Interconversion formulae for two-parameter AVO methods

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ABSTRACT

Most AVO studies use two-parameter inversion methods. These each yield a pair of estimated paramters, such as P-wave and S-wave velocity reflectivities, or intercept and gradient quantities. These methods are all approximations to a more general three-parameter method based on the Aki-Richards approximation. In addition to the pairs of parameters generated above, three-parameter inversion also yields a quantity related to density, but this inversion is often plagued by numerical instability.

For both theoretical and practical reasons, it is of value to understand the relationships between the various two-parameter methods. To this end, formal expressions are derived for the inversion errors of each method. Using these expressions, interconversion formulae are obtained, which allow one to convert the results of any two-parameter method to those of any other two-parameter method. The analysis demonstrates that the only requirement is that the maximum angle of incidence be at least a few degrees below any critical angle. The error expressions obtained also suggest the formulation of additional AVO tools that may be of use to industry.

INTRODUCTION

The goal in AVO inversion is to determine earth-property contrasts across an interface from the angle-dependence of seismic amplitudes. The starting point is $R_{\mathrm{PP}}(\theta_1)$, where R_{PP} is the P-wave reflection coefficient determined from seismic amplitudes and θ_1 is the angle of incidence at the interface. The objective is a set of relative contrasts of the form $\Delta x/x$, where $\Delta x = x_2 - x_1$ is the difference of property x across the interface, and $x = (x_1 + x_2)/2$ is its average. These can also be expressed as reflectivities, R_x, as shown in Table 1.

Table 1. Symbol definitions.

α_i	P-wave velocity of i^{th} layer
β_i	S-wave velocity of i^{th} layer
ρ_i	density of i^{th} layer
I_i	P-wave impedance $(= \rho\alpha)$ of i^{th} layer
J_i	S-wave impedance $(= \rho\beta)$ of i^{th} layer
μ_i	shear modulus $(= \rho\beta^2)$ of i^{th} layer
subscript 1	layer above interface
subscript 2	layer below interface
$\Delta x,\ x = \alpha, \beta, \rho, I, J$	$x_2 - x_1$
$x,\ x = \alpha, \beta, \rho, I, J$	$\frac{x_1+x_2}{2}$
$R_x,\ x = \alpha, \beta, \rho, I, J$	$\frac{1}{2}\frac{\Delta x}{x}$
γ	$(\beta_1 + \beta_2)/(\alpha_1 + \alpha_2)$
σ	$(\frac{1}{2} - \gamma^2)/(1 - \gamma^2)$
NI	R_I
PR = $\Delta\sigma/(1-\sigma)^2$	$8\gamma^2(R_\alpha - R_\beta)$
θ_1	angle of incidence and P-wave reflection
θ_2	angle P-wave transmission
θ	$(\theta_1 + \theta_2)/2$ $= \theta_1 + \sin^{-1}(\frac{\alpha_2}{\alpha_1}\sin\theta_1)$

The Aki-Richards approximation (Aki and Richards, 1980),

$$R_{\text{PP}}^{\text{A-R}}(\theta_1) = \frac{R_\alpha}{\cos^2\theta} - 8\gamma^2 \sin^2\theta R_\beta - (4\gamma^2\sin^2\theta - 1)R_\rho, \tag{1}$$

is a linearization of the Zoeppritz equations in the three parameters R_α, R_β, and R_ρ. Here θ is the average of incidence and P-wave transmission angles across the interface. Table 1 shows this to be a function of the incidence angle, θ_1, so we can still write $R_{\text{PP}}^{\text{A-R}}$ as function of θ_1, even the right hand side of equation 2 is expressed in terms of θ. Note that equation 1 implies a modeling perspective, in which earth-property reflectivities are known, and the angle-dependent reflection coefficient is approximated. For inversion the reverse is true, and one ought to write

$$R_{\text{PP}}(\theta_1) = \frac{R_\alpha^{\text{A-R}}}{\cos^2\theta} - 8\gamma^2 \sin^2\theta R_\beta^{\text{A-R}} - (4\gamma^2\sin^2\theta - 1)R_\rho^{\text{A-R}}. \tag{2}$$

The Aki-Richards approximation has been the starting point for most AVO inversion work. While the Zoeppritz equations give exact coefficients for idealized transmission, reflection, and conversion events, their complicated structure necessitates the use of non-linear inversion techniques to extract R_α, R_β, and R_ρ (MacDonald et al., 1987; Russell, 1988). By contrast, inversion with the Aki-Richards approximation is a one-step process, involving the least-squares solution of a set of linear equations.

In reality of course one requires some background parameters as input, even for a linear inversion. For instance, one requires an estimate of R_α, for use in raytracing to obtain θ_1, and an estimate of γ. These are required to set up the coefficients in the Aki-Richards equation.

In practical inversions, the three-parameter Aki-Richards approximation is itself usually set aside in favor of two-parameter approximations, which are the subject of this study. The best-known of these are the two-term Shuey expression (Shuey, 1985), which fits reflectivity curves to the expression

$$R_{\text{PP}}(\theta_1) = A + B\sin^2\theta \tag{3}$$

to generate intercept (A) and gradient (B) information; the Smith-Gidlow approximation (Smith and Gidlow, 1987),

$$R_{\text{PP}}(\theta_1) = \left(\frac{1}{\cos^2\theta} - \gamma^2\sin^2\theta + \frac{1}{4}\right) R_\alpha^{\text{S-G}} - 8\gamma^2\sin^2\theta R_\beta^{\text{S-G}}, \tag{4}$$

in which a differential form of Gardner's relation (Gardner et al., 1985) is used to replace R_ρ with R_α, and which generates estimates of R_α and R_β; the two-term approximation of Fatti et al. (1994),

$$R_{\text{PP}}(\theta_1) = \frac{R_I^{\text{Fatti}}}{\cos^2\theta} - 8\gamma^2\sin^2\theta R_J^{\text{Fatti}}, \tag{5}$$

in which the contribution of R_ρ is assumed to be negligible in comparison to that of the estimated parameters, R_I and R_J; and the method of Verm and Hilterman (1995),

$$R_{\text{PP}}(\theta_1) = \text{NI}^{\text{VH}}\cos^2\theta + \text{PR}^{\text{VH}}\sin^2\theta, \tag{6}$$

which predicts the Normal Incidence (NI) and Poisson Reflectivity (PR); and the method described by Goodway (2001) [see his equation (c) and discussion following] in which the Aki-Richards approximation is expressed in terms of ρ, α, and μ and the constant term $(\Delta\rho/2\rho)$ is dropped, yielding an approximation exact in the angular dependence:

$$R_{\text{PP}}(\theta_1) = (1 + \tan^2\theta) R_\alpha^{(\rho,\alpha,\mu)} - 4\gamma^2\sin^2\theta R_\mu^{(\rho,\alpha,\mu)}. \tag{7}$$

These five methods predict estimates for different quantities, yet we hypothesize that the results should in some manner be consistent with each other, being based on the same information. The purpose of this research is to develop the theory in a way that exposes the commonalities of all two-parameter methods, with the expectation that this will have practical implications for AVO.

The first step is to obtain analytic expressions for the inversion error of the common two-parameter methods; in other words, to show what quantities are truly being estimated by these methods. Once this goal has been achieved, it can then be shown that the results of all two-parameter methods contain equivalent information, in the sense that the results of one can be converted into the results of another. The perspective of this work also sheds light on a less known AVO approximation which combines the strengths of the methods of both Smith and Gidlow (1987) and of Fatti et al. (1994). This newer method should be of value in models featuring a large value of R_ρ. This work also shows a simple way to incorporate local calibration data into AVO results. A number of calculations are carried out which substantiate the above theoretical claims.

THEORY

What is actually calculated in two-parameter AVO?

To answer this question, let us assume that the exact reflectivities are given by inversion of the Aki-Richards approximation, equation 2, so that $R_\alpha^{\text{A-R}} = R_\alpha$, $R_\beta^{\text{A-R}} = R_\beta$, and $R_\rho^{\text{A-R}} = R_\rho$. Although this neglects nonlinear contributions, it will allow us to answer the question to linear order. We will consider a simple inversion with only two data points, at $\theta = 0$ and $\theta = \theta_{\max}$, as this can be carried out analytically. For the method of Smith and Gidlow (1987) this would mean equating the full three-term Aki-Richards expression (equation 2) to the Smith-Gidlow approximation (equation 4) for the two given values of θ, which then yields two equations:

$$R_\alpha + R_\rho = (5/4)R_\alpha^{\text{S-G}},$$

$$\frac{R_\alpha}{\cos^2\theta_{\max}} - 8\gamma^2\sin^2\theta_{\max}R_\beta + \left(1 - 4\gamma^2\sin^2\theta_{\max}\right)R_\rho = \left[\frac{1}{\cos^2\theta_{\max}} + \frac{1-4\gamma^2\sin^2\theta_{\max}}{4}\right]R_\alpha^{\text{S-G}} - 8\gamma^2\sin^2\theta_{\max}R_\beta^{\text{S-G}}.$$

Solving these for $R_\alpha^{\text{S-G}}$ and $R_\beta^{\text{S-G}}$ yields

$$R_\alpha^{\text{S-G}} = (4/5)(R_\alpha + R_\rho) = R_\alpha - (R_\alpha - 4R_\rho)/5, \tag{8}$$

$$R_\beta^{\text{S-G}} = R_\beta - \frac{1}{40}\left(4 + \frac{1}{\gamma^2\cos^2\theta_{\max}}\right)(R_\alpha - 4R_\rho). \tag{9}$$

Note that if the differential Gardner relation $(R_\alpha = 4R_\rho)$ is satisfied then the Smith-Gidlow results are exact (to linear order).

A similar exercise can be carried out for the rest of equations 3 through 7. From equations 2 and 5 we obtain the result

$$R_I^{\text{Fatti}} = R_I, \tag{10}$$

$$R_J^{\text{Fatti}} = R_J - \frac{1}{8}\left(4 - \frac{1}{\gamma^2\cos^2\theta_{\max}}\right)R_\rho. \tag{11}$$

This shows that the two-term Fatti approximation is exact if $R_\rho = 0$.

From equations 2 and 6 we find

$$\text{NI}^{\text{VH}} = \text{NI}, \tag{12}$$

$$\text{PR}^{\text{VH}} = \text{PR} + (1 - 4\gamma^2)R_I + \left(\frac{1}{\cos^2\theta_{\max}} - 4\gamma^2\right)R_\alpha. \tag{13}$$

These results are exact, for instance, if $\gamma = 1/2$ and $\theta_{\max} = 0$, or if $R_I = R_\alpha = 0$.

From equations 2 and 7 we find

$$R_\alpha^{(\rho,\alpha,\mu)} = R_\alpha + R_\rho, \tag{14}$$

$$R_\mu^{(\rho,\alpha,\mu)} = R_\mu + \frac{R_\rho}{4\gamma^2 \cos^2\theta}. \tag{15}$$

These results are exact for $R_\rho = 0$. What is more interesting, with a little manipulation it can be shown that these results are actually identical to those of the two-term Fatti approximation in equations 10 and 11 (aside from a factor of 2 in the R_μ result). Thus, although they have been derived in different ways, the fact that both methods employ the $R_\rho = 0$ approximation compels the final results to coincide.

Finally, from equations 2 and 3 we obtain the results

$$A = R_I, \tag{16}$$

$$B = \frac{R_I}{\cos^2\theta_{\max}} - 8\gamma^2 R_J + \left(4\gamma^2 - \frac{1}{\cos^2\theta_{\max}}\right) R_\rho. \tag{17}$$

Comparing this to the "exact" expression for B (to linear order) as given, for instance, in equation 1b of Ramos and Castagna (2001),

$$B^{\text{exact}} = R_\alpha - 4\gamma^2(2R_\beta + R\rho), \tag{18}$$

then equation 17 can also be written as

$$B = B^{\text{exact}} + \tan^2\theta_{\max} R_\alpha. \tag{19}$$

This shows how B in the two-term approximation becomes increasingly contaminated by the higher-order coefficient, $C^{\text{exact}} = R_\alpha$ (Shuey, 1985), as $\theta_{\max}$ increases.

Equations 8 through 17 show precisely what is being calculated by the Smith-Gidlow, Shuey, (ρ,α,μ), Verm-Hilterman, and Fatti approximations, at least for our simple two-point inversion. Calculations below will shed further light on their range of validity.

Interconversion formulae

With equations 8–17 we can also readily show that

$$R_I = A = NI = R_I^{\text{Fatti}} = \frac{5}{4} R_\alpha^{\text{S-G}} = R_\alpha^{(\rho,\alpha,\mu)}, \tag{20}$$

$$R_J^{\text{Fatti}} - R_\beta^{\text{S-G}} = \frac{1}{10}\left(1 + \frac{1}{4\gamma^2\cos^2\theta_{\max}}\right) R_I, \tag{21}$$

$$R_J^{\text{Fatti}} + B/(8\gamma^2) = \frac{R_I}{8\gamma^2\cos^2\theta_{\max}}, \tag{22}$$

$$R_\beta^{\text{S-G}} + B/(8\gamma^2) = \frac{1 - \gamma^2\cos^2\theta_{\max}}{10\gamma^2\cos^2\theta_{\max}} R_I, \tag{23}$$

$$R_\mu^{(\rho,\alpha,\mu)} - 2R_J^{\text{Fatti}} = 0, \tag{24}$$

$$\text{PR}^{\text{VH}} - B = R_I. \tag{25}$$

Thus if the values of γ and $\theta_{\max}$ used for the AVO inversion are known or can be estimated, these simple results predict that one can freely interconvert Shuey, Smith-Gidlow, (ρ, α, μ), Verm-Hilterman and Fatti inversion results.

Additional AVO tools

From equation 9 we see that the error term in $R_\beta^{\text{S-G}}$ is diminished by cancellation in the factor $4R_\rho - R_\alpha$ to the extent that the Gardner relation is satisfied. Similarly, from equation 11 we see that the error term in R_J^{Fatti} is diminished by partial cancellation in the factor $4 - \frac{1}{\gamma^2 \cos^2 \theta_{\text{max}}}$. Could we create a two-parameter method that has cancellation in both the angle and reflectivity factors? The method of Fatti et al. (1994) could be modified by replacing R_ρ with $R_I/5$ rather than setting R_ρ equal to zero (Larsen, 1999; Ursenbach and Stewart, 2001). We call this a "large R_ρ" approximation, as its usefulness should be most apparent when R_ρ is large. Analogous to equation 5 it is expressed as

$$R_{\text{pp}}(\theta_1) = \left(1 + \frac{4\gamma^2 \cos^2 \theta - 1}{5} \sin^2 \theta\right) \frac{R_I^{\text{L-}\rho}}{\cos^2 \theta} - 8\gamma^2 \sin^2 \theta R_J^{\text{L-}\rho}, \tag{26}$$

which leads to the result

$$R_J^{\text{L-}\rho} = R_J - \frac{1}{2}\left(1 - \frac{1}{4\gamma^2 \cos^2 \theta_{\text{max}}}\right)\left(R_\rho - \frac{R_I}{5}\right). \tag{27}$$

in analogy to equation 11. This result combines the strengths of both the Smith-Gidlow and Fatti approximations to minimize the R_ρ error term. The accurate estimation of R_J is uniquely desirable as it is used, for instance, in lambda-mu-rho analysis (Goodway et al., 1997)

A further improvement to the large-R_ρ method, and to the Smith-Gidlow method, can be made if the Gardner rule is locally calibrated, or specialized to a particular lithology (Wang, 2000; Ursenbach, 2002). For instance, suppose that well-log data is fit to a relation of the form $\rho = A\alpha^{1/g}$ (where $g = 4$ for the traditional Gardner relation). Employing this general Gardner relation would lead to the following generalizations of equations 8, 9 and 27:

$$R_\alpha^{\text{S-G,g}} = \frac{g}{g+1} R_I = R_\alpha - \frac{R_\alpha - gR_\rho}{g+1}, \tag{28}$$

$$R_\beta^{\text{S-G,g}} = R_\beta + \frac{1}{2(g+1)}\left(1 + \frac{1}{4\gamma^2 \cos^2 \theta_{\text{max}}}\right)(gR_\rho - R_\alpha), \tag{29}$$

$$R_J^{\text{L-}\rho\text{,g}} = R_J - \frac{1}{2}\left(1 - \frac{1}{4\gamma^2 \cos^2 \theta_{\text{max}}}\right)\left(R_\rho - \frac{R_I}{g+1}\right). \tag{30}$$

Combining these results with equations 8, 9 and 27 yields formulae which allow one to convert Smith-Gidlow or large-R_ρ results into the results that would have been obtained using a value other than $g = 4$ in Gardner's relation:

$$R_\alpha^{\text{S-G,g}} = \frac{5}{4}\frac{g}{g+1} R_\alpha^{\text{S-G}}, \tag{31}$$

$$R_\beta^{\text{S-G,g}} = R_\beta^{\text{S-G}} - \frac{1}{8}\left(1 + \frac{1}{4\gamma^2 \cos^2 \theta_{\text{max}}}\right)\frac{4-g}{1+g} R_\alpha^{\text{S-G}}, \tag{32}$$

$$R_J^{\text{L-}\rho\text{,g}} = R_J^{\text{L-}\rho} + \frac{1}{10}\left(1 - \frac{1}{4\gamma^2 \cos^2 \theta_{\text{max}}}\right)\frac{4-g}{1+g} R_I^{\text{L-}\rho}. \tag{33}$$

RESULTS

Test of interconvertibility

The interconvertibility decribed above can be effectively demonstrated using a well-known dataset due to Castagna and Smith (1994) which has been used in a number of studies (Castagna and Swan, 1997; Castagna et al., 1998; Smith and Sutherland, 1996; Smith and Gidlow, 2000; Ramos and Castagna, 2001). It consists of elastic parameters for 25 sets of co-existing brine-sand, gas-sand, and shale. The parameters are derived from well-log data and core measurements. Each set gives rise to five possible interfaces (shale-over-gas, shale-over-brine, gas-over-brine, brine-over-shale, and gas-over-shale), and thus the dataset describes 125 possible interfaces. Figure 1 displays Zoeppritz coefficient curves for 22 of the shale-over-brine interfaces and 22 of the shale-over-gas interfaces. [Three sets of data (#3, #4 and #6) contain unphysical gas sand parameters (W. Goodway, personal communication, October, 2004) and are excluded.] It is clear that this dataset contains a sampling of all AVO classes. Using curves such as those in Figure 1, AVO inversions have been carried out on synthetic datasets for all 110 possible interfaces. Each of the 110 synthetic datasets consists of reflection coefficients for 31 angles of incidence, $\theta_1 = 0^\circ, 1^\circ, 2^\circ, \ldots, 30^\circ$. In some cases random noise has been added to the data.

Of course the transformation relationships in equations 20–25, 31–33 are strictly true only when one is performing an exact inversion with only two offsets. Normally one uses many offsets from noisy data and applies least-squares techniques. How do such relationships hold up then? Figure 2 displays results from Fatti inversions for 110 interfaces, and compares these to results from Smith-Gidlow inversions that have been transformed to Fatti results using equations 20 and 21. Although the interconversion formulae were derived for only two points, each of these inversions is based on 31 synthetic data points with approximately a 5:1 signal to noise ratio (SNR). The transformation relationships are strongly verified. The only visible deviation occurs for interfaces with the largest values of $|R_\alpha|$ (note the horizontal scale). The explanation for this is that as $|R_\alpha|$ increases, the shape of the reflectivity curve below $\theta_{i,\max}$ becomes less parabolic; the relevant portion of the reflection coefficient curve is then less able to be modeled by two parameters. This is particularly true for positive R_α, for which $\theta_{i,\max}$ (which is chosen to be 30° in all cases) is approaching a critical angle. Additional tests (not shown) demonstrate that adding noise to the background values of R_α (used in calculating $\theta_{\max}$) and to γ, and both random and systematic error to the angles, $\{\theta_1\}$, does not degrade the correctness of the transformation, as long as the same noisy values are used in both the original inversion and in the transformation. Similar results are also obtained when testing interconversions involving the Shuey and Verm-Hilterman theories (equations 22–25). This does not mean that the inversion results themselves are accurate, only that the different methods can represent each other accurately. Therefore the principal condition for transforming results between different two-parameter methods is that $\theta_{i,\max}$ be a sufficient distance below the critical angle, apparently at least a few degrees.

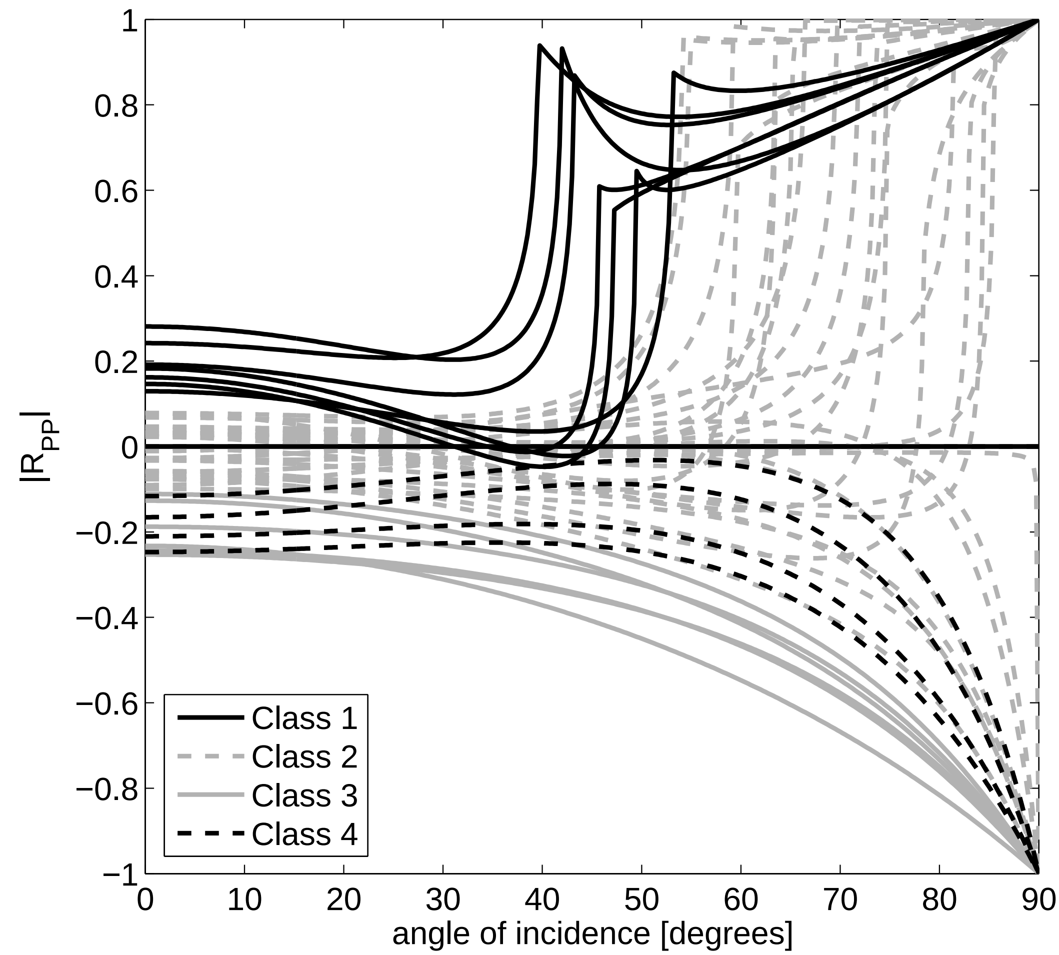

FIG. 1. 44 reflection coefficient curves. These contain examples from Class 1, 2, 3 and 4 systems, as indicated by the four line styles. (The detailed assignment to various classes is arbitrary, but is reasonable overall.) 66 other curves may also be generated for other interface lithologies, for a total of 110 curves used for the inversions in this study.

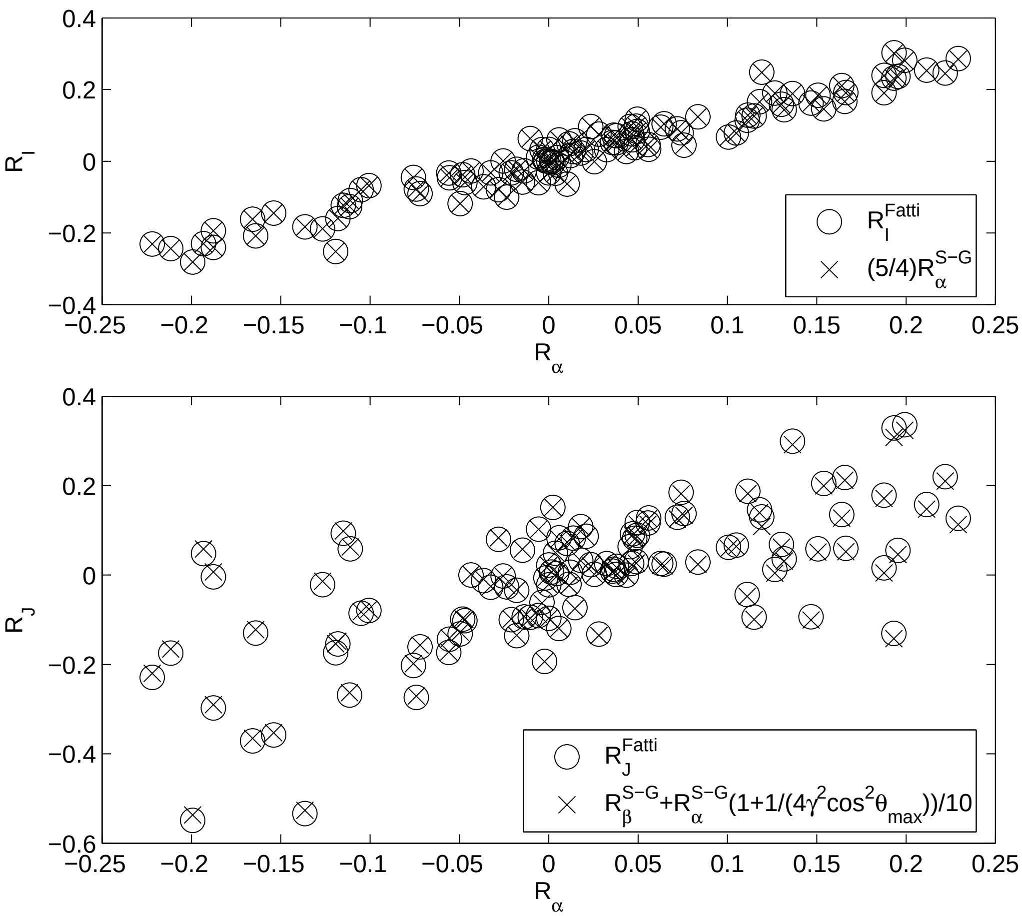

FIG. 2. Comparison of impedance reflectivities predicted by the method of Fatti et al. with those obtained by transformation of Smith-Gidlow inversion results. Note that the predictions are not claimed to be close to the exact values. It is simply shown that differing two-parameter methods yield interconnected results. The largest deviations occur with large values of $|R_\alpha|$, particularly as $\theta_{i,\max}$ approaches a critical angle.

Test of large-R_ρ method

As a simple test of the efficacy of equation 26, we perform inversions of the same synthetic data as was employed in the generation of Figure 2. In Figure 3 we compare the results for $R_J^{\text{L-}\rho}$ with those predicted by the method of Fatti et al. In this case we have plotted against the value of R_ρ, and we see that the largest discrepancies are for the extreme values of the horizontal coordinate. Next in Figure 4 we plot the *error* of the results from Figure 3 by subtracting off the exact value of R_J. Now it is clear that, in the cases featuring discrepancy between the two methods, the large-R_ρ method shows improvement over the method of Fatti et al. This is to be expected, as the Large-R_ρ method aims for more accurate treatment of the density term, which will only be significant for large R_ρ values. Note that in the spirit of interconvertibility, it is of course not necessary to actually carry out a large-R_ρ inversion; such results can be obtained from legacy inversion results employing formulae derived straightforwardly using the methods of this paper. For instance, it may be readily verified that

$$R_J^{\text{L-}\rho} = R_\beta^{\text{S-G}} + (1/4)R_\alpha^{\text{S-G}}. \tag{34}$$

Thus Smith-Gidlow inversion results (Smith and Gidlow, 1987) can be used to give a better estimate of R_J than is obtained from a Fatti inversion (Fatti et al., 1994).

CONCLUSIONS

The principal conclusion of this work is that all two-parameter AVO inversion methods yield results with equivalent information content. This does not mean that they give the same results, but rather that their results may be interconverted using derivable formulae. This conclusion does begin to break down at large offsets, particularly if one approaches a critical angle. For many practical cases though such conditions are not met, and interconversion promises to provide an efficient approach to multiple perspectives for interpretation, both on new datasets, and on legacy AVO inversions.

This work further shows that the classic methods of Fatti et al. (1994) and Smith and Gidlow (1987) can be combined to yield a more accurate estimate of shear-wave impedance reflectivity, particularly for cases where there is a relatively large density change across an interface.

Finally, a simple method is given for modifying legacy inversion results to account for local calibration of the Gardner relation (Gardner et al., 1985).

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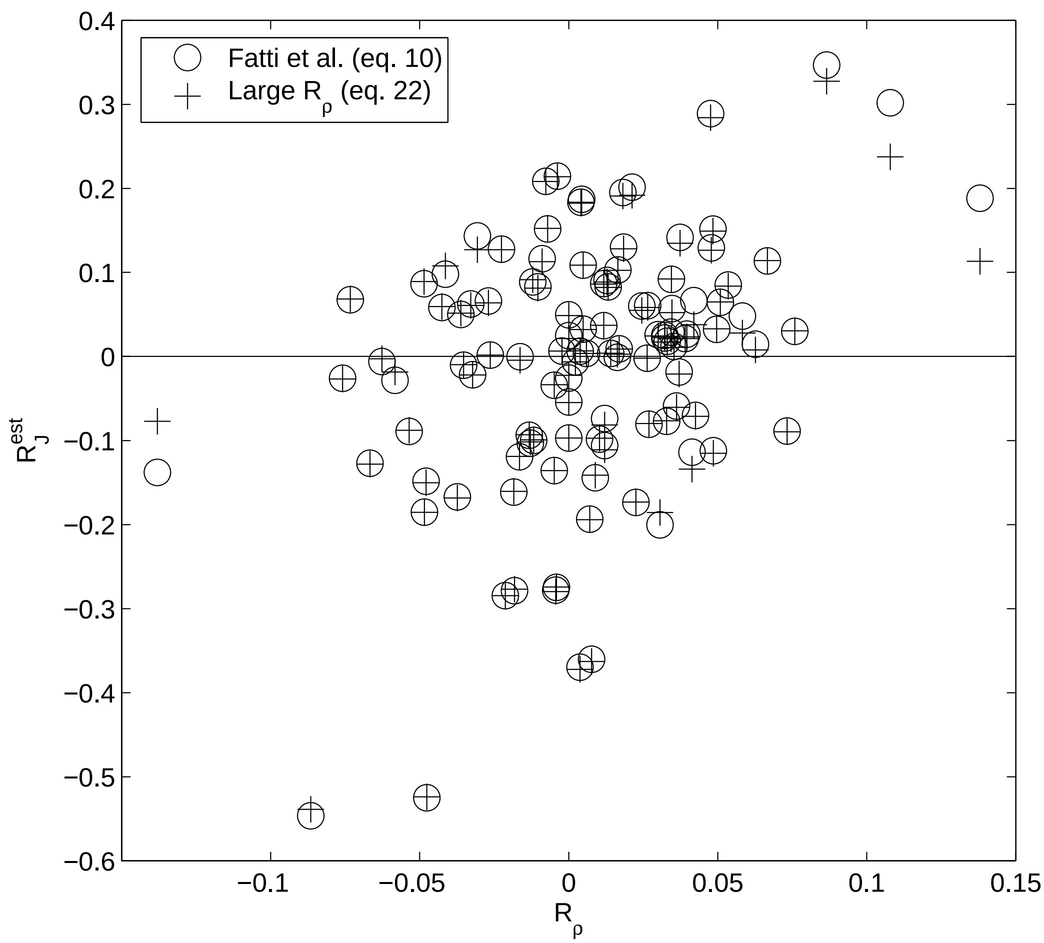

FIG. 3. Comparison of impedance reflectivities produced by the inversion method of Fatti et al. with those produced by the large-R_ρ method of equation 27. The largest discrepancies are seen for large values of R_ρ (note horizonal scale).

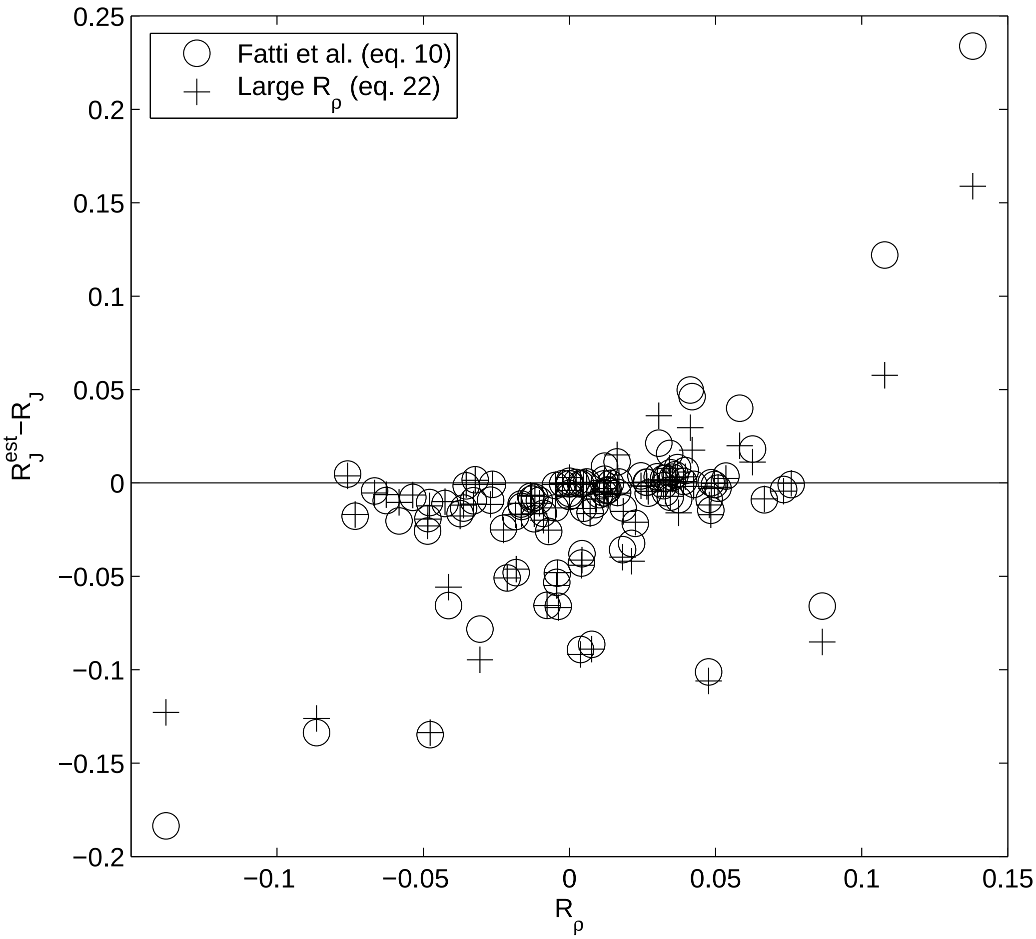

FIG. 4. Comparison of impedance reflectivity *errors*. Data for this plot is obtained by subtracting the exact value of R_J from each value plotted in Figure 3. The results produced by the methods of Fatti et al. and the large-R_ρ method of equation 27 are similar in most cases. For cases of large R_ρ though an improvement is seen with the new method.

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