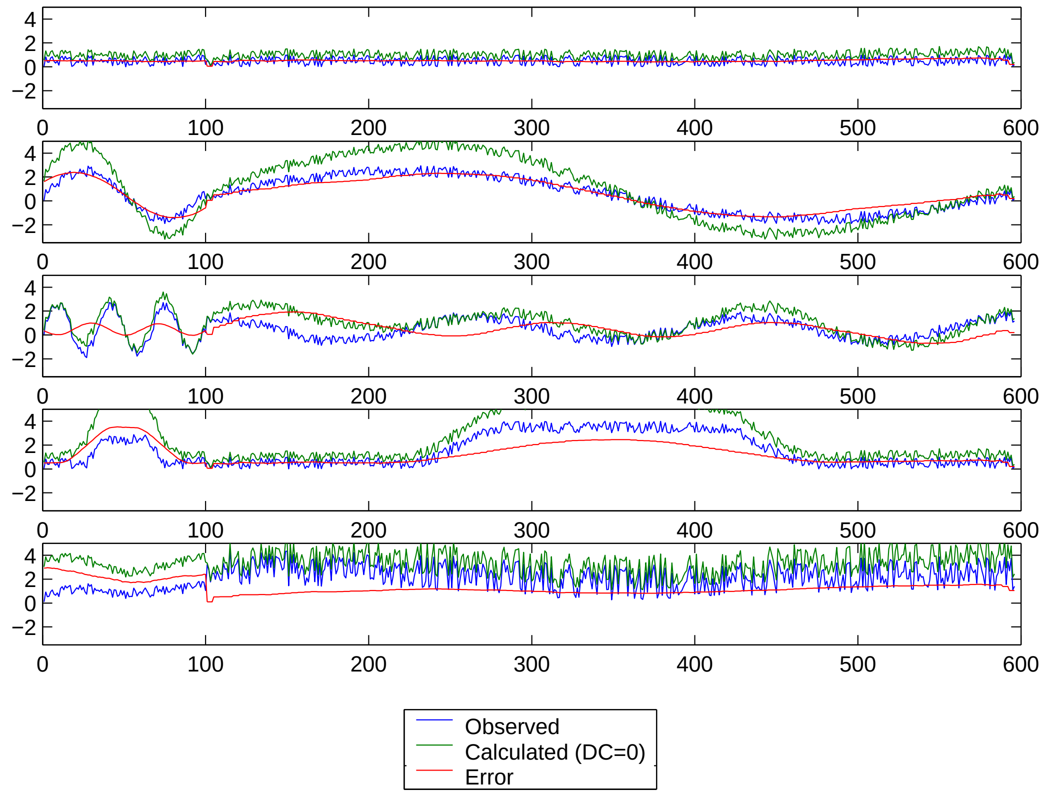

FIG. 2. The direct inversion results. The first 100 entries are source statics. Wavelengths above the spread length of 100 stations are not accurately calculated.

The resulting matrix is diagonal, so the inverse of the matrix is trivial to calculate, as it is just the inverse of the diagonal entries of the matrix.

By enforcing the condition that each gather has a zero average, we force the solution to not have any wavelengths longer than the spread. In Figure 2, the results of this fast inversion are depicted. A variety of models with long-wavelength statics were calculated. A random number was added to a smoothly varying function, to give source and receiver statics. This sum was used to calculate a time shift for each trace. Using only the time shifts and the geometry, an attempt was made to recover the separate source and receiver statics. For the direct inversion method, the longer wavelength components are not well represented. The survey parameters are for 100 shots, each with 100 live receivers, with a spread that advances by 4 stations with each shot.

Gauss-Seidel

Instead of forcing the DC term to zero and performing a direct inversion, an alternative approach is to solve this problem using an iterative method, such as Gauss-Seidel relaxation. By cycling through each value in the unknowns, and updating it using Equations 16 and 17, we can revise our estimation of the source and receiver statics. The Gauss-Seidel method converges very quickly to the solution for high frequencies. However, the solution after 15 iterations is barely distinguishable from the solution after 3 iterations, as the long wavelengths are corrected very slowly. To properly estimate the long-wavelength statics

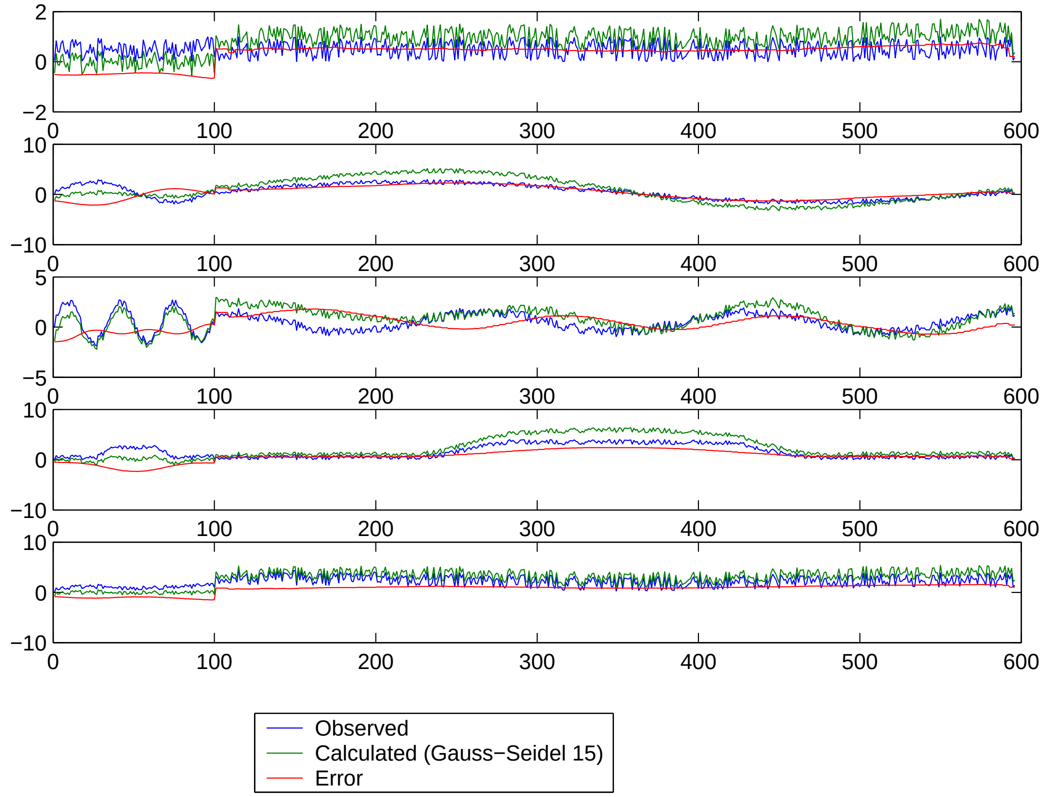

FIG. 3. The Gauss-Seidel inversion results. The first 100 entries are source statics. Wavelengths above the spread length of 100 stations are not accurately calculated.

using the Gauss-Seidel correction is highly impractical. Figure 3 shows the solution. The quality of the solution is similar to that of the direct inversion method.

Multigrid

Antialias-filtering the data groups adjacent shots (receivers) together. When we reduce the number of grid points down to a small number, what we are doing is averaging across multiple spread lengths to form the long-wavelength trends. Not only do we save computer time by doing a large part of the work on smaller systems of equations, we are also increasing the convergence rate of the iterative method employed. As can be seen in Figure 3, the Gauss-Seidel correction quickly attenuates error terms whose wavelength is near that of the grid spacing. Any trends in the data not attenuated are solved for at the coarser grid spacing. For more information on how the multigrid method works, see Millar and Bancroft (2003). The amount of computer effort required to produce the multigrid solution to the system is approximately twice that of one Gauss-Seidel correction (Bancroft and Millar, 2003). However, the results of the multigrid inversion are far superior, as can be seen in Figure 4.

CONCLUSIONS

Long-wavelength trends in residual statics can appear in a stacked section as structural artifacts. As well, errors in static predictions lead to errors in velocity analysis, and degrade

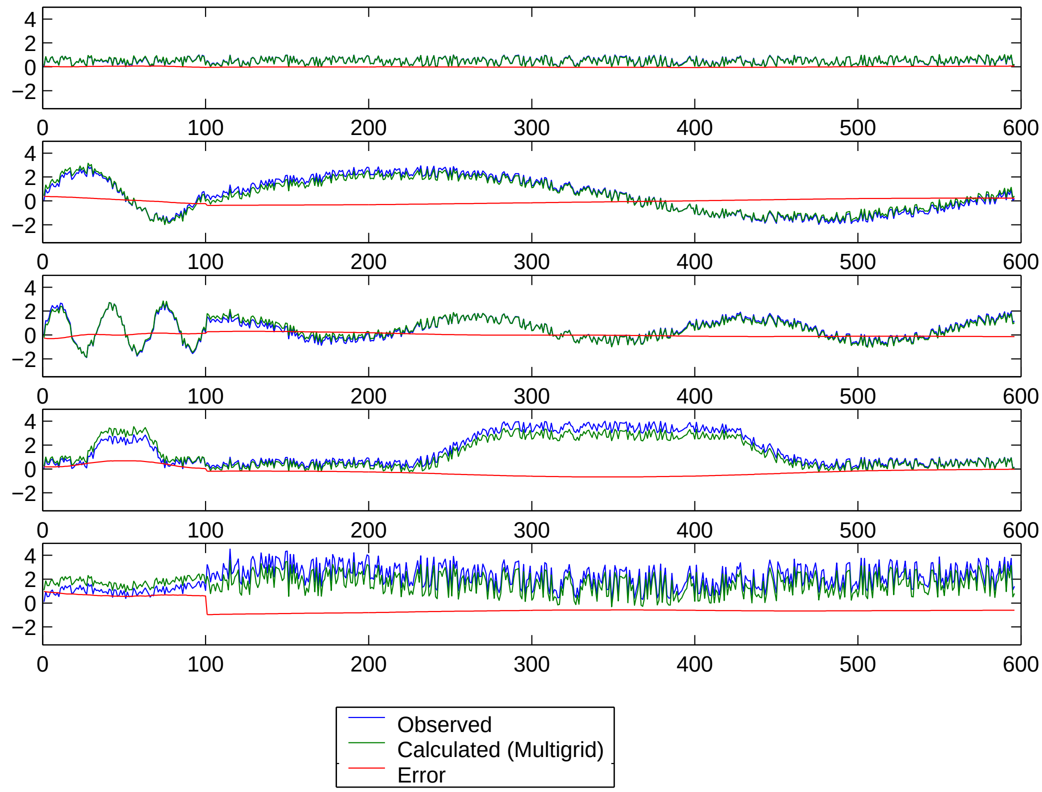

FIG. 4. The multigrid inversion results. The first 100 entries are source statics. All wavelengths are more accurately calculated than in the other examples.

image quality. It is preferred to model the statics as being surface consistent, as large CMP-oriented statics can greatly effect the apparent structure in a stack section.

Our analysis shows that it may be possible to recover these longer wavelength static corrections in the data, by using a multigrid method. The accuracy of the method on the synthetic data provided is far superior for multigrid methods, with a small reduction in the amount of necessary computer time. Further tests on field data are pending.

Multigrid methods are proving themselves to be fast, robust, and straightforward methods for solving a variety of problems in exploration geophysics. Over time it is planned to extend the method to include larger, non-linear systems, across multiple dimensions.

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