A hybrid method applied to a 2.5D scalar wave equation

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ABSTRACT

A variation of the acoustic scalar wave equation is used for the seismic modeling of compressional (P) wave propagation in a three dimensional structure. A combination of finite difference and finite integral transform methods are employed for this purpose. To reduce the complexity of the problem, the elastic parameters of the medium are assumed to be constant in one of the three Cartesian spatial dimensions. The homogeneous dimension is removed from the finite difference problem through the use of a finite integral transform. Homogeneity is not a necessary requisite but the resultant transformed problem is at a minimally uncomplicated computational level, while still allowing for some degree of 3D modeling capability. A fairly basic situation is provided from which computational issues may be addressed, that are also relevant in the more general fully inhomogeneous problem. True amplitudes are obtained as 3D geometrical spreading is inherent in the theory. The proper definition of the method presented here would be a 2.5D solution to a wave equation.

INTRODUCTION

A sampling of references in the geophysical literature on the topic of using a combination of finite integral transforms and finite difference methods are Alekseev and Mikhailenko (1980), Gazdag (1973, 1981), Kosloff and Baysal (1982), Mikhailenko and Korneev (1984) and Mikhailenko (1985), where additional references may be found. However, much of the relevant literature, at a fairly complex level, for this solution method is relatively inaccessible as it appears in journals and texts written in Russian. (Citations of these are not included here.) Papers in English by Russians scientists engaged in this research area are most often found in mathematical journals and contain little in the way of numerical implementation. For these reasons it was deemed appropriate to investigate the most basic problem of this type and deal with the more essential numerical considerations. A recent publication by Novais and Santos (2005), addresses a problem similar to that considered here. However, the method presented in this work follows a more analytic treatment in the initial stages of the problem using of a finite Fourier transform to remove the spatial dependence of one coordinate as opposed to an infinite Fourier transform, and the finite approximation thereof, employed in their paper. The use of a finite integral transform has the advantage of allowing for the determination of a number of quantities prior to running the program. A comparison of results will be made with a test model studied in the above cited work.

A combination of finite difference and finite integral transform techniques is employed here to obtain a numerical solution to a hyperbolic (wave) equation in three dimensions. This solution is more correctly called a 2.5 dimension (2.5D) solution. It was initially developed to compensate for hardware limitations, requiring less physical computer resources, with the trade off of a more complicated algorithm for modeling complex geological structures than by conventional finite difference methods alone. The elastic parameters in that spatial dimension in which a finite integral transform is applied, to reduce the dimensionality of the problem, are assumed to be homogeneous. This work may possibly be considered tutorial in nature, but the consideration of this simple problem within the context of hybrid solution techniques, has the ability to conveniently both introduce the method's concept and present computational measures that maximize the algorithm's efficiency, topics not usually dealt with in the literature. The transformed problem is highly parallel and is also a candidate for the application of vectorization techniques.

The use of variations on source and receiver patterns can enhance the 3D modeling capabilities of what is essentially a 2D solution with 3D kinematics and dynamics (geometrical spreading). As several lines of receivers parallel to the (x,z) plane that do not contain the source may be recorded, a type of rebinning, using traces from these individual lines may be used to produce receiver lines oriented at specific angles across the 2D array of surface receivers, within a single run. Rather than using the *P*-wave velocity, $v_P(\mathbf{x})$, as input, the elastic parameter $\lambda(\mathbf{x})$ and density $\rho(\mathbf{x})$ are used $\left(v_P^2(\mathbf{x}) = \lambda(\mathbf{x})/\rho(\mathbf{x})\right)$ for increased modeling flexibility. Additionally, if so required, a number of sources, at arbitrary positions, may be employed in a single run without any significant increase in run time.

As the computational speed of this algorithm is a major concern, the code has been written to take advantage of all possible enhancements of a number of specific Fortran compilers and available hardware. The motivation for this is that for moderately sized models, this code (or variations thereof) may possibly be utilized iteratively in applications such as inverse problems.

THEORY

Acoustic wave equation:

Assume a 3D acoustic medium in a Cartesian coordinate system, (x,y,z), that is inhomogeneous except in one spatial dimension, say the y – direction. The general acoustic wave equation for some scalar amplitude, $\phi = \phi(x,y,z,t)$, is given by

$$\nabla \cdot \left[\lambda(\mathbf{x})\nabla\phi(\mathbf{x},t)\right] - \rho(\mathbf{x})\partial_t^2\phi(\mathbf{x},t) = \delta(\mathbf{x}-\mathbf{x}_0)\, f(t) \tag{1}$$

where $\rho(\mathbf{x})-$density, $\lambda(\mathbf{x})-$is the elastic parameter, such that the acoustic *P*-wave velocity is defined as $v_P^2(\mathbf{x}) = \lambda(\mathbf{x})/\rho(\mathbf{x})$, $\delta(\mathbf{x})-$ indicates a point source of acoustic waves and $f(t)$ is a band limited wavelet, which will be discussed in more detail later, with t being time in the digitized interval $(0 \le t \le t_{\max})$, $t_{\max}$ being the length of the computed synthetic trace. The 3D Cartesian coordinate system is chosen so that the vertical coordinate, *z*, is positive downwards.

As the density, ρ, and elastic parameter, λ, have been assumed independent of the spatial coordinate y, equation (1) becomes

$$\partial_x\left(\lambda\partial_x\phi\right)+\lambda\partial_y^2\phi+\partial_z\left(\lambda\partial_z\phi\right)-\rho\partial_t^2\phi=\delta\left(\mathbf{x}-\mathbf{x}_0\right)f\left(t\right) \qquad (2)$$

with $\lambda=\lambda\left(x,z\right)$ and $\rho=\rho\left(x,z\right)$. The initial value problem is fully specified by introducing the conditions

$$\phi\big|_{t=0}=\partial_t\phi\big|_{t=0}=0\,. \qquad (3)$$

Wave propagation, due to a point source excitation of $P-\text{waves}$, will be assumed to be confined to the spatial geological volume $\left(0\le x\le a;0\le z\le b;0\le y\le c\right)$. Only four of these six boundaries will initially be taken to be perfectly reflecting, $\left(x=0\text{ and }a\right)$ $\left(z=0\text{ and }b\right)$ as the finite integral transform may require that other conditions be specified at $y=0\text{ and }c$. These preliminary boundary conditions require that some measures such as absorbing boundaries (Clayton and Enquist, 1977 and Reynolds, 1978) or attenuating boundaries (for example, Cerjan et al., 1985) be incorporated in the solution method so that spurious reflections from them will not contaminate, to any significant extent, the wave field propagating within the spatial volume and recorded at the receivers. The y spatial dimension is to be temporarily removed employing a finite cosine transform, which requires the specification of explicit boundary conditions at $y=0$ and $y=c$ in the transform procedure. As a consequence, it is not possible to redefine the boundary conditions that could introduce attenuation at these boundaries. A schematic of a simple geological model is shown in Figure 1.

Specious reflections from the boundaries at $y=0$ and $y=c$ can be eliminated by setting the source at a sufficient distance from both of these boundaries so that reflections from them do not arrive at the receivers within the specified time window $\left(0\le t\le t_{\max}\right)$ of the synthetic trace at any receiver location. As will be shown later, the number of terms required to approximate the infinite cosine inverse series summation increases linearly with the value of c, indicating that some consideration should be given to its choice. The reason for this is that the 2D finite difference portion of the algorithm must be computed for each of the terms in the inverse series summation.

The y direction was chosen to have no spatial dependence so that a simple finite integral transformation could remove this coordinate leaving only the fairly standard $\left(x,z,t\right)$ finite difference computation. If this were not done and the geology was allowed to vary in the y direction the transform of the elastic parameters in this direction must be included in the solution, complicating matters to the extent that what was to be discussed here would become a secondary issue. It will be further assumed that the 2D finite difference algorithm used is accurate as a consequence of its frequent use since its development, so that discussion of this aspect will not be undertaken here.

The topic of finite integral transforms that involve the variation of elastic parameters in the transformed coordinate has been dealt with in the literature for a 2D medium and its extension to 3D is the topic of ongoing work. The limitations on the modeling capabilities of the resultant computer code presented here are fairly severe, but do allow for a least a minimal amount of latitude for dealing with an acoustic type of wave propagation in a 3D model at a moderate expenditure of computer resources. The y spatial dimension is removed temporarily through the application of a finite cosine transform, with the solution (x, y, z, t) space being recovered by applying the finite cosine transform inverse.

Finite cosine transform:

If the function $\phi(y)$ satisfies the Dirichlet conditions in the interval $(0, c)$ and if in this interval the relation

$$\Phi(n) = \int_0^c \phi(y)\,\cos\left(\frac{n\pi y}{c}\right) dy \tag{4}$$

is valid at all points in the interval $(0, c)$, where the function $\phi(y)$ is continuous, the following equality

$$\phi(y) = \frac{\Phi(0)}{c} + \frac{2}{c}\sum_{n=1}^{\infty}\Phi(n)\cos\left(\frac{n\pi y}{c}\right) \equiv \frac{2}{c}\sum_{n=0}^{\infty}\Phi(n)\cos\left(\frac{n\pi y}{c}\right) \tag{5}$$

holds. It is understood that the $n = 0$ term has been included in the summation for convenience of notation. Some upper bound on the summation must be determined that adequately approximates the infinite series. This number, apart from the previously mentioned linear dependence on the distance *c*, will be shown to be related to the spectral content of the source wavelet, the reason for requiring it to be band limited.

Applying the cosine transform to equation (2), with the assumption of stress free conditions at $y = 0$ and c, $\partial_y \phi\big|_{y=0} = 0$ and $\partial_y \phi\big|_{y=c} = 0$, having to be made, results in

$$\begin{aligned} &\partial_x(\lambda\partial_x\Phi) - \frac{n^2\pi^2}{c^2}\lambda\Phi + \partial_z(\lambda\partial_z\Phi) - \rho\,\partial_t^2\Phi = \\ &\qquad \delta(x - x_0)\cos\left(\frac{n\pi y_0}{c}\right)\delta(z - z_0)\,f(t) \end{aligned} \tag{6}$$

with $\Phi = \Phi(x, n, z, t)$ and the source position, y_0, located such that $0 < y_0 < c$. Neglecting the source term for the moment, the finite difference analogue, accurate to second order in both space and time may be written as

$$\Phi_{i,j}^{m+1} = \left(\frac{\delta t^2}{\rho_{i,j}\delta s^2}\right)\left[a_{i+1,j}\Phi_{i+1,j}^{m} + a_{i,j}\Phi_{i-1,j}^{m} + b_{i,j+1}\Phi_{i,j+1}^{m} + b_{i,j}\Phi_{i,j-1}^{m}\right] - \Phi_{i,j}^{m-1} + \left[2 - \frac{n^2\pi^2\delta t^2}{c^2\rho_{i,j}}\lambda_{i,j} - \frac{\delta t^2}{\delta s^2\rho_{i,j}}\left[\left(b_{i,j+1} + b_{i,j}\right) + \left(a_{i+1,j} + a_{i,j}\right)\right]\right]\Phi_{i,j}^{m} \tag{7}$$

In the above equation, the spatial sampling rates in the $(x,z)\rightarrow(i,j)$ directions were both taken to be δs. The time step, for which some stability conditions are yet to be established, was denoted as $\delta t \rightarrow (m)$. The quantities $a_{p,q}$ and $b_{p,q}$ may be determined from derivations which appear in Ames, (1969) or Mitchell, (1977), among others, as

$$a_{p,q} = \frac{2\lambda_{p,q}\lambda_{p-1,q}}{\lambda_{p,q} + \lambda_{p-1,q}} \tag{8}$$

$$b_{p,q} = \frac{2\lambda_{p,q}\lambda_{p,q-1}}{\lambda_{p,q} + \lambda_{p,q-1}} \tag{9}$$

The boundary at the surface of the model, $z = 0$, is assumed to be perfectly reflecting, a not unrealistic assumption, based on the condition that $\phi(x,y,z,t)\big|_{z=0}$ be continuous, and the half space $z < 0$ is taken to be a vacuum, making the model boundary perfectly reflecting there. If required, it may also be made absorbing, thus removing any surface multiples.

The saving in space, having to use only 2D arrays to specify the potential, ϕ, elastic parameter, λ, and density, ρ, requires the additional expenditure in computational time, compared with the 2D case, as equation (7) must be solved for those values of n, $(0 \le n \le n_{\max})$, where $n_{\max}$ is the number of terms in the inverse cosine series that reasonably approximates the infinite series. However, as shown by Novais and Santos (2005), the time requirements are significantly less than if 3D finite difference methods are employed. They further showed, as might be predicted, that the time requirements diverge, favoring the method discussed here, as the number of grid points increases.

The estimate of the number of terms required to approximate the inverse series summation is discussed in the next section. As previously mentioned, the 2D finite difference computations must be undertaken $n_{\max}$ times, the utmost use of parallel processing options is indicated.

Number of terms in inverse transform series:

A plane wave dependence of ϕ on y is assumed so that $\phi(y) = Ae^{ik_y y}$, for some constant amplitude *A*, leading to

$$\partial_y^2 \phi(y) = A\left(ik_y\right)^2 e^{ik_y y} \tag{10}$$

The wave number in the y direction is defined in terms of the circular frequency, ω, and the y component of horizontal slowness, p_y, as

$$k_y = \omega p_y . \tag{11}$$

Comparing this with the wave number used in the finite cosine transform results in

$$k_y = 2\pi f p_y = n\pi/c . \tag{12}$$

It follows that

$$2 f p_y = n/c \tag{13}$$

and subsequently

$$n = 2 f p_y c . \tag{14}$$

The quantity $f_{\max}$, which is assumed to be known, is that frequency such that the interval $\left(0 \le f \le f_{\max}\right)$ contains 100% of the frequency content of the source pulse, and hence any associated seismic traces. In practice, this requirement is relaxed to 100% of the *numerical* content. It is for this reason that a band limited source pulse is assumed, as it was easier to obtain a finite range of frequencies that covers the trace/wavelet spectrum for what is essentially a wavenumber method of solution in the transformed coordinate. The Gabor wavelet, which will be used here and is of the specified type, is defined in the time domain

$$f(t) = \cos\left(2\pi f_0 t\right) \exp\left[-\left(\frac{2\pi f_0 t}{\gamma}\right)^2\right] \tag{15}$$

which after a Fourier time transform may be written in the frequency domain as

$$F(\omega) = \frac{\pi^{1/2}}{\omega_0} \exp\left[-\frac{\gamma^2}{4}\left(1+\omega/\omega_0\right)^2\right] \cosh\left(\frac{\omega\gamma^2}{\omega_0}\right) \tag{16}$$

where f_0 is the predominant frequency of the source wavelet and the dimensionless quantity γ $\left(\gamma \approx 4-5\right)$ controls the side lobes in the time domain and is also a measure of the width of the Gaussian term in the frequency domain. An initial estimate of $f_{\max}$ is approximately $2 f_0$ for $\gamma = 4$. A more exact determination may be obtained by numerically integrating equation (16), at a suitably fine sampling rate, Δf. More about this will be said shortly. Figure (2) shows the time and frequency spectrum values of $f(t)$ for $f_0 = 30 Hz$ for $\gamma = 4$ and 5. In the time domain panel the wavelets have been shifted such that their normalized maximum amplitudes coincide. It should be restated that the value used for here for $f_{\max}$ $\left(f_{\max}=2f_0\right)$ is generally valid only for the Gabor wavelet with $\gamma=4$. For $\gamma\neq4$ or for some other wavelet type, $f_{\max}$ must be obtained by other means, most often numerical. It was thought that the use of a reasonably accurate estimate for $f_{\max}$, in terms of a relevant quantity, might simplify this discussion somewhat.

The quantity $\left(p_y\right)_{\max}$ is given by $\sin\left(\pi/2\right)/v_{\min}\approx1/v_{\min}$, with $v_{\min}=\left[\sqrt{\lambda/\rho}\right]_{\min}$, the minimum velocity in the $\left(x,z\right)$ model plane. Thus, a fairly accurate estimate for $n_{\max}$ is

$$n_{\max}\approx4f_0\left(p_y\right)_{\max}c\approx\frac{2f_{\max}c}{v_{\min}} \tag{17}$$

where $n_{\max}$ is such that in the frequency domain the range of frequencies considered is such that it covers the spectrum of the wavelet.

It should be note here that if too few terms are initially used in the infinite series approximation, which would be apparent upon viewing the synthetic traces, additional terms may be added to the original solution without recomputing the original traces using the initial estimate of $n_{\max}$ so that an acceptable result is produced. However, it is usually best to err on the cautious side and overestimate the quantity $n_{\max}$.

Also, from equation (17), it should also be recognized that

$$f_{\max}=\frac{n_{\max}v_{\min}}{2c} \tag{18}$$

and further that

$$f_{\max}=n_{\max}\Delta f \tag{19}$$

for some frequency step, Δf, dependent on the width of the wavelet spectrum in the frequency domain. From this it follows from (18) that

$$\Delta f_{\max}=\frac{v_{\min}}{2c}. \tag{20}$$

This frequency increment, denoted as $\Delta f_{\max}$, is the maximum value of the frequency domain sampling rate, recommended for use in the numerical integration of the Gabor frequency domain spectrum. For the Gabor wavelet, the relation $f_{\max}\approx2f_0$ has been assumed as a reasonable guess to obtain the quantity $n_{\max}$. The use of $\Delta f_{\max}$ in a numerical integration of the spectrum of the wavelet to determine $n_{\max}$ can be viewed as a cautious check on the accuracy of this quantity.

This method has been referred to as a *wavenumber summation method* by Müller (1995) when referring to similar methods presented by Alekseev and Mikhailenko (1980). This is substantiated by the relation, obtained from equations (12) and (20), that

$$k_y\Big|_\ell = 2\pi\ell\Delta f_{\min}\left(p_y\right)_{\max} = \ell\Delta\omega_{\min}\left(p_y\right)_{\max} \approx \frac{\pi\ell}{c} \qquad \left(0 \le m \le n_{\max}\right) \tag{21}$$

where k_y is the out of plane horizontal component of the wavenumber vector. The inverse cosine series summation is then a summation over the discrete set of wavenumbers $k_y\Big|_\ell = \pi\ell/c$.

Stability conditions

As is common practice, the von Neumann criterion to obtain stability conditions will be derived under the assumption of constant elastic parameters and density. Thus equation (7) reduces to

$$\begin{aligned}\Phi_{m,n}^{\ell+1} = \left(\frac{\delta t^2 v_{\max}^2}{\delta s^2}\right)\left[\Phi_{m+1,n}^{\ell} + \Phi_{m-1,n}^{\ell} + \Phi_{m,n+1}^{\ell} + \Phi_{m,n-1}^{\ell}\right] - \Phi_{m,n}^{\ell-1} + \\ \left[2 - \frac{n^2\pi^2\delta t^2 v_{\max}^2}{c^2} - \frac{4\delta t^2 v_{\max}^2}{\delta s^2}\right]\Phi_{m,n}^{\ell}\end{aligned} \tag{22}$$

where $v_{\max}^2$ is the square of the maximum $P-$wave velocity encountered on the 2D finite difference grid. The von Neumann stability condition is used, as harmonic decomposition of equation (22) may be performed and consequently it will be shown that the error at a given grid point and time step may be determined. The amplitude $\Phi_{m,n}^{\ell}$ containing the amplification factor, η, can then be written as (Mitchell, 1977)

$$\Phi_{m,n}^{\ell} = \eta^{\ell} e^{i\beta_x(m\delta s)} e^{i\gamma_z(n\delta s)}. \tag{23}$$

The wave numbers β_x and γ_z are arbitrary. To determine the conditions required so that error does not increase with increasing time it is required to find a solution of the finite difference analogue such that $|\eta| \le 1$. Substituting equation (23) into equation (22) results in

$$\eta^2 - \sigma\eta + 1 = 0 \tag{24}$$

whose solution is

$$\eta = \frac{\sigma \pm \sqrt{\sigma^2 - 4}}{2} \tag{25}$$

and σ may be written as

$$\sigma=\left(\frac{\delta t^2 v_{\max}^2}{\delta s^2}\right)\left(4\sin^2\left(\frac{\beta_j\delta s}{2}\right)+4\sin^2\left(\frac{\gamma_j\delta s}{2}\right)\right)-2+\frac{n^2\pi^2\delta t^2 v_{\max}^2}{c^2}. \tag{26}$$

and finally to

$$\sigma=8\left(\frac{\delta t^2 v_{\max}^2}{\delta s^2}\right)-2+\frac{n^2\pi^2\delta t^2 v_{\max}^2}{c^2} \tag{27}$$

as $\sin^2\left(\beta_j\delta s/2\right)\leq 1$ and $\sin^2\left(\gamma_j\delta s/2\right)\leq 1$.

As stability requires that $|\eta|\leq 1$ which is equivalent to $|\sigma|\leq 1$ *(Mitchell*, 1977) the following results

$$\frac{\delta t^2 v_{\max}^2}{\delta s^2}\left[1+\frac{n^2\pi^2\delta s^2}{c^2}\right]\leq\frac{3}{8} \tag{28}$$

leading to

$$\delta t\leq\sqrt{\frac{3}{8}}\frac{\delta s}{v_{\max}}\left[1+\frac{n_{\max}^2\pi^2\delta s^2}{c^2}\right]^{-1/2} \tag{29}$$

specifies the stability condition for the problem given in equation (2). It may be seen from equation (29) that δt has a different value at each n and as $\left[1+n^2\pi^2\delta s^2/c^2\right]^{1/2}$ is always greater than unity attaining its maximum value at $n=n_{\max}$, it is that value which is used to specify δt. For the transformed 2D finite difference scheme that is accurate to the second order in both space and time. This choice also ensures a constant global time step for all roots.

Points per wavelength (grid dispersion)♦

There are a number of other numerical issues on the use of finite difference methods that could be addressed. However, the major topic that has not yet been dealt with is specifying the number of grid points per wavelength. There are a number of ways in which a wavelength may be defined when considering a problem such as that dealt with here. Probably the most useful is

$$WL=\frac{v_{\min}}{f_{\max}} \tag{30}$$

where WL – wavelength and $v_{\min}$ and $f_{\max}$ have been previously defined. From equation (18) it should be noted that the following also holds (for the Gabor wavelet used here)

♦ From Mikhailenko (1980), $q=\left(v_{\min}\delta t\right)/\delta s\approx 0.4$ is used to assess the error at or in the vicinity of the grid points displaying the minimal velocity of the medium.

$$WL = \frac{v_{\min}}{f_{\max}} = \frac{2c}{n_{\max}}. \tag{31}$$

As stated in Mufti (1990), and others, a finite difference scheme accurate to the second order should have at least 10 $points/WL$ $(n_{WL} \geq 10)$. It is further stated the grid spacing used in the second order accurate spatial finite difference analogue, so as not cause excessive dispersion of energy requires that

$$\delta s \leq \frac{v_{\min}}{n_{WL}\ f_{\max}} = \frac{2c}{n_{WL}\ n_{\max}} \tag{32}$$

where n_{WL} is the number of points per wavelength.

As a final comment in this section, some indication of accuracy should be addressed. Being that the exact solution of the scalar wave equation is known for an infinite medium with constant velocity, a comparison can be done with method presented here for the same medium type. This exercise produced results that varied in the order of $1-3\%$ at $20WL$ from the source. These are in agreement with those indicated by Mikhailenko (1980) for a transformed 3D medium with radial symmetry.

NUMERICAL RESULTS

One model has been chosen and computed using this method to test the accuracy of the method presented here with the results for a model similar to model 1 in the paper of Novais and Santos (2005) and shown in Figure (4). For consistency, equation (1) in this paper must be replaced by

$$v_P^2(\mathbf{x})\nabla^2\phi(\mathbf{x},t) - \partial_t^2\phi(\mathbf{x},t) = \delta(\mathbf{x}-\mathbf{x}_0)\ f(t) \tag{33}$$

the standard 3D acoustic wave equation, which can be done with minimal program modifications. The spatial increment used here is $\delta s = 5m$, half of that used by Novais and Santos (2005), as in that paper they use finite difference code accurate to fourth order in spatial coordinates and second order in time while the pseudo – code presented here is accurate to second order in both space and time. An explosive point source of $P-$waves on the surface at $x = 800m$ the center of the $1600m$ range with 41 receivers at the surface at a $40m$ spacing. The source location in y direction is at $y = y_0 = 1000m$ with $y = c = 2000m$ provides a reasonable time window $(0 \leq t \leq t_{\max})$ such that receiver lines in the vicinity of $y = 1000m$, say $(750m < y_0 = 1000m < 1250m)$ are not affected by unwanted arrivals from the boundaries at $y = 0$ and c. The Gabor wavelet (equation (15)) is used here, so that with $f_0 = 30Hz, \gamma = 4$, that $f_{\max} \approx 60Hz$. The minimum and maximum velocities in the model are $v_{\min} = 3000\,m/s$ and $v_{\max} = 4000\,m/s$. A schematic of the model is given in Figure (4).

From equation (17), $n_{\max} = 80$ for $f_0 = 30Hz, \gamma = 4$, and $n_{\max} = 160$ for $f_0 = 60Hz, \gamma = 4$. This is in contrast to the number of points in the summation used by

Novais and Santos (2005) where the summation was taken over the wavenumber range $(0, 0.22)m^{-1}$ at a step of $0.0005m^{-1}$ which results in 440 points required to be considered in the inverse series summation. Results are presented in Figures (5) and (6). A fairly crude measuring system was used to compare the results computed here with those in the abovementioned paper and it was found that there was a reasonable fit, given the measuring technique. As a final note, the run times were 221 seconds for $f_0 = 30Hz$ and $\gamma = 4$ and 511 seconds for $f_0 = 60Hz, \gamma = 4$ on a PC with a 1.7GHz processor and a Linux operating system.

CONCLUSIONS

The solution of a 3D scalar equation using a combination of finite difference and finite integral transform methods is presented. It is assumed that the medium of propagation is independent of one of the Cartesian coordinates defining the model; in the case considered here, the y coordinate. This coordinate is removed from the finite difference process utilizing a finite Fourier integral (cosine) transform. The resultant problem is the solution of the scalar wave equation in 2.5D, that is, the amplitudes recorded on the synthetics are what could be termed true amplitudes (dynamic properties) as 3D geometrical spreading is incorporated.

The run times for this 2.5D problem and a pure 3D finite difference problem are not similar. Also, only two dimensional arrays are required when this method is used, which provides a significant saving in computer resources. The trade off is that strict 3D modeling is quite limited. However, for 2D seismic lines in complex geometries this method produces proper kinematic and dynamic properties for the reflected, refracted and diffracted arrivals.

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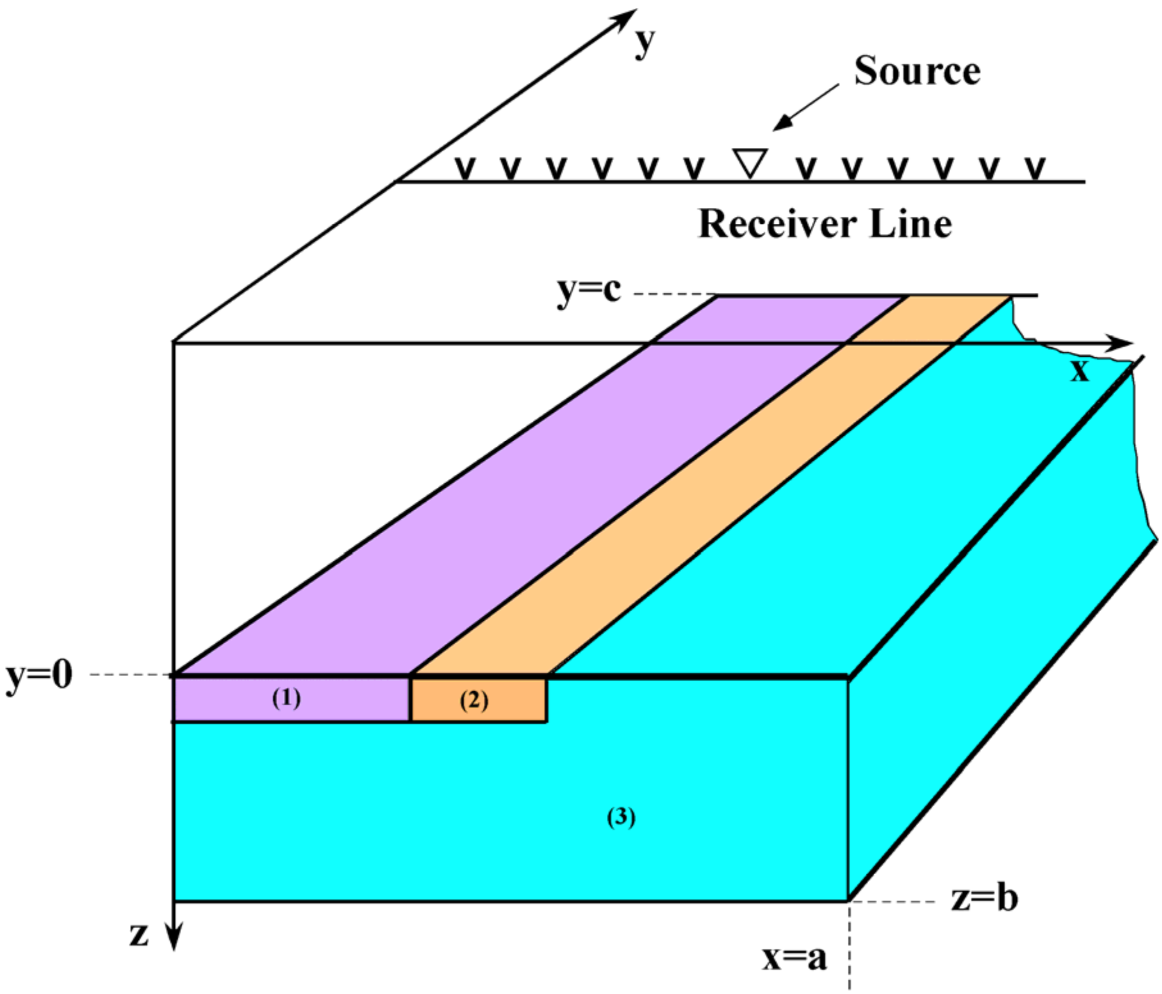

FIG. 1. Schematic of an example model that may be treated using the method described here. The computational model employed is similar with a source located at $z_S = 0$, $0 < x_S < a$, $0 < y_S < c$.

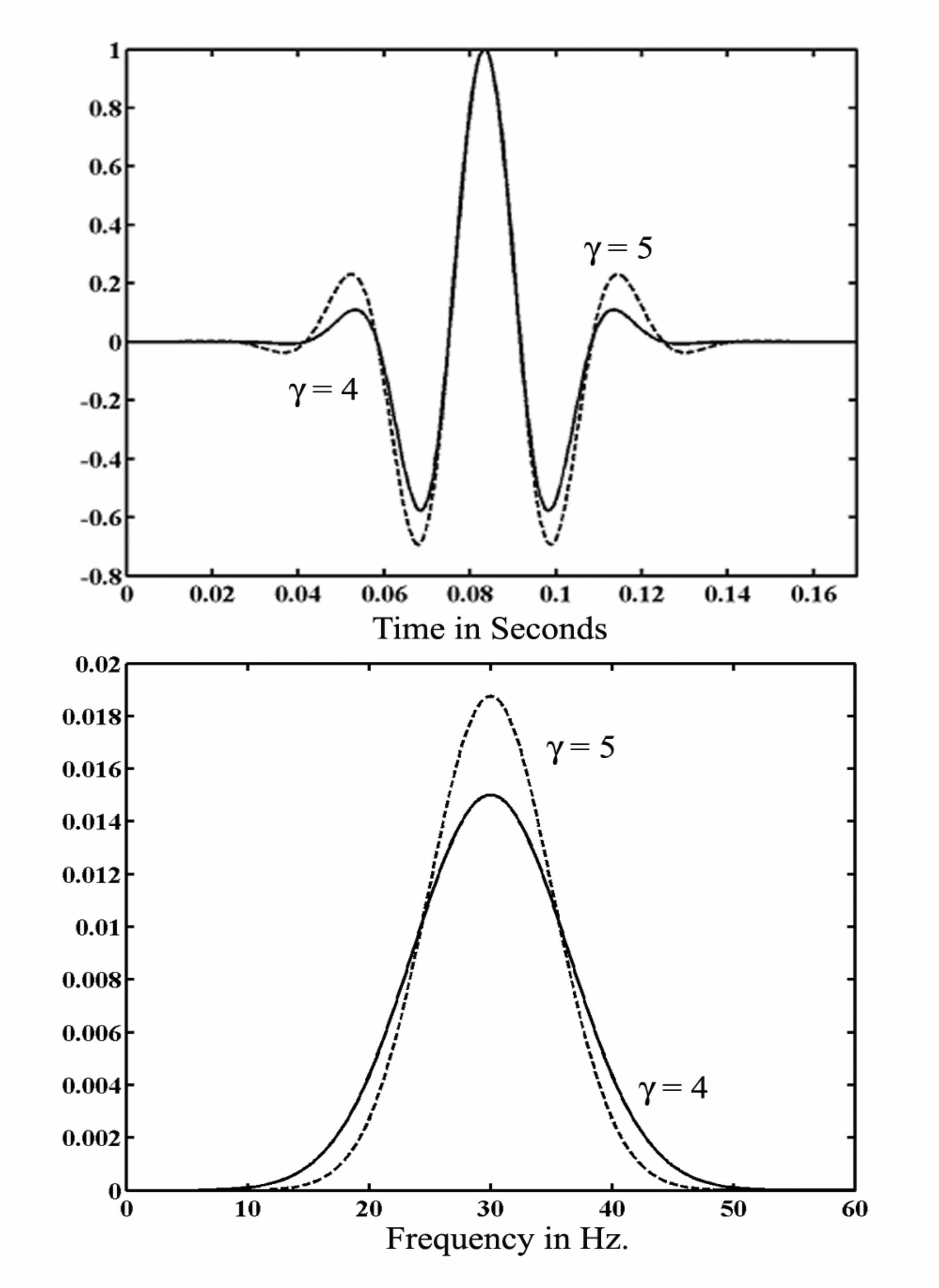

FIG. 2. Time and frequency plots of the Gabor wavelet for $f_0 = 30$ and $\gamma = 4$ and 5.

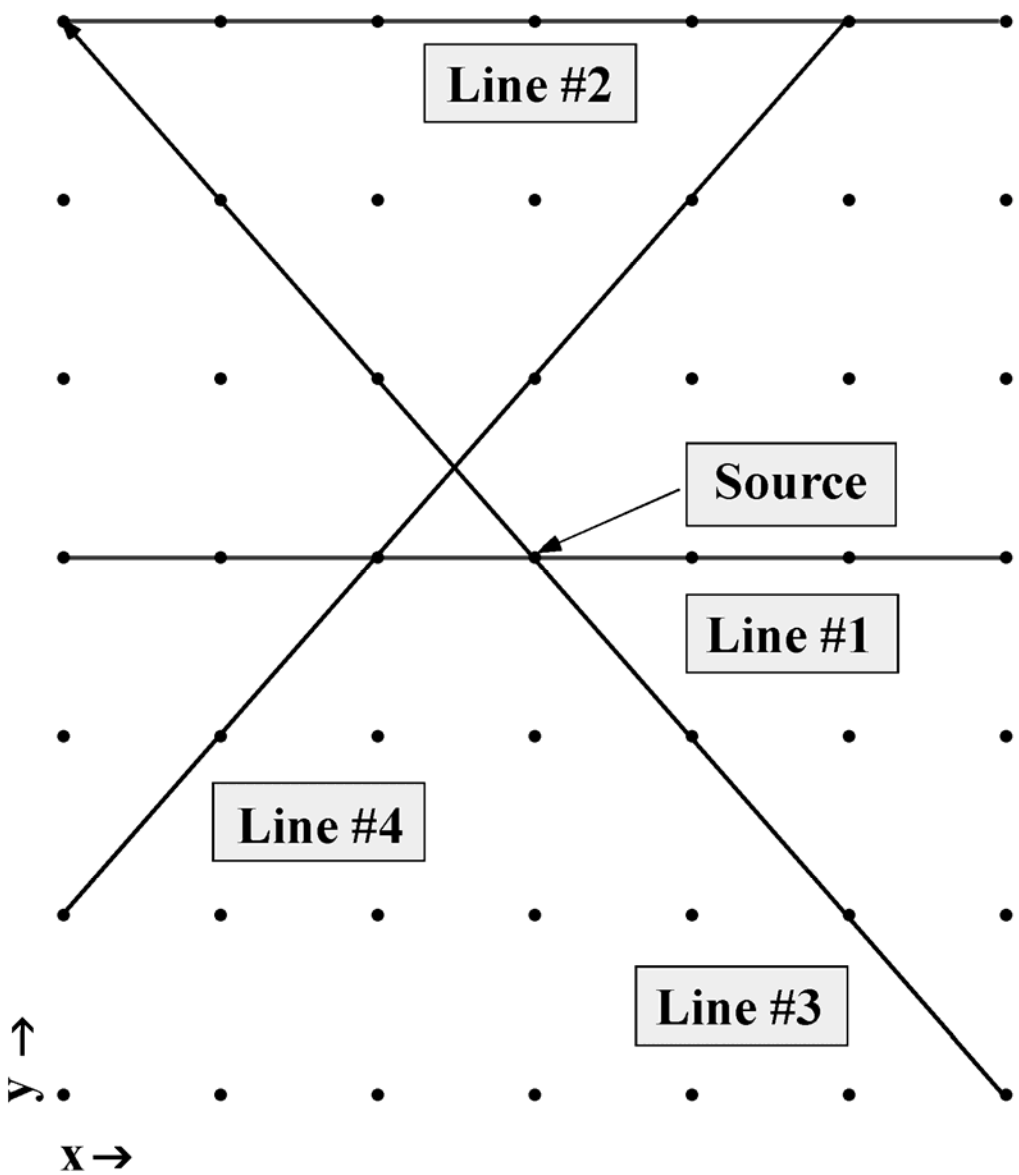

FIG. 3. Schematic of shooting geometry. A single source as indicated and a $n \times n$ surface array of receivers with $\Delta x = \Delta y = \delta s$. Other possible receiver lines are shown. Note that the receiver lines at an angle to the (x, y) spatial points have different spatial increments between receivers.

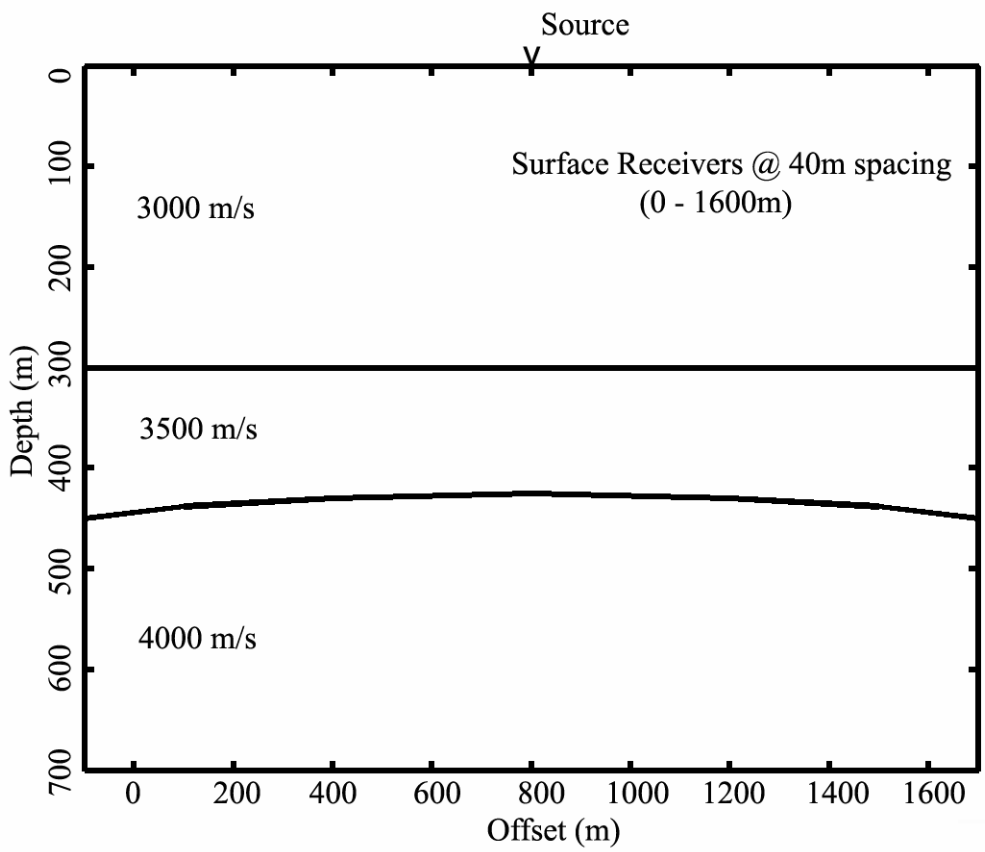

FIG. 4. Model 1 after Novais and Santos (2005). See details in text.

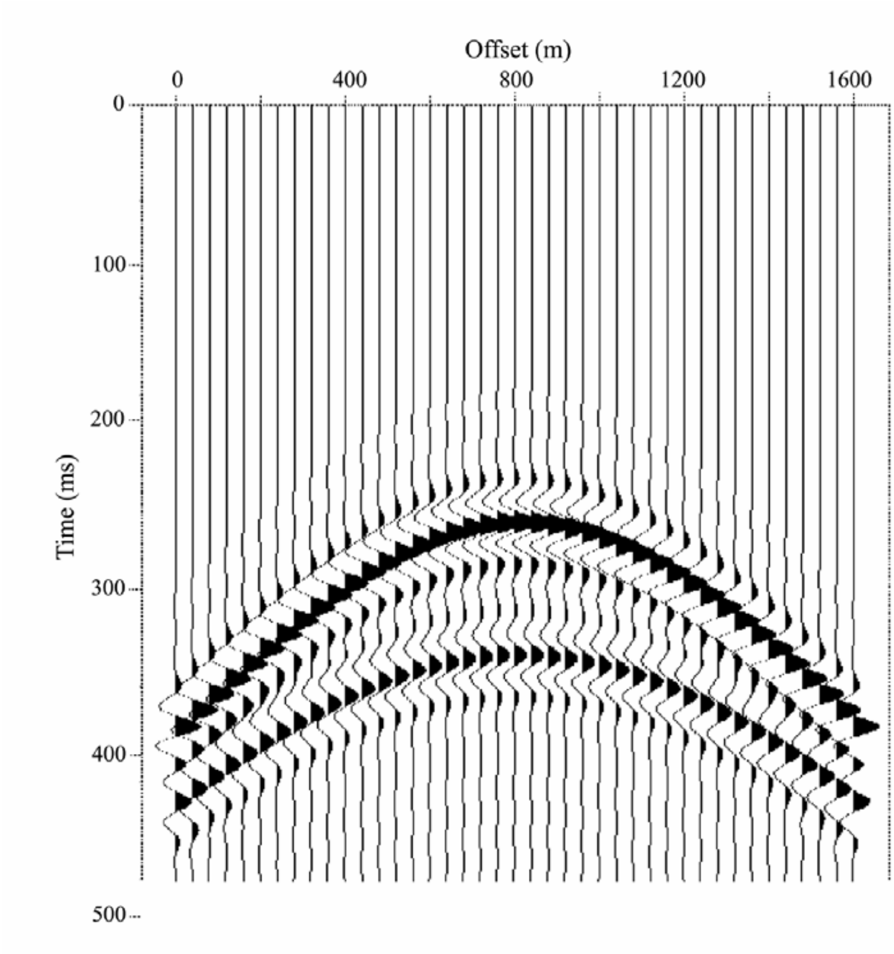

FIG. 5. After Novais and Santos (2005), Model 1, with Gabor wavelet $f_0 = 30Hz$ and $\gamma = 4$.

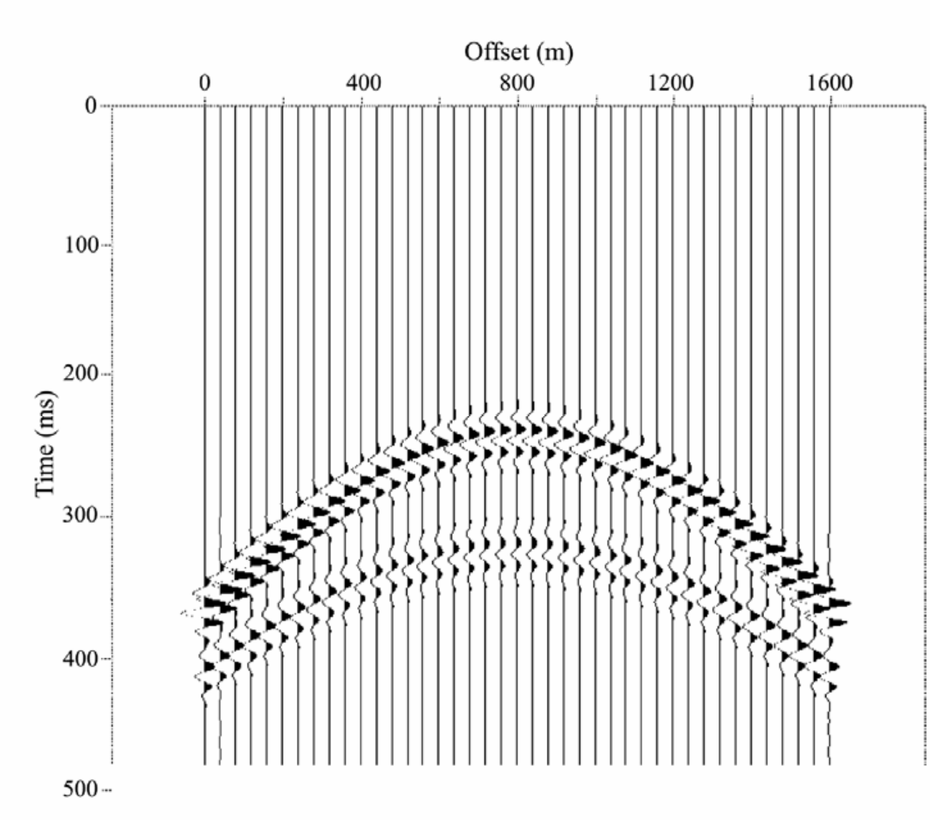

FIG. 6. After Novais and Santos (2005), Model 1, with Gabor wavelet $f_0 = 60Hz$ and $\gamma = 4$.