Wavefield separation in acoustic media from normalized energy flux density

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ABSTRACT

The analytic separation of upgoing and downgoing wavefields in acoustic media is summarized, and explicit relations are given for use in prestack-depth migration of seismic data under the acoustic assumption. The linearized wave equations for acoustic media are assumed to be valid, and planewave decomposition of pressure and displacement is used to generate a set of coupled differential equations for pressure and vertical displacement. A wavefield-separation operator is then postulated whose effect is to separate upgoing waves from downgoing waves, and time averaged (normalized) energy flux density is used to determine coefficients for this operator.

INTRODUCTION

Though improvements in computational throughput have allowed for increased sophistication in seismic imaging, 3D prestack depth migration is based, often, on an acoustic model of the earth (rather than the true elastic model). Based on the acoustic wave equations, then, many imaging operators are designed for depth migration in elastic media. As pointed out by Zhang and Zhang (2005), conventional acoustic operators lose amplitude fidelity in even the simplest media. As a remedy, Zhang and Zhang (2005) provide a source function for use in forward modelling and migration in acoustic media. Their approach improves amplitude fidelity of acoustic operators, and therefore seismic images, for high wavenumbers.

An alternative remedy for amplitude fidelity is proposed by Pedersen et al. (2007) in notes based on time averaged (normalized) energy flux density. Though not explored in this paper, there is expectation of improved amplitude performance when the separation operators of (Pedersen et al., 2007) are employed in seismic imaging.

This paper begins with the definition of total energy for a volume within an elastic medium. Kinetic energy is given in terms of displacement and density, and potential energy is given in terms of elasticity and 21 independent coefficients. The elastic medium is then assumed to be acoustic so that 1 coefficient plus density completely describes the medium. To derive a conservation relation for total energy in terms of hydrophone and geophone measurements, the wave equations of linear acoustics, pressure and particle velocity are introduced. then, through analysis of individual planewaves, energy flux density time-averaged (normalized) over the duration of a passing wave yields a conservative relation ξ that is given in terms of pressure and particle velocity.

An operator for wavefield separation is then postulated based on the system of linearized wave equations for acoustics, and ξ is used to solve for the coefficients. The result is a set of explicit definitions of the upgoing components and downgoing components of acoustic media.

THEORY

Total energy E in volume V within an elastic medium is the sum of kinetic energy W and potential energy Φ

$$= \int_V [W + \Phi]\, dV, \tag{1}$$

where kinetic energy W is associated with density ρ, particle displacement $\mathbf{U}$, and velocity $v = \mathbf{v} \cdot \mathbf{v} = \sum_{i=1}^{3} \dot{U}_i \, \dot{U}_i$ according to Easley and Brown (1990)

$$W = \frac{1}{2}\,\rho\,\mathbf{v} \cdot \mathbf{v} = \frac{1}{2}\,\rho\,\dot{U}_i\,\dot{U}_i, \tag{2}$$

and repeated indices indicate summation. For elastic coefficient tensor $\mathbf{C} \Leftrightarrow C_{ijkl}$, potential energy Φ is

$$\Phi = \frac{1}{2}\,C_{ijkl}\,U_{i,j}\,U_{k,l}, \tag{3}$$

where indices to the right of the comma indicate differentiation. For an acoustic medium, $\mathbf{C}$ simplifies to a single parameter λ

$$C_{ijkl} = \lambda\,\delta_{ij}\,\delta_{kl}, \tag{4}$$

where δ is the Kronecker delta-function. Substitute equation 4 for $\mathbf{C}$ in equation 3 and it reduces to

$$\Phi = \frac{1}{2}\,\lambda\,U_{i,j}\,U_{k,l}. \tag{5}$$

To simplify manipulation of equations 2 and 5, we may write them in vector notation as

$$W = \frac{1}{2}\,\rho\,\dot{\mathbf{U}} \cdot \dot{\mathbf{U}}, \tag{6}$$

and

$$\Phi = \frac{1}{2}\,\lambda\,\left(\nabla \cdot \mathbf{U}\right)^2. \tag{7}$$

where $\nabla\cdot$ is the divergence operator. According to equations 1, 6, and 7, then, total energy E for an acoustic medium is

$$E = \frac{1}{2}\int_V \rho\,\left[\dot{\mathbf{U}} \cdot \dot{\mathbf{U}} + c^2\,\left(\nabla \cdot \mathbf{U}\right)^2\right] dV, \tag{8}$$

where $c = \sqrt{\lambda/\rho}$ is the speed of sound.

In seismology, we record hydrostatic-pressure P, and displacement velocity vector $\mathbf{v}$, so we want E in terms of $\mathbf{v}$ and P. For linear acoustics, then, we have the following wave equations in terms of P, $\mathbf{v} = \dot{\mathbf{U}}$, and $\dot{\mathbf{v}} = \ddot{\mathbf{U}}$:

$$\dot{P} + \lambda\,\nabla \cdot \dot{\mathbf{U}} = 0, \tag{9}$$

from conservation of mass and, from conservation of force,

$$\rho\,\ddot{\mathbf{U}} + \nabla P = 0, \tag{10}$$

where ∇ is the gradient operator. The rate of change of E in the volume is the temporal derivative of equation 8 (Ursin, 1984; Easley and Brown, 1990) according to

$$\frac{\partial E}{\partial t} = \int_V \rho \left[\ddot{\mathbf{U}} \cdot \dot{\mathbf{U}} + c^2 \left(\nabla \cdot \ddot{\mathbf{U}} \right)^2 \right] dV, \tag{11}$$

and, according to equation 10, replace $\ddot{\mathbf{U}}$ with $-\frac{1}{\rho}\,\nabla P$ so that equation 11 becomes

$$\frac{\partial E}{\partial t} = \int_V \rho \left[-\frac{1}{\rho}\,\nabla P \cdot \dot{\mathbf{U}} + c^2 \left(\nabla \cdot \ddot{\mathbf{U}} \right)^2 \right] dV. \tag{12}$$

From the first theorem of Green,

$$-\int_V \nabla P \cdot \dot{\mathbf{U}} dV = \int_V P\,\nabla \cdot \mathbf{U}\,dV - \int_S P\,\dot{\mathbf{U}} \cdot \hat{\mathbf{n}}\,dS, \tag{13}$$

where S is the surface enclosing V, and $\hat{\mathbf{n}}$ is a unit vector normal to S. Replace the volume integral of $\nabla P \cdot \dot{\mathbf{U}}$ in equation 12 with equation 13 to get

$$\frac{\partial E}{\partial t} = \int_V \nabla \cdot \mathbf{U}\ [P + \lambda\,\nabla \cdot \mathbf{U}]\,dV - \int_S P\,\dot{\mathbf{U}} \cdot \hat{\mathbf{n}}\,dS. \tag{14}$$

From equation 9, and from the basic definition of integration, we have

$$P + \lambda\,\nabla \cdot \mathbf{U} = \int \dot{P} + \lambda\,\nabla \cdot \dot{\mathbf{U}}\,dt = \epsilon, \tag{15}$$

where ϵ is a constant, and equation 14 is reduced to

$$\frac{\partial E}{\partial t} = \int_V \left(\frac{\epsilon - P}{\lambda} \right) \epsilon\,dV - \int_S P\,\dot{\mathbf{U}} \cdot \hat{\mathbf{n}}\,dS. \tag{16}$$

To determine a value for ϵ, consider that, prior to the passage of an acoustic wave, $P = 0$ and total energy $E = 0$, so if we integrate equation 16 over t we have

$$\int_t \frac{\partial\,E}{\partial t}\,dt = E = 0 = \int_t \int_V \frac{\epsilon^2}{\lambda}\,dV\,dt. \tag{17}$$

For time invariant λ, equation 17 is reduced to a volume integral

$$E = 0 = [t + \varepsilon] \int_V \frac{\epsilon^2}{\lambda}\,dV = \int_V \frac{\epsilon^2}{\lambda}\,dV, \tag{18}$$

and from equation 18, then, for $t > 0$ and $\lambda << \infty$, $\epsilon \equiv 0$, equation 15 becomes

$$P + \lambda\,\nabla \cdot \mathbf{U} = 0. \tag{19}$$

According to equation 19, $\partial E/\partial t$ (equation 16) is reduced to the following surface integral (Ursin, 1984):

$$\frac{\partial E}{\partial t} = \int_S \lambda\,\nabla \cdot \mathbf{U}\,\dot{\mathbf{U}} \cdot \hat{\mathbf{n}}\,dS. \tag{20}$$

Because seismic data is multidimensional, plane-wave decomposition is used to simplify data analysis. It is natural, therefore, that we consider E of individual plane-waves according to

$$\mathbf{U} = [U_1\,\hat{\mathbf{e}}_1 + U_2\,\hat{\mathbf{e}}_2 + U_3\,\hat{\mathbf{e}}_3]\; e^{i[\omega\,t-\mathbf{k}\cdot\mathbf{x}]}. \tag{21}$$

where ω is temporal frequency, $\mathbf{x} = x_1\,\hat{\mathbf{e}}_1 + x_2\,\hat{\mathbf{e}}_2 + x_3\,\hat{\mathbf{e}}_3$ and $\mathbf{k} = k_1\,\hat{\mathbf{e}}_1 + k_2\,\hat{\mathbf{e}}_2 + k_3\,\hat{\mathbf{e}}_3$ are space and wavenumber vectors respectively, and

$$k_3 = \frac{\omega}{c}\sqrt{1-k_1^2-k_2^2}. \tag{22}$$

Regardless of our model of displacement-vector $\mathbf{U}$, however, $\partial E/\partial t$ is real valued, and energy flux density $\nabla\cdot\mathbf{U}\,\dot{\mathbf{U}}$ from the integrand of equation 20, therefore, is also real valued. Real-valued $\nabla\cdot\mathbf{U}\,\dot{\mathbf{U}}$ is written (Ursin, 1984)

$$\Re\left\{\nabla\cdot\mathbf{U}\,\dot{\mathbf{U}}\right\} = \frac{1}{4}\left[\nabla\cdot[\mathbf{U}+\mathbf{U}^*]\,\left[\dot{\mathbf{U}}+\dot{\mathbf{U}}^*\right]\right], \tag{23}$$

where $*$ indicates complex conjugate. Factor equation 23 so that it has two terms according to:

$$\Re\left\{\nabla\cdot\mathbf{U}\,\dot{\mathbf{U}}\right\} = \frac{1}{4}\left[\dot{\mathbf{U}}\,\nabla\cdot\mathbf{U}+\dot{\mathbf{U}}^*\,\nabla\cdot\mathbf{U}^*\right]+\frac{1}{4}\left[\dot{\mathbf{U}}^*\,\nabla\cdot\mathbf{U}+\dot{\mathbf{U}}\,\nabla\cdot\mathbf{U}^*\right]. \tag{24}$$

Notice that complex exponentials in the second term cancel as in, for example, $\mathbf{U}^*\,\nabla\cdot\mathbf{U}$ where, using equation21, it is reduced to

$$\dot{\mathbf{U}}^*\,\nabla\cdot\mathbf{U} = -\omega\,[k_1\,U_1+k_2\,U_2+k_3\,U_3]\,\left[U_1\,\hat{\hat{\mathbf{e}}}_1+U_2\,\hat{\hat{\mathbf{e}}}_2+U_3\,\hat{\hat{\mathbf{e}}}_3\right]. \tag{25}$$

Complex exponentials in the first term do not cancel as in, for example, $\dot{\mathbf{U}}\,\nabla\cdot\mathbf{U}$ where, using equation 21,

$$\dot{\mathbf{U}}\,\nabla\cdot\mathbf{U} = -\dot{\mathbf{U}}^*\,\nabla\cdot\mathbf{U}e^{2\,i\,[\omega\,t-\mathbf{k}\cdot\mathbf{x}]}, \tag{26}$$

and $\dot{\mathbf{U}}^*\,\nabla\cdot\mathbf{U}$ is given by equation 25 (Ursin, 1984).

If we compute the time average of $\partial E/\partial t$ for the period $t_1\to t_2$, where $t_1>0$, $t_2>0$, and if t_2-t_1 is large, the first term of equation 24 cancels. For example, the time integral of $\dot{\mathbf{U}}\,\nabla\cdot\mathbf{U}$ according to

$$\frac{1}{t_2-t_1}\int_{t_1}^{t_2}\dot{\mathbf{U}}\,\nabla\cdot\mathbf{U} = \frac{1}{t_2-t_1}\,\dot{\mathbf{U}}^*\,\nabla\cdot\mathbf{U}\,e^{-2\,i\,[\mathbf{k}\cdot\mathbf{x}]}\int_{t_1}^{t_2}e^{-2\,i\omega t}\,dt = 0, \tag{27}$$

is zero because of the complex exponential in t. Normalized energy flux density, therefore, is

$$\xi = \frac{1}{t_2-t_1}\int\lambda\,\nabla\cdot\mathbf{U}\,\dot{\mathbf{U}}\,dt = \frac{1}{4}\lambda\,\left[\dot{\mathbf{U}}^*\,\nabla\cdot\mathbf{U}+\dot{\mathbf{U}}\,\nabla\cdot\mathbf{U}^*\right]+\epsilon, \tag{28}$$

where ϵ is a constant. Again, from steady-state considerations, $\epsilon=0$, and

$$\xi = -\frac{1}{4}\left[\dot{\mathbf{U}}^*\,\tilde{P}+\dot{\mathbf{U}}\,\tilde{P}^*\right], \tag{29}$$

where, from equation 19, $\nabla \cdot \mathbf{U} = -\tilde{P}/\lambda$ has been substituted, and $\tilde{P}$ is the plane-wave decomposition of pressure P according to

$$\tilde{P} = \frac{1}{(2\,\pi)^3} \int_{-\infty}^{\infty} P\, e^{i\,[\mathbf{k}\cdot\mathbf{x} - \omega\, t]}\, d\mathbf{x}. \tag{30}$$

From $\dot{\mathbf{U}} = \tilde{\mathbf{v}}$, where $\tilde{\mathbf{v}}$ is the plane-wave decomposition of the particle-velocity vector according to

$$\tilde{\mathbf{v}} = \frac{1}{(2\,\pi)^3} \int_{-\infty}^{\infty} \mathbf{v}\, e^{i\,[\mathbf{k}\cdot\mathbf{x} - \omega\, t]}\, d\mathbf{x}, \tag{31}$$

normalized energy flux (equation 29) is (Ursin, 1984)

$$\xi = -\frac{1}{4} \left[\tilde{\mathbf{v}}^*\, \tilde{P} + \tilde{\mathbf{v}}\, \tilde{P}^* \right]. \tag{32}$$

Fourier domain linear acoustic equations

For conventional acquisition of seismic data, a recording plane is established where $x_3 = \bar{x}_3$ is constant nominally. In this case, plane-wave decomposition of P and $\mathbf{v}$ become

$$\tilde{P} = \frac{1}{(2\,\pi)^2} \int_{-\infty}^{\infty} P\, e^{i\,[k_1\, x_1 + k_2\, x_2 - \omega\, t]}\, dx_1\, dx_2, \tag{33}$$

and

$$\tilde{\mathbf{v}} = \frac{1}{(2\,\pi)^2} \int_{-\infty}^{\infty} \mathbf{v}\, e^{i\,[k_1\, x_1 + k_2\, x_2 - \omega\, t]}\, dx_1\, dx_2. \tag{34}$$

In the Fourier domain, according to equations 33 and 34, the linear, acoustic wave equations of equations 9 and 10 become

$$-i\,\omega\, \tilde{P} + \lambda \left[i\, k_1\, \tilde{v}_1 + i\, k_2\, \tilde{v}_2 + \frac{\partial}{\partial x_3} v_3 \right] = 0, \tag{35}$$

and

$$\rho\, [-i\,\omega]\, \tilde{\mathbf{v}} + \left[i\, k_1\, \tilde{P}\, \hat{\mathbf{e}}_1 + i\, k_2\, \tilde{P}\, \hat{\mathbf{e}}_2 + \frac{\partial}{\partial x_3} \tilde{P}\, \hat{\mathbf{e}}_3 \right] = 0. \tag{36}$$

Equation 36 is a vector equation that provides the following system of three equations:

$$\begin{aligned} \rho\,\omega\,\tilde{v}_1 &= k_1 \tilde{P} \\ \rho\,\omega\,\tilde{v}_2 &= k_2 \tilde{P} \\ \rho\,(i\,\omega)\, v_3 &= \tfrac{\partial}{\partial x_3} \tilde{P}, \end{aligned} \tag{37}$$

So, from equation 37 we have the following differential equation

$$\frac{\partial}{\partial x_3}\, \tilde{P} = i\,\omega\,\rho\, v_3. \tag{38}$$

to relate $\tilde{P}$ and v_3, where v_3 is particle velocity in the normal to the recording plane. Further, use equation 37 to eliminate $\tilde{v}_1$ and $\tilde{v}_2$ in equation 35 to give (Ursin, 1984)

$$\frac{\partial}{\partial x_3} v_3 = \frac{i\omega\, p_3}{\rho}\, \tilde{P}, \tag{39}$$

where

$$p_3 = \sqrt{\frac{1}{c^2} - \left(\frac{k_1}{\omega}\right)^2 - \left(\frac{k_2}{\omega}\right)^2}. \tag{40}$$

Separation of up-going and down-going waves

Equations 38 and 39 provide the following system of coupled wave equations:

$$\frac{\partial}{\partial x_3} \begin{bmatrix} \tilde{P} \\ v_3 \end{bmatrix} = i\,\omega \begin{bmatrix} 0 & \rho \\ \frac{p_3^2}{\rho} & 0 \end{bmatrix} \begin{bmatrix} \tilde{P} \\ v_3 \end{bmatrix}. \tag{41}$$

In reflection seismology, we are interested in up-going reflections from boundaries of interest, and down-going waves are a component of seismic noise. We seek, therefore, filter $\mathbf{L}$ such that (Amundsen, 2001)

$$\begin{bmatrix} \tilde{P} \\ v_3 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} U \\ D \end{bmatrix}, \tag{42}$$

where

$$\mathbf{L} \Leftrightarrow \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}, \tag{43}$$

and U and D are the up-going and down-going wavefields respectively. If $\mathbf{L}$ can be determined, and if it's inverse $\mathbf{L}^{-1}$ exists, then U and D can be separated.

Assume, then, that $\mathbf{L}$ is independent of x_3 in homogeneous media and substitute equation 42 for $[UD]^T$ in equation 41, and then left multiply by $\mathbf{L}^{-1}$ to get

$$\frac{\partial}{\partial x_3}\mathbf{W} = i\,\omega\, \left[\mathbf{L}^{-1}\,\mathbf{A}\right] \left[\mathbf{L}\,\mathbf{W}\right], \tag{44}$$

where

$$\mathbf{W} \Leftrightarrow \begin{bmatrix} U \\ D \end{bmatrix}, \tag{45}$$

and

$$\mathbf{A} \Leftrightarrow \begin{bmatrix} 0 & \rho \\ \frac{p_3^2}{\rho} & 0 \end{bmatrix}. \tag{46}$$

Apply the associative law of matrices to equation 44 to get

$$\frac{\partial}{\partial x_3}\mathbf{W} = i\,\omega\, \left[\mathbf{L}^{-1}\,\mathbf{A}\,\mathbf{L}\right]\, \mathbf{W}. \tag{47}$$

Eigenvalue-value decomposition of $\mathbf{A}$ gives (Amundsen, 2001)

$$\mathbf{A} = \mathbf{B}\,\mathbf{D}\,\mathbf{B}^{-1}, \tag{48}$$

where

$$\mathbf{B} = [\mathbf{e}_1|\mathbf{e}_2]\,, \tag{49}$$

has columns made up of the eigenvectors $\mathbf{e}_1$ and $\mathbf{e}_2$ of $\mathbf{A}$, and

$$\mathbf{D} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}, \tag{50}$$

is a diagonal matrix made up of the eigenvalues λ_1 and λ_2 of $\mathbf{A}$. Substitute equation 48 for $\mathbf{A}$ in equation 47 to get

$$\frac{\partial}{\partial x_3}\mathbf{W} = i\,\omega\,\left[\mathbf{L}^{-1}\,\mathbf{B}\,\mathbf{D}\,\mathbf{B}^{-1}\,\mathbf{L}\right]\,\mathbf{W}. \tag{51}$$

From the requirement that

$$\det\{\mathbf{D} - \mathbf{A}\} = 0, \tag{52}$$

we have

$$\mathbf{D} = \begin{bmatrix} \lambda_1 = -p_3 & 0 \\ 0 & \lambda_2 = p_3 \end{bmatrix}. \tag{53}$$

Then, from equations 51 and 53, wave equation 41 becomes (Amundsen, 2001)

$$\frac{\partial}{\partial x_3}\mathbf{W} = i\,\omega\,\mathbf{D}\,\mathbf{W}. \tag{54}$$

Equation 54 implies $[\mathbf{L}^{-1}\,\mathbf{B}\,\mathbf{B}^{-1}\,\mathbf{L}] = \mathbf{I}$, and this is satisfied by the following definition (Amundsen, 2001):

$$\mathbf{B} \equiv \mathbf{L} = [\mathbf{e}_1|\mathbf{e}_2]\,. \tag{55}$$

Eigenvectors $\mathbf{e}_1$ and $\mathbf{e}_2$ are computed according to (Amundsen, 2001)

$$[-p_3\,\mathbf{I} - \mathbf{A}]\,\mathbf{e}_1 = \mathbf{0}, \tag{56}$$

and

$$[p_3\,\mathbf{I} - \mathbf{A}]\,\mathbf{e}_2 = \mathbf{0}, \tag{57}$$

where $\mathbf{I}$ and $\mathbf{0}$ are the identity matrix and the zero matrix respectively. Equations 56 and 57 have no explicit solution, however, but they do restrict the elements with in $\mathbf{e}_1$ and $\mathbf{e}_2$ as follows:

$$\mathbf{e}_1 \Leftrightarrow e_{11}\begin{bmatrix} 1 \\ -\frac{p_3}{\rho} \end{bmatrix}, \tag{58}$$

and

$$\mathbf{e}_2 \Leftrightarrow e_{21}\begin{bmatrix} 1 \\ \frac{p_3}{\rho} \end{bmatrix} \tag{59}$$

where e_{11} and e_{21} are unknown (for now) scalars, and

$$\mathbf{L} = [\mathbf{e}_1|\mathbf{e}_2] \Leftrightarrow \begin{bmatrix} e_{11} & e_{21} \\ \frac{-e_{11}}{Z} & \frac{e_{21}}{Z} \end{bmatrix}, \tag{60}$$

where $Z = \frac{\rho}{p_3}$. For a hydrostatic medium, normalized energy flux density in the x_3 direction is given by Pedersen et al. (2007)

$$\xi_3 = -\frac{1}{4}\left[\tilde{P}\, v_3^* + \tilde{P}^*\, v_3\right]. \tag{61}$$

From equation 42, equation 61 is computed

$$\xi_3 = -\frac{1}{4}\left[\begin{array}{c} (L_{11}\, U + L_{12}\, D) \\ (L_{21}\, U + L_{22} D) \end{array}\right]^T \left[\begin{array}{c} (L_{21}\, U + L_{22}\, D)^* \\ (L_{11}\, U + L_{12} D)^* \end{array}\right]. \tag{62}$$

From equation 60, Substitute $L_{11} = e_{11}$, $L_{12} = e_{21}$, $L_{21} = -e_{11}/Z$, and $L_{22} = e_{21}/Z$ in equation 62, and then collect terms of $U\,U^*$, $U\,D^*$, $D\,U^*$, and $D\,D^*$ to get

$$\xi_3 = \frac{1}{2\,Z}\left[e_{11}\, e_{11}^*\, U\, U^* - e_{21}\, e_{21}^*\, D\, D^*\right], \tag{63}$$

where we assume $Z = Z^*$. (Note, $Z = Z^*$ corresponds to real valued ρ and non-evanescent propagation.) According to equation 60, we are free to choose e_{11} and e_{21}, so

$$\xi_3 = \left[U\, U^* - D\, D^*\right], \tag{64}$$

is satisfied for (Pedersen et al., 2007)

$$e_{11} = e_{11}^* = e_{21} = e_{21}^* = \sqrt{2\,Z}. \tag{65}$$

From equation 60, we may now write $\mathbf{L}$ explicitly as

$$\mathbf{L} \Leftrightarrow \sqrt{2}\left[\begin{array}{cc} \sqrt{Z} & \sqrt{Z} \\ -\frac{1}{\sqrt{Z}} & \frac{1}{\sqrt{Z}} \end{array}\right], \tag{66}$$

and for $\mathbf{L}^{-1}$ we have

$$\mathbf{L}^{-1} \Leftrightarrow \frac{\sqrt{2}}{4}\left[\begin{array}{cc} \frac{1}{\sqrt{Z}} & -\sqrt{Z} \\ \frac{1}{\sqrt{Z}} & \sqrt{Z} \end{array}\right]. \tag{67}$$

From equation 42 and 67, then, we have for up-going waves (Pedersen et al., 2007)

$$U = \frac{1}{2\sqrt{2\,Z}}\left[\tilde{P} - v_3\, Z\right], \tag{68}$$

and for down-going waves (Pedersen et al., 2007)

$$D = \frac{1}{2\sqrt{2\,Z}}\left[\tilde{P} + v_3\, Z\right]. \tag{69}$$

DISCUSSION

Equations 68 and 69 provide a starting point from which prestack depth migration may be developed. Operator D (equation 69) is now available to derive the source wavefield of depth migration, and U (equation 68) is now available to condition the recorded wavefield. Because both D and U are developed strictly for the acoustic case, they should perform well in imaging for smooth media variations. Future application to prestack depth migration are planned. The expectation is that, for media that vary with depth, amplitude fidelity to 90 degrees will be preserved using U and D in the provision of starting wavefields. Unclear, for the present, is accuracy in media that vary in all coordinates.

CONCLUSIONS

Operators for the separation of up and downgoing wavefields are developed. These operators are valid for acoustic media and so they are suitable for use in the provision of starting wavefields for common methods of prestack imaging. The operators are developed from the linearized wave equations of acoustic media with planewave decomposition of pressure and displacement. The resulting set of coupled differential equations for pressure and vertical displacement are then compared to a conservative relation derived from normalized energy flux density.

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