Ray generation and the PS_V primary arrival

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ABSTRACT

It has become common practice, when considering a plane parallel structure for seismic modelling purposes, to treat the primary PS_V and PP reflected arrivals in a similar fashion. For many of the aspects of these investigations the two arrivals may be studied using the same procedures. However, a major area of difference, more crucial in a model with a large number of layers, is the dynamics or amplitude properties of the PS_V arrival. In a many layered medium there is only one ray in the PP case, having two P ray segments in each layer for a given source-receiver offset. In the PS_V case there is the so called primary ray composed of P ray segments down to the reflector and S_V ray segments up to the receiver. This arrival has only one conversion, the reflected conversion at the deepest interface. In addition, there are a number of other multiplied converted waves, each with one P and one S_V ray segment per layer, which have identical kinematic properties (travel times) as the primary PS_V arrival and may also have different conversion points on the reflecting interface. Due to the increase in the number of conversions in these other rays their amplitudes will be substantially less than that of the primary. However, as the number of layers increases, the number of these extra rays increases significantly. This effect is what will be considered here.

INTRODUCTION

The generation of a subset of the infinite number of rays which adequately describes the total wave field response at a surface receiver due to a point source at the surface in a medium composed of plane parallel layers has not received as much attention in the literature as is probably warranted. There are a number of constraints which may be imposed on a ray generation scheme, such as the number of mode conversions, the number of multiples within a given layer, and a condition on the number of layers in which the previous two conditions will be honoured, after which only primary rays, rays with a single conversion and rays with no multiple conversions will be considered in the construction of synthetic traces.

Any form of limiting the number of rays to produce what has come to be known as a *partial ray expansion* (Hron et al., 1973) requires the imposition of the bias of the author in the ray generation algorithm. Even if several numerical checks are done on the velocity distribution within the layered structure prior to the actual partial ray expansion construction, which utilizes the knowledge gained in this preliminary analysis of the model parameters and shooting geometry, the inclusion of all rays of significance is not guaranteed. However, this manner of proceeding usually produces a reasonable (and manageable) approximate subset of the infinite number of rays required to construct the total wave field. This method of ray generation has its limitations with respect to the number of layers that may be practically considered. The current number of layers is of the order $10^1 - 10^2$. Beyond this, computation time and other computer resources required are extreme even with the current generation of workstations and PCs.

There are very few references given here regarding ray generation in general. The reason for this is that those that appear in the literature were based on basic combinatorial methods (Hron, 1971 and Hron, 1972) or on the series expansion of some generating function. From Hron et al. (1974) *"... no accurate estimate of the accuracy of the partial ray expansion was possible because the convergence of the complete expansion had not been established. This difficulty was removed by Cisternas, Betancourt and Leiva (1973) who showed that the complete ray expansion can be obtained for a solid layered medium by expanding the Rayleigh matrix rather than the Rayleigh determinant into a power series in which each term can be interpreted as a physical ray."* Both of these approaches have been superseded by a more sophisticated combinatorial approach using the theory of partitions. None of this theory, unfortunately, was ever published in the general geophysical or mathematical literature, apart from some proprietary material in the form of technical reports, whose status is unknown. A very specific set of rays will be considered in what follows, for a very specialized purpose, in which some of the basic essentials of ray generation are presented.

THEORY

Partial ray expansions for approximating the total geometrical arrival wave field are based on algorithms of a combinatorial nature, but, as mentioned above, have gone through several generations of improvement since publication of the original algorithms. Unconverted primaries can serve as the bases for these, or if conversions and multiples are to be considered, it is usually fastest to consider a layer at a time, starting with the first and increase the number of P and S_V segments in each layer until a specified upper bound on the number of ray segments and mode conversions within a layer is reached. As the PP and PS_V primaries are often the only arrivals used to construct synthetic traces for a plane layered medium composed of a large number of layers, it is only these two ray types which will considered here. The PP primary from any given interface has only one element in the kinematic set, and thus only one element in the dynamic set. The PS_V primary will be given the most attention in this report.

Before proceeding it may be useful to introduce the concept of kinematic and dynamic sets by examining the simplest case of a kinematic set, which may be partitioned into more than one dynamic set. This ray consists of 4 ray segments in both layers 1 and 2 of a two layered isotropic homogeneous model. As shown in Figure 1, there are 3 elements in this kinematic set denoted *N*, implying that all three rays have the same kinematic properties, namely travel time, and as a consequence of the medium specification, the same geometrical spreading. As rays 1 and 2 of this kinematic set also employ the same number and type of reflection and transmission coefficients, given the previous statement, they will have identical amplitudes. Thus their combined effect at receiver is just twice the amplitude of one of the elements, which is generally complex. Ray 3 of this kinematic set differs from rays 1 and 2 in the reflection and transmission coefficients used to obtain its amplitude. It must be considered separately at the receiver. However, there are many terms common to the two dynamic sets comprising the kinematic set, which reduces the computation time to obtain both amplitudes.

The case of the PS_V primary propagating in a similar medium type as above consisting of *n+1* plane layers will now be considered. No constraints will initially be imposed on the direction of propagation, either up or down, of a P or S_V ray segment within a given layer. In this case the total number of rays comprising the kinematic set of all rays reflected from the $(n+1)^{th}$ interface, the $\mathrm{zero}^{\mathrm{th}}$ interface corresponding to the

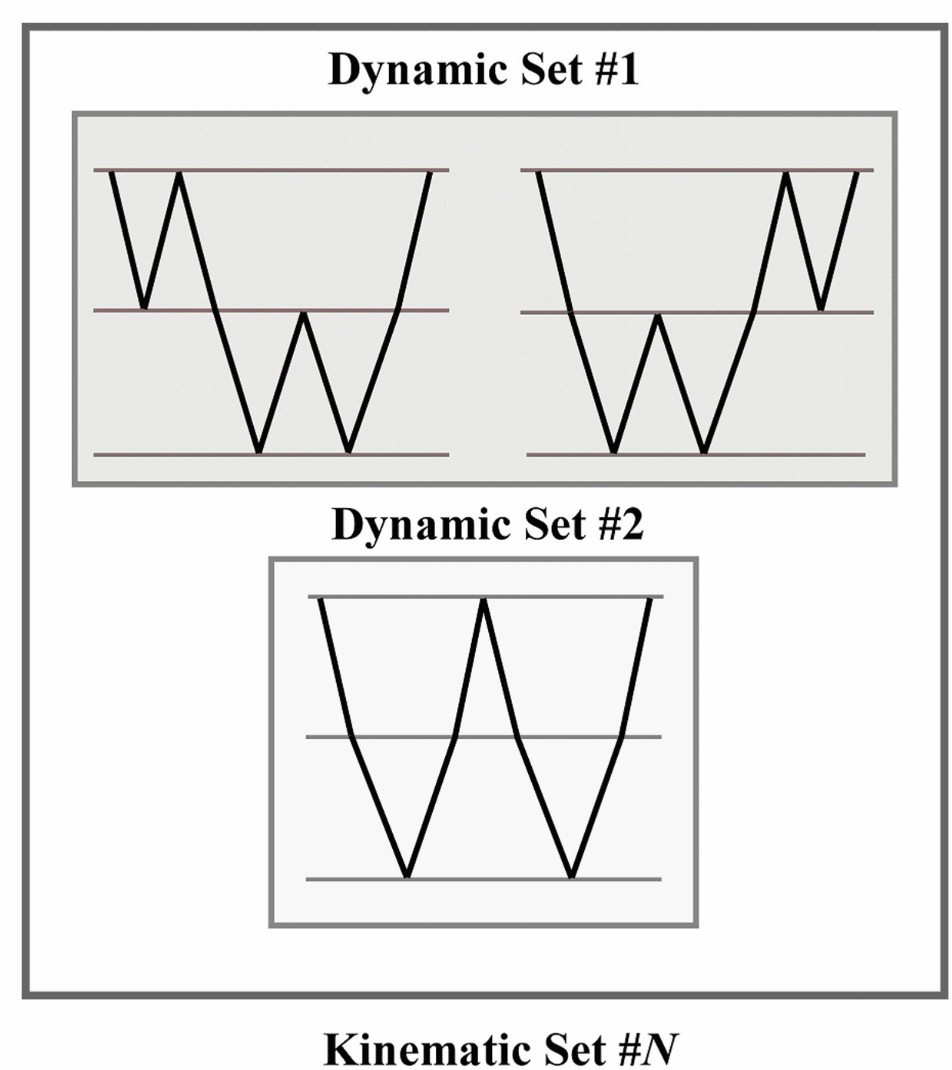

FIG. 1. The most basic kinematic ray set (with no mode conversions) with more than one dynamic set. The 2 rays in the first dynamic set have identical, generally complex, amplitudes. The third ray belongs to a second dynamic set of different amplitude. There are many similar quantities computed that are common to both dynamic sets, including geometrical spreading.

free surface, with both and only one P and S_V ray segment in each layer, may be formally given in terms of combinatorial theory:

Theorem: Given two distinct sets each of *n+1* objects, the number of ways in which they may be uniquely arranged, two at a time, where order is not of importance, using one of each of the two sets of objects and using all $2(n+1)$ objects, is 2^{n+1}.

What has been done here is to arbitrarily choose *n+1* objects, corresponding to the downward propagating ray, and use the condition that the type of ray segment in any layer in the upward propagating ray is opposite in type to the downward propagating ray segment. All $2(n+1)$ objects (ray segments) will be used in this construction.

The above is demonstrated schematically in Figure 2 for the first two layers of the PS_V $(S_V P)$ ray set. The instances of the first ray segment being S_V and the last segment being *P* are included in the figure.

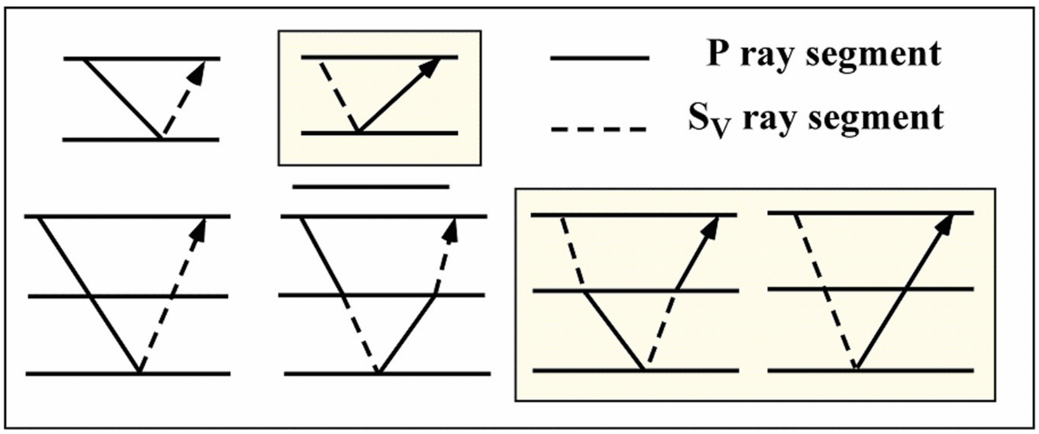

FIG. 2. All of the possible rays in a 2 layered medium with one *P* ray segment and one S_V segment per layer. The constraint that the source ray segment is of the *P* type and the receiver segment of the S_V type eliminates the rays in the shaded areas.

What is being considered here is the PS_V primary reflected from the $(n+1)^{th}$ interface. The constraint that the source ray segment be *P* and receiver ray segment S_V will be imposed. In this case the number of elements in the PS_V kinematic set may be shown using the above definition to be 2^n. One element of this number has *P* ray segments propagating from the source down to the $(n+1)^{th}$ interface and S_V ray segments up to the surface receiver. This ray has only one mode conversion and is the only member of this constrained set to have this attribute. The other $2^n - 1$ rays comprising this set are not usually considered in the construction of simple ray synthetics. As they have 3 or more mode conversions, resulting in small amplitudes relative to the first member of this set, an argument can be made to not include these in the construction of synthetic traces. However, when *n* is large, the other $2^n - 1$ rays should at least be given a cursory examination as collectively they may be of importance from a dynamic (amplitude) perspective. Define a ray within the kinematic group to be the PS_V

secondary ray. As in the PS_V primary case, it is composed of a P ray segment at the source and an S_V ray segment at the receiver, both of which are assumed to be located in layer 1. The difference is that S_V ray segments will propagate through layer 2 down to the $(n+1)^{th}$ interface and be reflected back to the interface between layer 2 and layer 1 as a sequence of P ray segments. This ray will have 3 mode conversions, making it much smaller in amplitude than the PS_V primary ray.

The number of all possible PS_V rays with one P and one S_V ray segment per layer propagating in an $(n+1)$ layered medium, including those that don't satisfy the constraint that the source segment is P and the receiver segment is S_V, is

$$m_{rays}^{(n+1)} = \sum_{j=0}^{n} 2^{j+1} . \tag{1}$$

Removing the $(n+1)$ once converted (primary) PS_V rays from this set leaves

$$\hat{m}_{rays}^{(n+1)} = \sum_{jj=0}^{n} 2^{j+1} - (n+1) \tag{2}$$

rays. As an example, for $(n+1)=5$ this results in

$$\hat{m}_{rays}^{(n+1)} = \hat{m}_{rays}^{(5)} = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 - (n+1) = 1+2+4+8+16-5 = 28 . \tag{3}$$

In contrast, the PP primary reflected from the $(j+1)^{th}$ interface which separates layer $(j+1)$ from layer $(j+2)$, the set of rays consisting of identical kinematic properties (travel times), consists of one element. In other words, in an $(n+1)$ layered medium there are only $[(n+1)=5]$ PP rays to be considered.

For a 5 layered model $[(n+1)=5]$ two rays of the PS_V kinematic set are shown in Figure 3 These rays are the PS_V primary and the previously defined PS_V secondary with the constraint imposed that the first and last ray segments be P and S_V, respectively. The model parameters used are defined in Table 1. As a consequence of Snell's Law, these two rays are the limiting rays for this kinematic set. That is, the conversion points on the 5^{th} interface of all other rays in this set, of which there are 14, must lie on this reflecting interface between points delineated by the above two limiting rays. The proof of this maybe obtained by switching a down going P ray segment with an up going S_V segment in any layer, say the j^{th}, $2 \geq j \geq n+1$. This has the effect of shifting the conversion point towards the source as a consequence of the now down going S_V ray segment having a velocity less than its associated P ray segment resulting in a decrease in the horizontal distance travelled by this ray segment in the j^{th} layer. The proof that the secondary ray is the other limit is obtained in a similar manner.

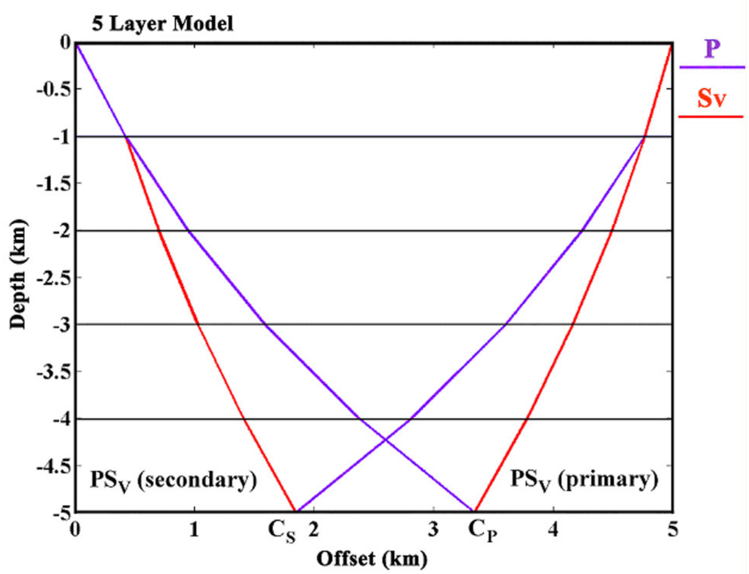

FIG. 3. The primary and secondary PS_V reflected arrivals in a 5 layer model. The corresponding conversion points are denoted as C_P and C_S, respectively. As there are 16 rays in this kinematic set, there are 14 more conversion points between the points C_S and C_P. The model used is defined in table 1.

RAY GENERATION

In this section a method for generating all the rays comprising the $(n+1)$ PS_V kinematic sets for all of the possible reflected PS_V sets from the $(n+1)$ interfaces in a plane layered model will be presented. This is accomplished by generating all rays reflected from the $(n+1)^{th}$ interface, and then taking subsets of this total set for the layers n to 1. Each subset is a subset of the preceding subset. An example is shown in Figure 4 for a 4 layered model. A binary tree is constructed in the manner shown. A *"1"* corresponds to a P ray segment and a *"0"* to an S_V segment. The tree is parsed in order of descending significant number of P segments, that is, the parsing is done in descending binary values and hence decimal value of the ray code. This gives the downward propagating portion of the total ray. The up going portion of the ray is just the 2's complement of the down going ray. For example if the down going portion is described by the string *1001*, the upward propagating ray is designated as *0110*. As this code does not take into account the layer in which each ray segment propagates, the following code is used to fully describe this ray:

$$\{+1,-2,-3,+4,-4,+3,+2,-1\}. \tag{4}$$

The integer indicates the layer of propagation and the sign the type of ray segment, "$+$" or "$-$" for P and S_V segment respectively.

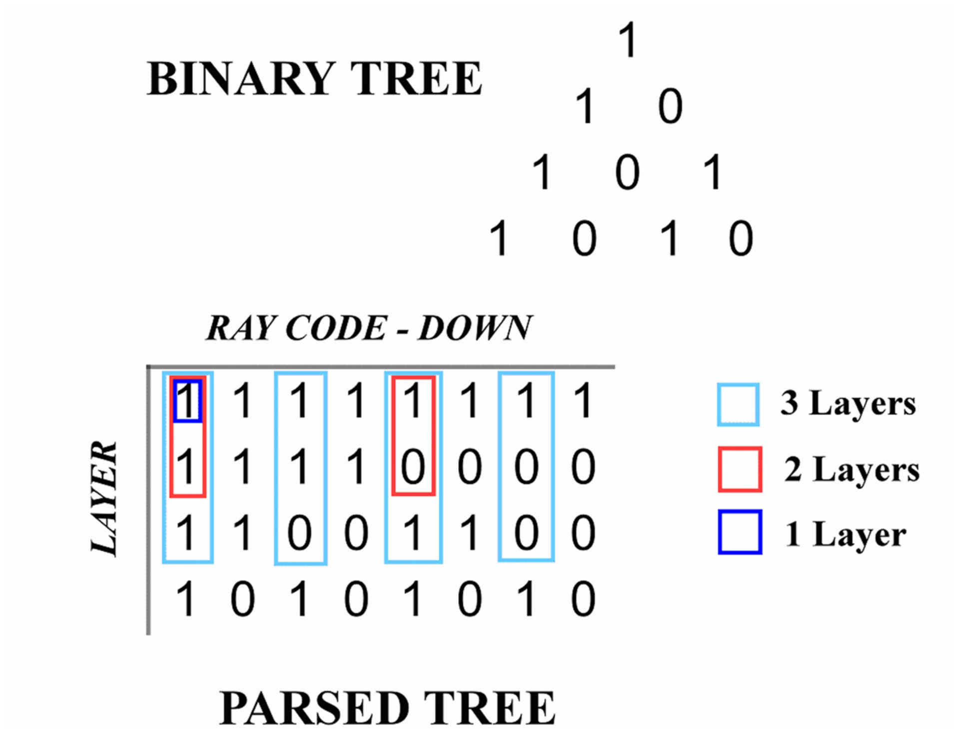

FIG. 4. An example of ray generation for a 4 layer model required in this report. The binary tree, if parsed in the proper manner, allows for the rays in all layers in the model to be obtained in an orderly fashion. It should be noted that the decimal values of the ordered binary ray code is $(15, 14, \ldots 9, 8)$ for the 4 layered model, $(7,6,5,4)$ for the 3 layers, $(3,2)$ for 2 layers and (1) for 1 layer.

As indicated in Figure 4, if the binary tree is parsed in the proper manner for rays reflected from the deepest, $(n+1)^{th}$ interface, the rays for all of the subsequent j layers $(n+1>j\geq 1)$ are just modulo 2 of the previous subset, with the binary character string truncated at length j.

As previously established, the PS_V primary has only one mode conversion, while the secondary ray undergoes 3 mode conversions. This would be an indication that its amplitude would be less than that of the PS_V primary. Other rays with 3 or more mode conversions, which make up this kinematic set, would be expected to behave in a similar manner.

As it has been stated that two rays with equal travel times between source and receiver will have diminished amplitudes with respect to one another depending on the number of mode conversions each has undergone, it is useful to partition a kinematic set into subsets according to the number of conversions along the ray. For a ray of the PS_V type being discussed here that propagates through *n+1* layers and is reflected from the $(n+1)^{th}$ interface, the maximum number of conversions it can undergo is $(2n+1)$. This count is independent of the constraints that the first and last ray segments are of type P and S_V, respectively. It is not difficult to ascertain that the number of rays in the subset with the maximum number of conversions is 1, if the constraints are imposed. As previously noted, the number of rays in the subset with one conversion (constraints in place) is also 1. It may also be observed that for this ray type, all rays within the kinematic set will have an odd number of conversions. Hence, the number of members of a kinematic set with constraints in place reflected from the $(n+1)^{th}$ interface may be written as

$$\hat{m}_{rays}^{(n+1)} = \left(C_1, C_3, C_5, \ldots, C_{2n+1}\right) \tag{5}$$

where it has been established that $C_1 = C_{2n+1} = 1$. What remains is the determination of the number of elements in the other subsets,

$$\left[C_j,\ j = 3, \ldots, 2n-1\right]. \tag{6}$$

It is not difficult to argue the point that the number of elements in the subset consisting of those rays belonging to the PS_V kinematic set that travel to the interface below the $(n+1)^{th}$ layer, and undergo 3 conversions, is *n*. This may be seen by considering the PS_V primary in lowest layer in which the ray propagates and switching the upgoing and downgoing ray segments in that layer. Leaving the rays switched, proceed to the next highest layer and switch the ray segments here. For an $(n+1)$ layered medium the number of possibilities for this is *n*. It is required that $n \geq 3$ and that the first and last ray segment type ray segment constraints are if effect. By the same reasoning that is used above the number of elements in the subset consisting of those rays that travel to the interface below the $(n+1)^{th}$ layer and undergo $(2n-1)$ conversions is also *n*. The starting point in this case is the ray with $(2n+1)$ conversions. The number of possible conversions in media types composed of 1, 2, and 3 layers may be easily determined. With the knowledge that the total number of rays in the 4^{th} layer is $2^4 = 16$, it is not difficult to fill in the blank in line 4 of Figure 5.a. It could also be noticed that this number could be obtained by summing the two integers spanning the unknown space in the line immediately above it.

If at this point the number of conversions subsets for the first three layers together with the partial knowledge obtained regarding the remaining layers $(n>3)$ in the $(n+1)$ layered structure regarding those ray subsets with 1, 3, $(2n-1)$ and $(2n+1)$ are written in triangular form, a pattern should be recognized (Figure 5). What in fact is being constructed is Pascal's triangle. The proof of this follows from observing that the elements in the $(2j-1)$ and $(2j+1)$ conversion partition subsets in the m^{th} layer are the only possible contributors to the $(2j+1)$ conversion partition subset in the $(m+1)^{th}$ layer, $3 \geq m \geq n+1$. The contributors from the $(2j+1)$ conversion partition subsets in the m^{th} layer to the $(2j+1)$ conversion partition subset in the $(m+1)^{th}$ layer is obtained by introducing a P segment and an S_V segment in the $(m+1)^{th}$ layer in such a manner that no conversions are added.

1
1 1
1 2 1
1 3 3 1 **(a).**
1 4 ? 4 1
1 5 ? ? 5 1
1 6 ? ? ? 6 1

1
1 1
1 2 1
1 3 3 1 **(b).**
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

FIG. 5. Partitioning of kinematic ray set in terms of the elements in a conversion subset. In (a) is shown what can be inferred from observation while filling in the blank entries as in (b) requires it be observed that (a) is very much like Pascal's triangle and in all probability, the entries would correspond. Minimal argument is required to show that this is so.

The addition of 2 conversions to the $(2j-1)$ conversion partition subset in the m^{th} layer is accomplished by adding two ray segments in the $(m+1)^{th}$ layer. These two segments are the opposite of those in the m^{th} layer. That is, if the ray segments in the m^{th} layer are a down going P segment and an up going S_V segment a down going S_V segment and an up going P segment are inserted in the $(m+1)^{th}$ layer, and vice versa.

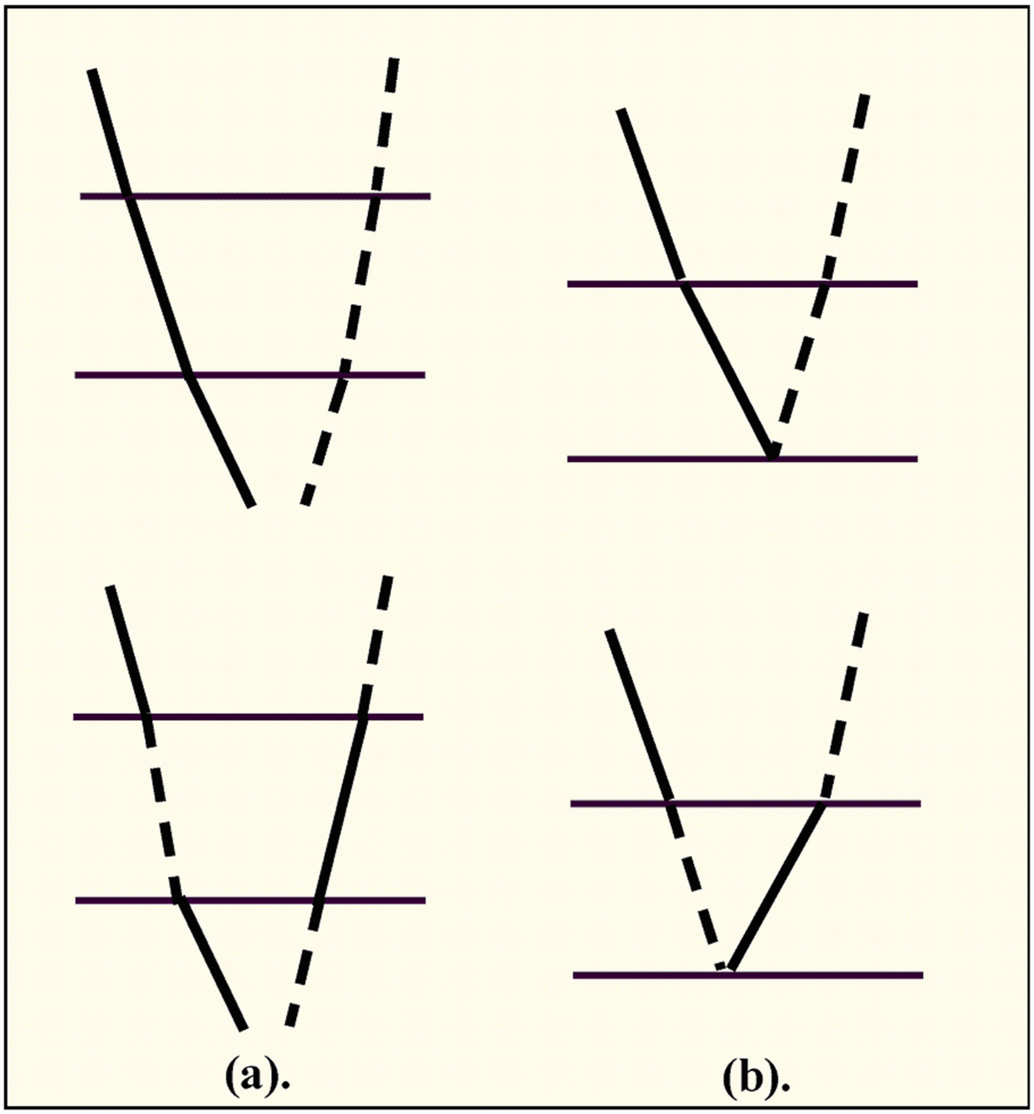

FIG. 6. Schematic of rays used in Figures 7 and 8.

As mentioned above, the first 3 rows of the triangle may be obtained by direct observation without much difficulty. For any other layer, the ray code reflected from that layer can be generated and then scanned in a fashion that allows for the partitioning of the specific kinematic set of rays into subsets corresponding to the number of conversions. This has been done up to a layer count that far exceeds anything that should be encountered in practice. The corresponding row of Pascal's triangle never fails to be the numerical output.

COMPARISON OF RAY AMPLITUDES

In order to ascertain the nature of the differences in amplitudes that may allow the replacement of the total PS_V kinematic set by the single PS_V primary ray. Two numerical experiments will be done, both involving modifying the PS_V primary and comparing the amplitudes with that of the reference ray. Phases will not be displayed, only amplitudes without surface conversion coefficients. The model used will be that described in Table 1. The first 10 layers will be used with the 11^{th} taken to be the half space.

The first comparison involves switching the P and S_V ray segments in the bottom $\left(10^{th}\right)$ layer in the reference ray. This has the effect of introducing 2 extra conversions, for a total of 3, into the amplitude computation. The third conversion in the reference ray case is the PS_V reflection at the interface between layer 10 and the half space. In the modified ray this third conversion is the $S_V P$ reflection at this same interface. This is shown schematically in Figure 6.b. The resulting amplitudes are presented in Figure 7.

The second comparison is done after switching the P and S_V ray segments in some layer, not the bottom reflecting layer, say the 4^{th} layer (Figure 6.a). The effect of this is to introduce 4 more conversions along the ray path from surface source to surface receiver. The other conversion in both the reference ray and the modified ray is the PS_V reflection from the top of the half space. Results of this numerical experiment, again displaying only amplitudes, are shown in Figure 8.

It is fairly clear that even the introduction of 2 extra conversions in a ray (plus one modified conversion) has significant effects on amplitude recorded at the surface. The more severe amplitude decline when 4 conversions are introduced provides credibility that the PS_V primary alone can adequately provide the reflected response of the whole kinematic set, no matter haw many layers are involved.

What has not been mentioned to this point is that in the pre-critical rays the phases of the individual rays within a kinematic set differ following a very particular design yielding equal numbers of rays with positive and negative amplitudes, having the effect of cancelling one another. More is said about this in the subsequent section.

NUMERICAL RESULTS

The model that will be used for demonstration purposes is a 16 layer model over a half space. It is specified in Table 1. Three separate cases of ray propagation will be considered; a 5 layered model, a 10 layer model and a 16 layer model. The same velocity model is used for all cases with the 6^{th} and 11^{th} layers being the half spaces for the first and second situations above. Zero order asymptotic ray theory is employed to compute the generally complex amplitudes.These are shown in Figures 9 through 11. There are two curves in each plot, one giving the amplitudes of each of the dynamic elements within a kinematic set at each offset. The moduli amplitudes plotted versus the offset range of 0 to 20 km for all three models sets being considered are $2^4 = 16$, $2^9 = 512$ and $2^{15} = 32768$, respectively. It is clear that the number of rays within a kinematic set increases dramatically as he integer denoting the reflecting interface increases.

Table 1. Elastic parameter definitions of the layers comprising the 16 layers over a half space model, the amplitude of the primary PS_V ray, and the sum of the amplitudes of all other rays comprising the kinematic set.

Layer	**P – Velocity**	**S – Velocity**	**Density**	**Thickness**
1	2.5	1.44	1.65	1.0
2	3.0	1.73	1.72	1.0
3	3.5	2.02	1.79	1.0
4	4.0	2.31	1.85	1.0
5	4.5	2.60	1.90	1.0
6	5.2	3.00	1.98	1.0
7	6.0	3.46	2.05	1.0
8	6.5	3.75	2.09	1.0
9	5.0	2.87	1.96	1.0
10	6.4	3.70	2.08	1.0
11	8.2	4.73	2.22	1.0
12	8.5	4.91	2.24	1.0
13	8.7	5.02	2.25	1.0
14	9.0	5.20	2.27	1.0
15	9.5	5.48	2.30	1.0
16	9.7	5.60	2.31	1.0
H – Space	9.8	5.66	2.32	∞

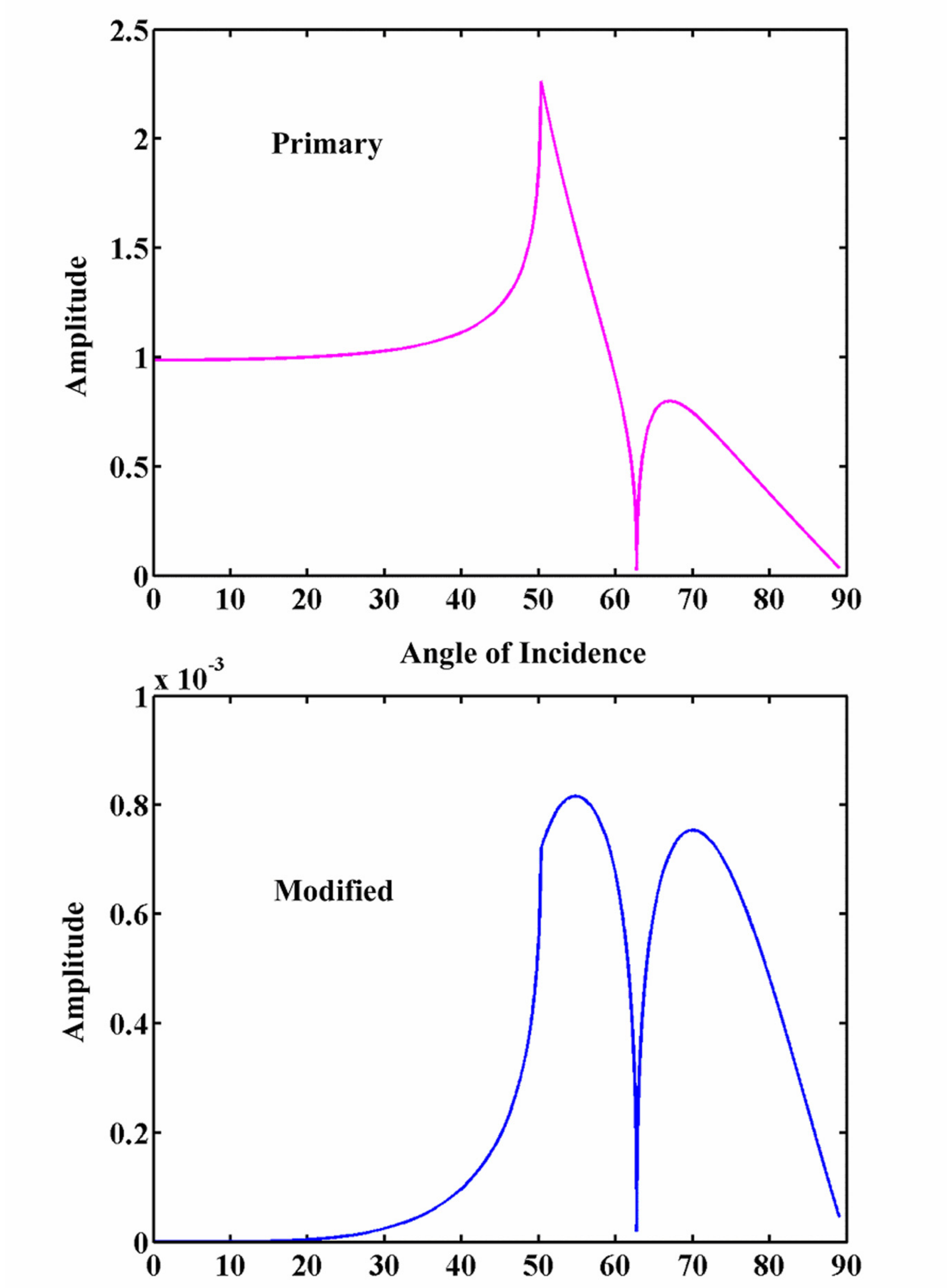

FIG. 7. Amplitude comparison #1.

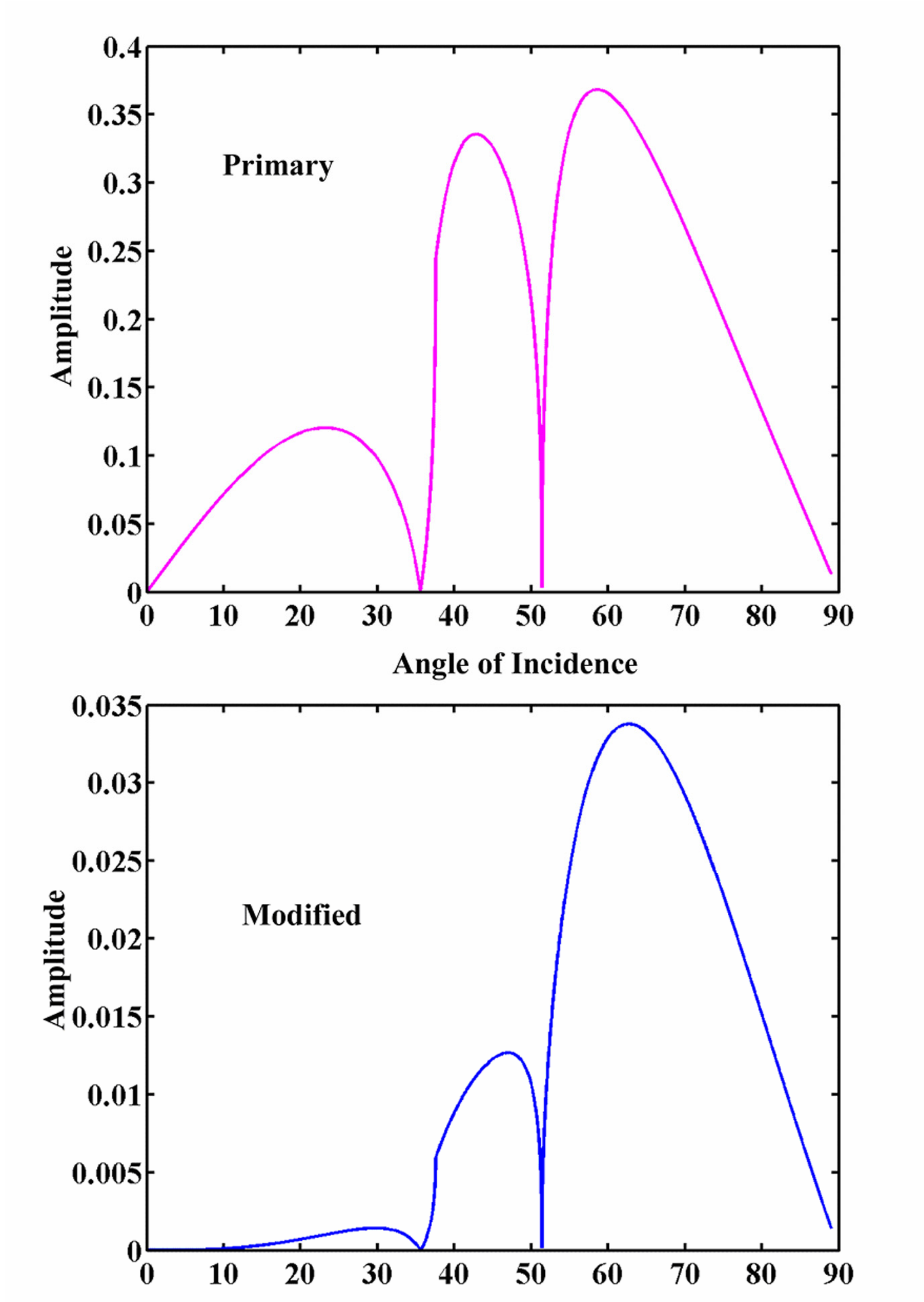

FIG. 8. Amplitude comparison #2.

What appears problematic upon viewing Figures 9 to 11 is that the PS_V primary dominates the total amplitude of the total kinematic set. As a numerical example, for the 16 layer model, the amplitude of the PS_V primary at the sub-critical offset $r = 5.0km$ is

$3.54289515 \times 10^{-3}$ while the sum of the amplitudes of those rays in the kinematic set whose amplitude is negative is $-9.07682651 \times 10^{-8}$ and those whose amplitude is positive, $2.63100546 \times 10^{-5}$. At $r = 10.0km$ these values are, $-5.04518354 \times 10^{-3}$, $1.39309256 \times 10^{-4}$ $-1.78632709 \times 10^{-6}$ and, at $r = 15.0km$ they are $-3.667927074 \times 10^{-3}$, $-5.04407170 \times 10^{-6}$ and $1.99664150 \times 10^{-4}$. This last case is the only instance where the sum of the amplitudes of the *other* rays even approaches a value of consequence when compared to the PS_V primary, to the order of about 7%. Figures 9 to 11 all show this pre-critical behaviour.

CONCLUSIONS

A discussion of the kinematic set of rays consisting of one P ray segment and one S_V ray segment per layer with the constraint that the first ray segment be of the P type and the last ray segment being of type S_V. The primary PS_V ray, which consists of P rays segments down to a reflecting interface and S_V ray segments back up to the surface, considered in the literature is contained within this set. It is not often mentioned that there are other rays associated with this specific ray that have identical kinematic properties (travel times). The reason for this can be surmised to be because for a small number of layers the amplitudes of these other members of a kinematic set are negligible when compared to the primary arrival. Using a simple numerical experiment this assumption has apparently been shown to be valid no matter what the value the cardinal number of the reflecting interface. More study would be indicated.

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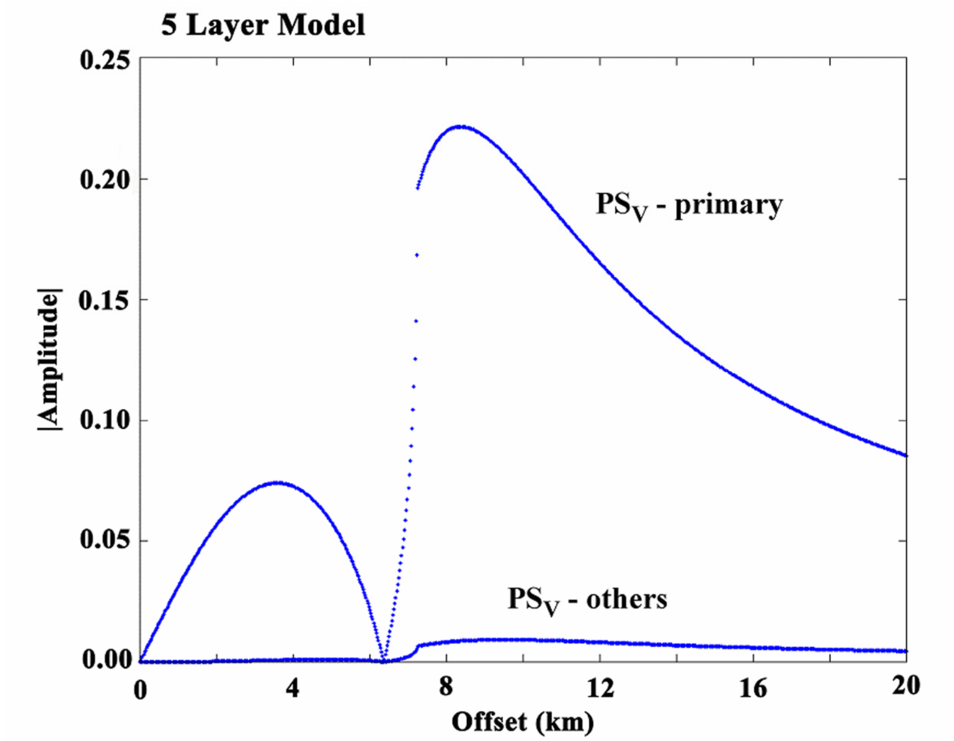

FIG. 9. Amplitude – distance curves for the PS_V primary and the sum of the other 15 members of kinematic set. Velocity – depth model is given in Table 1.

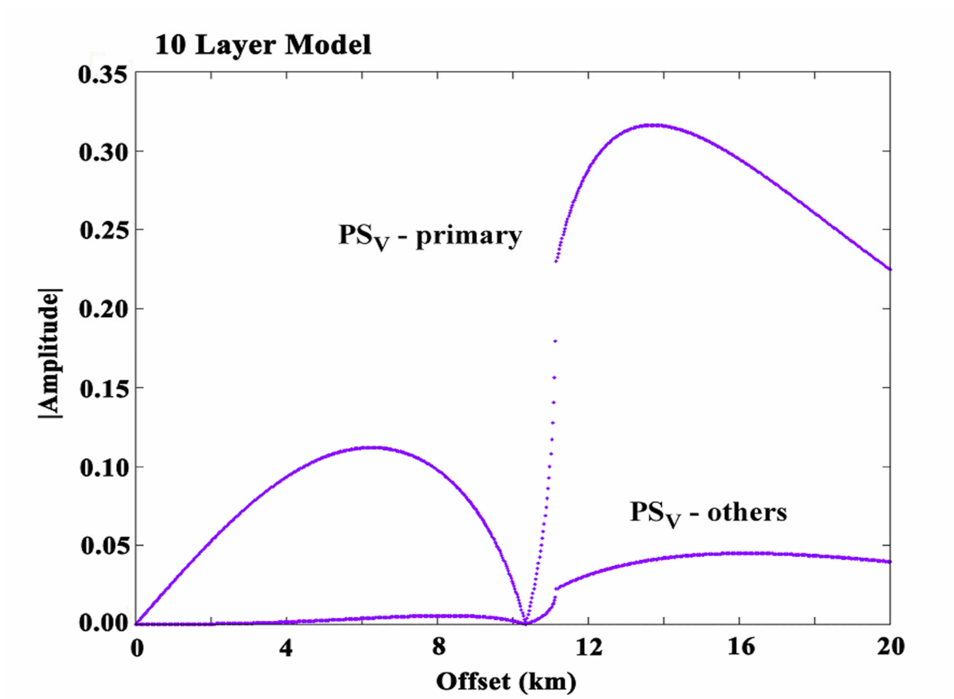

FIG. 10. Amplitude – distance curves for the PS_V primary and the sum of the other 511 members of kinematic set. Velocity – depth model is given in Table 1.

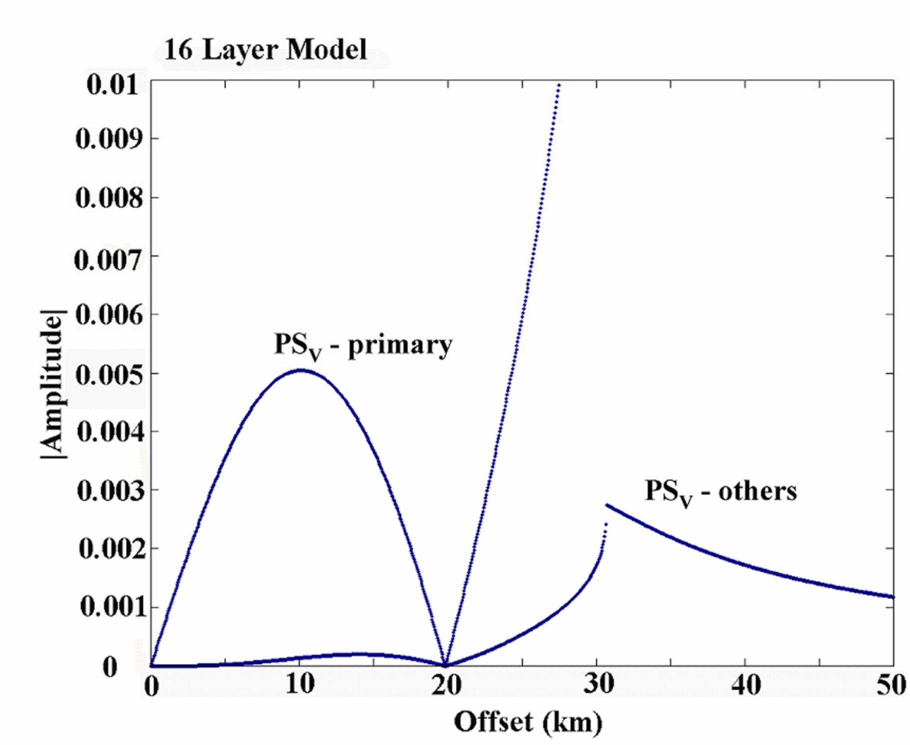

FIG. 11. Amplitude – distance curves for the PS_V primary and the sum of the other 16383 members of kinematic set. Velocity – depth model is given in Table 1.