Multigrid surface consistent deconvolution

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ABSTRACT

This paper addresses the relative ability of Gauss-Seidel, conjugate gradient and multigrid methods in solving the surface consistent equations. Multigrid methods provide superior solutions, especially in the intermediate wavelengths in the frequency domain (Millar and Bancroft, 2004), (Millar and Bancroft, 2006).

The importance of the errors in these intermediate wavelengths in the deconvolution of seismic data is demonstrated on synthetic data. Multigrid and Gauss-Seidel surface consistent operators improve on trace by trace deconvolution when ground roll is included in the model.

Adding offset and midpoint consistent terms in the equations helps further remove the effect of ground roll from the surface consistent deconvolution operators. The multigrid deconvolution operators are more stable, and produce a more white spectrum than Gauss-Seidel operators. This leads to improved resolution throughout the seismic section.

The intent of this paper is to encourage the use of four term surface consistent deconvolution of seismic data aquired on land. Additionally, we suggest that the calculation of the operators used for surface consistent deconvolution can be improved using a multigrid method, as opposed to a Gauss-Seidel or conjugate gradient method.

INTRODUCTION

The surface consistent equations are a useful tool for the processing of any seismic data acquired on land. The equations decompose several types of noise and signal interference into separate channels, base on the surface location of the source and receiver. By projecting the data into separate channels where specific types of noise can be concentrated, this noise can be more precisely estimated and eliminated (Taner et al., 1974), (Taner and Koehler, 1981). Associating a data correction with a physical location and condition in the near surface of the earth gives us more confidence when we make corrections to the the data that we are not arbitrarily changing the signal (Bancroft et al., 2000).

The surface consistent method assumes that most of the signal degradation is confined to the weathering layer at the surface of the Earth. The quality of the signal that is recorded is a combination of a unique source signature, and the conditions immediately surrounding the geophone.

In the case of finding a static correction, an automated process (autocorrelation or a max stack power optimization) provides an estimate of the time shift to apply to each trace. The surface consistent equations are used to separate this total static into components specific

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to the source and receiver. The static shift applied to each trace is the sum of the source and receiver pair that contribute to the trace.

For estimating deconvolution operators, a system of equations is solved to determine the amplitude of each frequency in the signal.

By combining the surface consistent solutions for each frequency, an amplitude signature for each unique source and receiver is provided. The amplitude spectrum of the total deconvolution operator for a trace is a product of the 2 channels, determined by the source and receiver pair that contributes to the recorded trace.

The surface consistent equations are singular, non-diagonally dominant, and difficult to solve. A direct solution is not available, and iterative methods are subjected to instabilities that harm the solution (Taner et al., 1974). Traditional commercial seismic processing software uses an iterative method , either Gauss-Seidel or conjugate gradients, to solve the system. Gauss-Seidel and conjugate gradient are limited in their ability to resolve wavelengths in the solution longer than the width of their operator (Trottenberg et al., 2001), (Press et al., 1992).

For the purpose of resolving the intermediate wavelengths in the solutions we employ a multigrid method. Using a combination of filtering and down-sampling, the system of equations can be reduced to a smaller number of equations and unknowns. We solve for the longer wavelengths in the solution on this coarse grid, then interpolate the result to the original sample rate, and use it as an initial estimate of the solution for an iterative method.

The goal of this paper is to illustrate the need for improved solutions to the surface consistent equations, and demonstrate that these improved solutions can be attained using multigrid methods.

Figure 1 shows the basic synthetic model used througout this paper. All of the receivers on the right hand side of the small survey are planted into a low velocity layer. The large sudden change would reflect a seismic survey entering an area such as a lake, or a bog. There is no sudden change associated with the source.

The ability of our solution method to resolve the large step function in the receiver term will be shown to impact the quality of the data.

After introducing the equations and the solution methods, this paper first shows that the Gauss-Seidel or conjugate gradient methods only resolve the short wavelengths in the solution. Multigrid methods improve the intermediate wavelengths in the solution.

The longest wavelengths (on the order of the length of the entire survey) are impossible to solve for, due to instabilities caused by singularities in the system, (Millar and Bancroft, 2006). Intermediate wavelengths (the cable length or longer) that are not resolved by the mentioned methods do contribute to the resolution of the stacked section. To demonstrate how an error in the medium wavelength of the solution can manifest itself in seismic data example, a statics solution is modelled from the errors and applied to a synthetic common midpoint stacked seismic section.

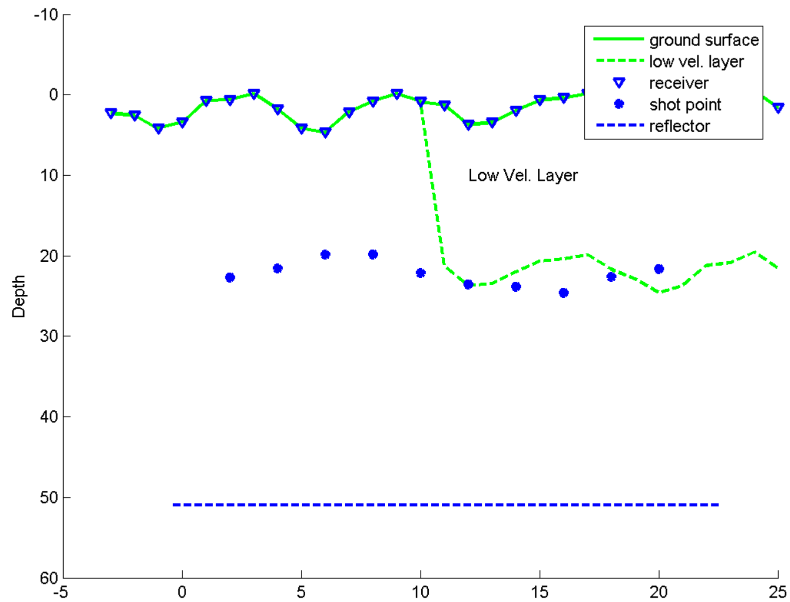

FIG. 1. A diagram of the physical model that we use for the synthetic surface consistent system

Following this, a surface consistent deconvolution model is developed. A separate source signature is calculated for each shot, and a Q attenuation factor is assigned based on the receiver location. The amplitude spectrum of each trace in the synthetic data set is the product of the source signature and receiver attenuation factor for the source receiver pair that contribute to the trace.

The first model is a noise free section with a single reflector at constant time. This model contains a large surface consistent anomaly. In the absence of noise the trace by trace deconvolution provides an optimal solution.

The utility of using surface consistent deconvolution (as opposed to trace by trace deconvolution) is its ability to reject coherent noise that acts in a surface consistent manor. A second model is presented, whereupon a modest amount of ground roll is introduced to interfere with the deconvolution operators. Ground roll is an offset consistent noise component (Cary and Lorentz, 1993) that effects trace by trace deconvolution operators, but can be accumulated in an offset component in surface conistent deconvolution. This keeps ground roll from influencing the deconvolution operators.

Finally, a third and fourth term are added to the surface consistent equations. The ground roll tends to collect in the offset consistent term (Cary and Lorentz, 1993). Giving coherent noise its own channel removes the influence it may have on the calculation of shot and receiver operators used for the deconvolution. Furthermore, a small amount of random noise is added to see its effect on the data.

THE SURFACE CONSISTENT EQUATIONS

To derive the surface consistent equations we assume that the amplitude of a particular frequency in the embedded wavelet $A_w(f)$, in a particular trace is the product of the amplitude of the source wavelet $A_s(f)$ and the attenuation factor due to the receiver $A_r(f)$ at that frequency.

$$A_w(f) = A_s(f) \times A_r(f). \tag{1}$$

To express this product as a sum, take the natural logarithm of each side of equation 1.

$$\log A_w(f) = \log A_s(f) + \log A_r(f). \tag{2}$$

For each trace in the seismic survey we write a form of equation 2 , and assemble all of the equations into a matrix operation,

$$\mathbf{Ax} = \mathbf{b}. \tag{3}$$

To do this, assign each of the unknown shot and receiver amplitudes a column in the matrix. Each row corresponds to a different equation of the form 2. All of the unknown values (A_s and A_r) are factored out, and lined up in the unknown vector $\mathbf{x}$. Each row of the matrix $\mathbf{A}$ contains a 1 in the column corresponding to the shot and receiver that each contribute to the trace. The source term $\mathbf{b}$ is the measured amplitude value from the trace.

There are far more traces (equations) than there are unknowns (shot and receiver stations), so a least squares solution is sought.

$$\mathbf{A^T Ax} = \mathbf{A^T b}. \tag{4}$$

The value in the i^{th} index of the vector $\mathbf{A^T b}$ is the total value of all of the traces that contribute to the i^{th} unknown value in the vector $\mathbf{x}$. For instance, the first entry of $\mathbf{A^T b}$ is the sum of all values in the traces that contribute to the first shot gather, directly across from the first unknown $\mathbf{x}$, the first shot unknown.

The matrix $\mathbf{A^T A}$ is rank defficient by 1. This restricts the number of available methods we can use to solve it. Direct methods such as Gaussian elimination will not provide a result. Iterative methods will provide a solution, but there will be a degree of uncertainty left in the solution.

To perform a surface consistent deconvolution, we solve equation 4 for each frequency. Combining all of the frequencies that correspond to a particular shot leads to an evaluation of the amplitude spectrum of the source. Likewise, we also get an approximation of the filtering effect each receiver has on all traces that are measured by that receiver. By combining the effect due to the source and due to the receiver for each trace we get the surface consistent deconvolution operator.

SOLVING THE SYSTEM

The transpose estimate is the initial estimate which is based on the approximation

$$\mathbf{D^{-1}A^{T}A} \approx \mathbf{I}, \tag{5}$$

where $\mathbf{D}$ is a matrix whose non-zero coefficients are confined to the main diagonal, and whose main diagonal is equal to the main diagonal of $\mathbf{A^{T}A}$.

The surface consistent statics routine packaged in Seismic Un*x, as well as the time variant surface consistent Gabor deconvolution proposed by Montana et al. (2006) use an averaging process equivalent the transpose estimate. Both routines calculate the source consistent correction by calculating the mean value for all of the traces in the shot gather, and likewise for the receiver operator.

Traditionally the solution is refined using a Gauss-Seidel method. The Gauss-Seidel method is an iterative method. This means that rather than finding an exact solution, an approximate solution is repeatedly refined either a selected number of times, or until it falls within some error tolerance.

Expanding the i^{th} equation in the system in equation 4,

$$\tilde{x}_{n+1}(i) = \frac{b - \mathbf{A}(i,1)\tilde{x}_{n+1}(1) - \mathbf{A}(i,2)\tilde{x}_{n+1}(2) - ... - \mathbf{A}(i,N)\tilde{x}_{n}(N)}{\mathbf{A}(i,i)}. \tag{6}$$

The Gauss-Seidel method cycles through each of the unknowns, and corrects the previous estimate of the solution based on equation 6. The subscript $n+1$ on the right hand side of equation 6 is there because with each application of the iterative method, we use the updated values as soon as they become available.

Gauss-Seidel operators, as well as other iterative methods, can have difficulty solving the long wavelength terms in a solution (Press et al., 1992). Multigrid methods are an attractive option for resolving the long wavelength termss of a solution.

Multigrid methods work by antialias filtering all of equation 4, and downsampling. After a low pass filter is applied to the source term $\mathbf{A^{T}b}$, the vector can be re-sampled at a more coarse rate. The solution $\mathbf{x}$ and matrix $\mathbf{A^{T}A}$ are both similarly filtered and re-sampled.

This process, called *restriction*, reduces the number of unknowns and increases the effectiveness of the Gauss-Seidel operator on the longer wavelengths in the solution.

A basic multigrid implentation would apply several Gauss-Seidel corrections to the reduced system. This coarse grid estimate would capture the long wavelengths in the solution. This coarse estimate can then be interpolated to the original resolution, and used as an initial estimate for a normal Gauss-Seidel correction.

At its most basic level, multigrid methods are a way of applying an iterative method such as Gauss-Seidel, on several scales. This produces superior solutions, without sacrificing a great amount of computer time.

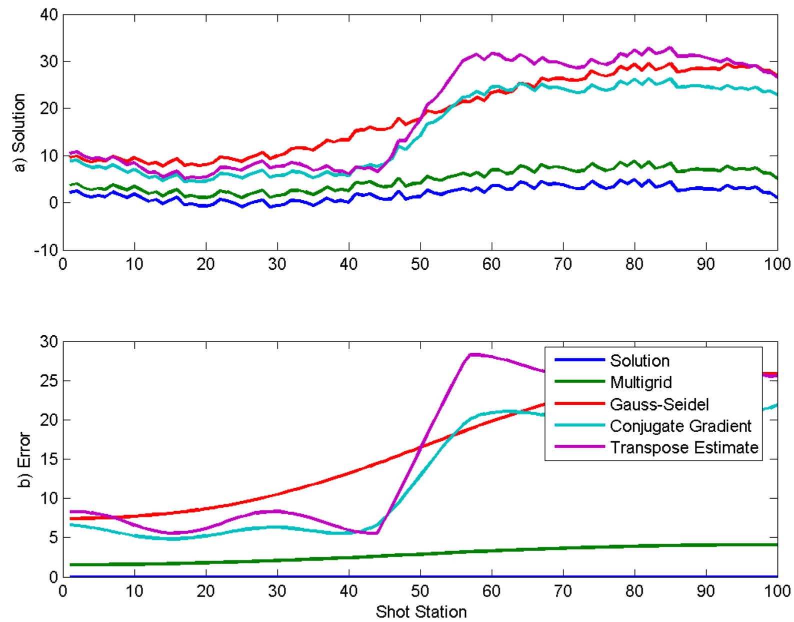

FIG. 2. The calculated source consistent terms in our model

SURFACE CONSISTENT SOLUTIONS

The forward model assigns a value to each of the shots and receivers. To test the algorithms, in particular the ability of the algorithms to evaluate the long wavelengths in the solution, we add a strong step function to the receivers. This simulates the survey crossing a boundary where there is a large low velocity or absorptive layer covering half of the survey, such as a marsh, or sand dune. This is diagrammed in Figure 1. The model is designed to emphasize the important role the intermediate wavelengths play in the resolution of the stack section.

The synthetic survey we use has 100 separate shots. Each shot has 51 live receivers centered on the shot. Every shot moves forward 4 stations at a time. This leads to 450 separate receiver stations.

In Figures 2 and 3, the solutions and the error for a number of methods are both plotted. Figure 2 shows the shot consistent solutions and errors, and Figure 3 shows the receiver consistent solutions and errors.

In Figure 2 the transpose estimate has the worst error. This error is caused by the jump in the receiver term. All of the shots below the low velocity zone are wrongly influenced by the step found in the receiver term. In Figure 3 the transpose estimate does manage to do a good job of predicting the receiver term accurately, in the absence of any significant long wavelength changes in the source term.

The following applications of the iterative methods all have the effect of smoothing this error out across the rest of the solution. The error in the source term of the transpose estimate is quickly pushed into the receiver term as well, as the solution is refined by the

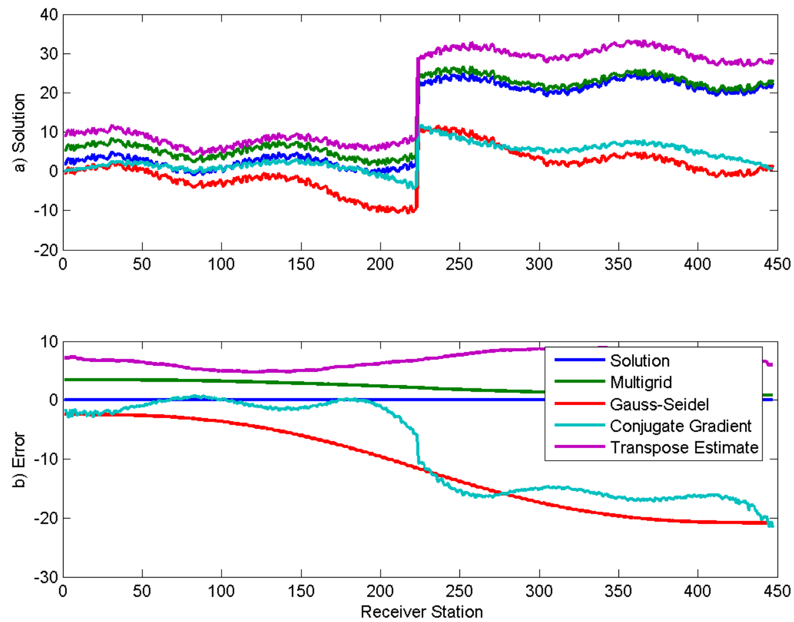

FIG. 3. The calculated receiver consistent terms in our model

first few iterations. Once this error stabilizes across the entire domain, it is smoothed out by the iterative methods. The high frequency components are attenuated quickly, while the long wavelength component of the error is persistent, and is only reduced very slowly. This limitation is due to their spectral performance (Millar and Bancroft, 2006).

The conjugate gradient method performs the worst. Gauss-Seidel produces a large but smooth error, and the multigrid method provides the least error of all the methods.

The amplitude spectrum of the error is plotted in Figure 4. The multigrid scheme reduces all wavelengths of error by nearly an order of magnitude over the other methods.

The relationship between the shot and receiver error is shown in figure 5. The shot error has a tendency to negatively reflect the receiver error. On first consideration in may look as though the two errors would cancel. However, the result shows that there is a large difference between how two traces with a positive offset and an equal but negative offset, both with the same midpoint coordinate handle the trace. The errors in the positive and negative offset directions do not cancel, but instead are additive.

If we were to model this total error as an error in a surface consistent static correction, the resulting stack section is shown in Figure 6. Each prestack trace is a 40 hertz ricker wavelet time shifted by the sum of the source and receiver error.

A midpoint gather can have a different error for each of the positive and negative offsets. This causes the wavelet in a midpoint gathered stack to be smeared across a time, proportional to the size of the surface consistent anomaly in the receiver term. The width of the smeared part of the section is roughly the width of the live spread of receivers. This synthetic survey had 51 live receivers for each shot, and the damage to the section seems

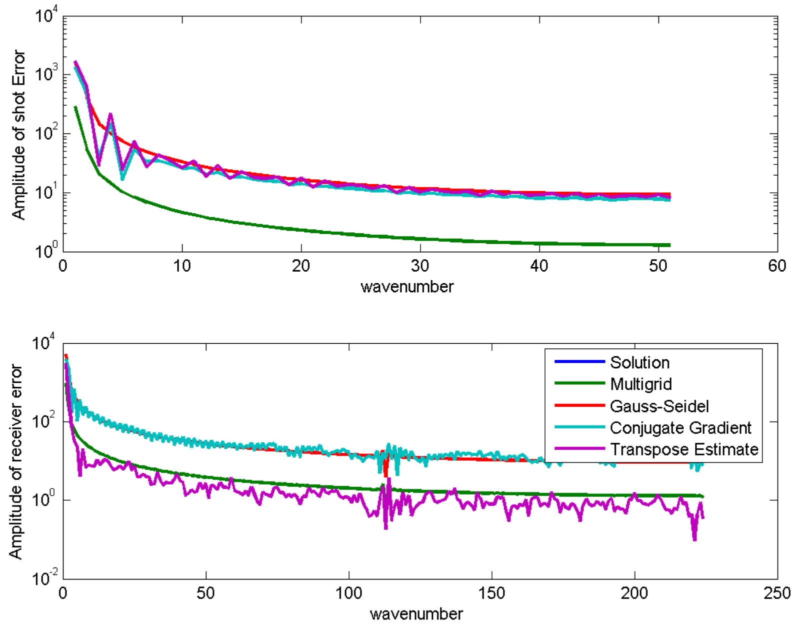

FIG. 4. The amplitude spectrum of the shot (top) and receiver error (bottom) of the 2 term surface consistent model

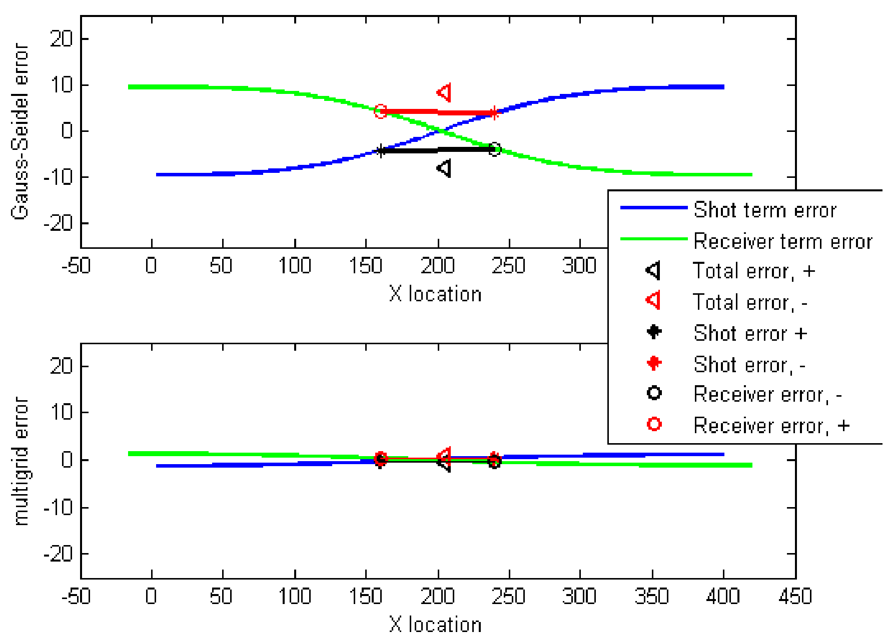

FIG. 5. The shot and receiver error terms for the surface consistent model

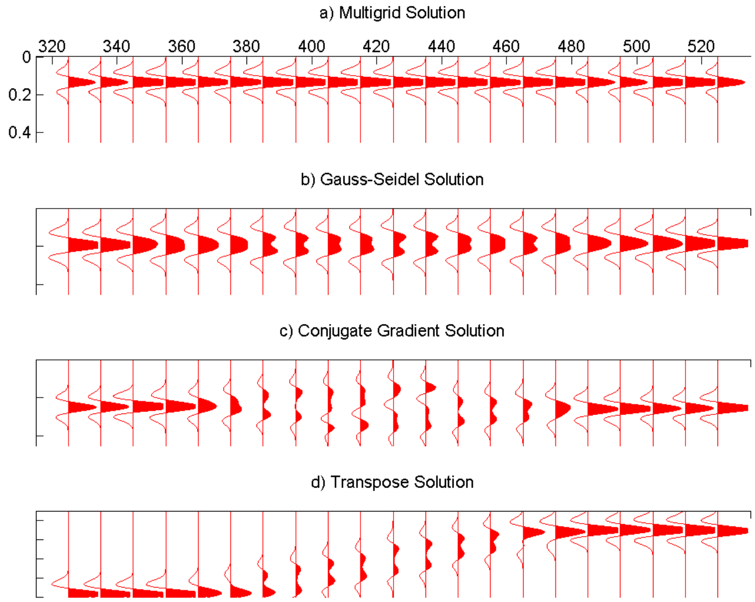

FIG. 6. The effect on the error in our surface consistent static solutions on a midpoint stack section. The x axis labels denote the midpoint station. Not all midpoints are plotted.

to cover about 100 midpoints.

Adding a constant to all shot statics in the solution causes a bulk shift of all of the traces across the entire stack, with no change in structure or resolution throughout. It is the sharp changes in the error across the length of the live spread that contribute to a loss of resolution. The quality of the stack is related to the derivative of the error.

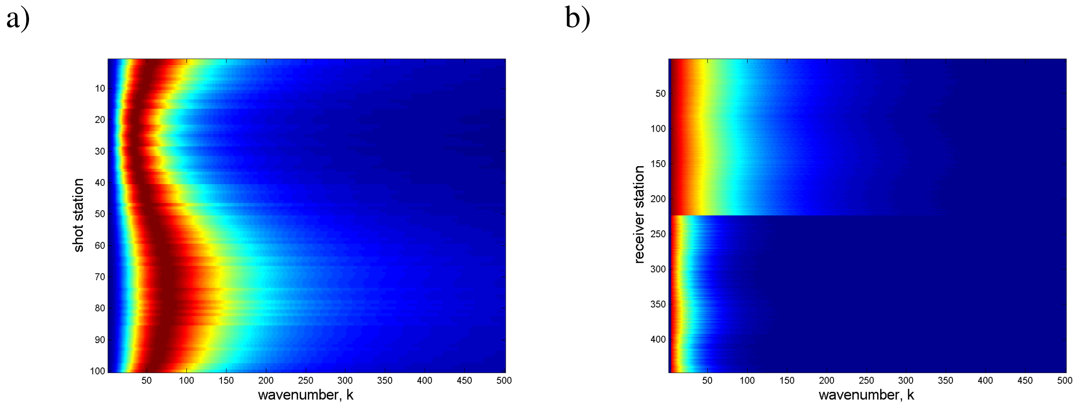

FIG. 7. The frequency spectrum of the shot consistent (a) and receiver consistent (b) solution we use to forward model our surface consistent deconvolution problem

DECONVOLUTION

We wish to test how our results behave themselves in a surface consistent deconvolution process.

The source signature A_s is calculated using the CREWES Matlab routine *tntamp.m*. It uses the formula

$$A_s(f) = \frac{1 - e^{-\frac{f^2}{f_{dom}^2}}}{1 + \frac{f^2}{f_{dom}^2}}. \tag{7}$$

A different dominant frequency f_{dom} is supplied by the surface consistent forward model for each independent source. An amplitude is calculated for each frequency f, and the resulting function is the source spectrum.

For each receiver, the model assigns a value of Q, from which a constant Q filter is calculated using

$$A_r(f) = e^{-f/Q}. \tag{8}$$

This filter is unique to each receiver, and is applied to all traces recorded by a particular receiver.

Each seismic trace in the forward model is generated by convolving a Dirac delta function at a constant time by the shot wavelet, and then filtered using the corresponding Q filter from the receiver. The Q values that we use suffer from a similar jump as the receiver terms in the previous model. The ability of the solution methods to resolve the intermediate wavelengths in the step function, and its effect on the spectrum of the deconvoluted trace is studied.

The true solutions used to forward model the seismic data are displayed in Figure 7. Each multigrid or Gauss-Seidel solution, those like the solutions shown in Figure 2, would comprise a line parallel to the y axis of Figure 7. A line parallel to the x axis through Figure 7 is the frequency domain operator that we use to model a particular source or receiver.

A shot gather showing the raw and deconvolved synthetic data is displayed in Figure 8.

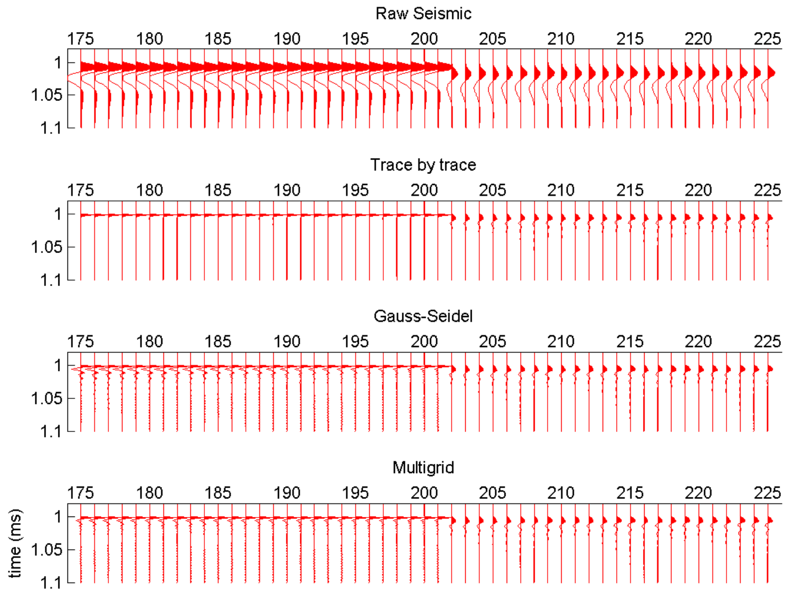

FIG. 8. A shot gather, from shot station 50, showing the results of our different deconvolution methods.

The horizontal axes labels denote the receiver station number for each trace.

The effects of our forward model can be seen in the top axis in the raw synthetic data. The shot record chosen sits directly over top of the receiver consistent jump. All of the positive offset traces can be seen to be suffering from a much stronger attenuation effect. The deconvolution results will be discussed further later in this section.

The surface consistent deconvolution first constructs a matrix $\mathbf{A}^{\mathbf{T}}\mathbf{A}$, and calculates a separate vector $\mathbf{A}^{\mathbf{T}}\mathbf{b}$ for each frequency. The vector $\mathbf{A}^{\mathbf{T}}\mathbf{b}$ is smoothed from frequency to frequency (but not from unknown to unknown) to ensure a smooth spectrum for the wavelet.

Solutions like the ones displayed in Figures 2 and 3 are calculated for each frequency of $\mathbf{b}$.

Once the source and receiver operators are calculated, the process cycles through each trace in the seismic survey. For each seismic trace, the corresponding source and receiver operators are convolved together to form the design trace. The seismic trace and the design trace are input into the CREWES Matlab routine *deconw.m*. This is a standard Weiner Levinson deconvolution routine, that creates a deconvolution operator based on the design trace, and applies the updated operator to the seismic trace. The output of *deconw.m* is the deconvolved seismic trace.

For the trace by trace deconvolution, the seismic trace is used as the design trace, so the Weiner Levinson deconvolution operator is based only on the current input trace.

In the Gauss-Seidel operators, displayed in Figure 9, there is a long wavelength insta-

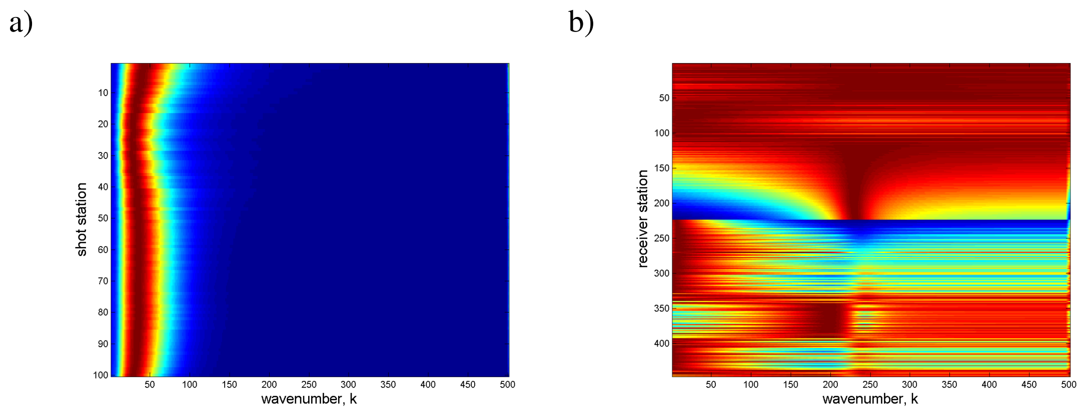

FIG. 9. The frequency spectrum of the shot consistent (a) and receiver consistent (b) solution calculated using the Gauss-Seidel method

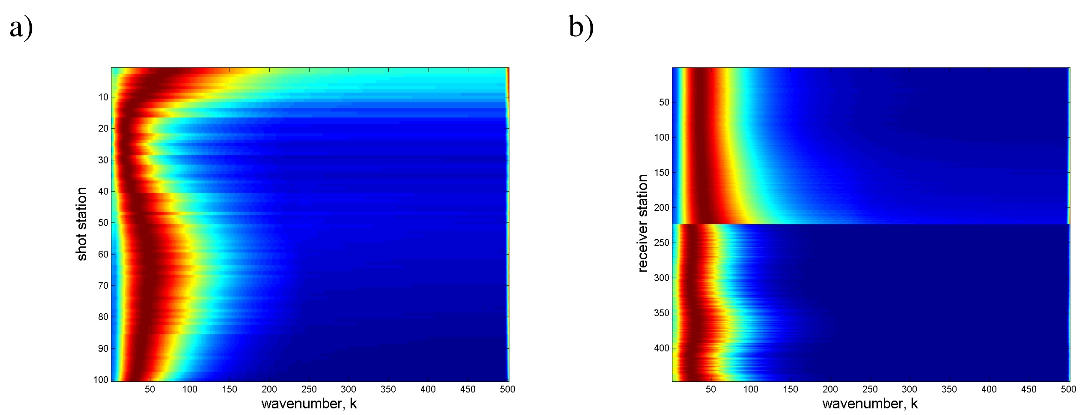

FIG. 10. The frequency spectrum of the shot consistent (a) and receiver consistent (b) solution calculated using the multigrid method

bility in the receiver term. The shot spectrum is still quite smooth, and captures in general the shot operator, but underestimates it. Most of the error in this problem is pushed into the receiver term. This is because the Gauss-Seidel operation addressed the shot operator first, forcing it to be a smooth function. Once the general shape of the shot operator is determined in the first iteration, the Gauss-Seidel method cannot smooth out the medium wavelengths in the receiver term. The error in the shot consistent term spreads out into the receiver term with subsequent iterations.

The shot and receiver operators calculated by the multigrid method are displayed in Figure 10. The shot and receiver terms are both significantly improved by the multigrid method. Even the sharp jump in the receiver term is well imaged.

Looking at a common midpoint gather, in Figure 11, we can judge the effectiveness of the deconvolution algorithms. This is a midpoint gather around station 445. The Gauss-Seidel and multigrid surface consistent deconvolution are very close. Both are outperformed by the trace by trace deconvolution. Offsets 6 and 7 in the midpoint gather demonstrate that the multigrid operators near the surface consistent anomaly are better able to whiten individual traces than the Gauss-Seidel method. This error in the statics example caused the smearing of the wavelet in the stacked section. The effect of these errors on the

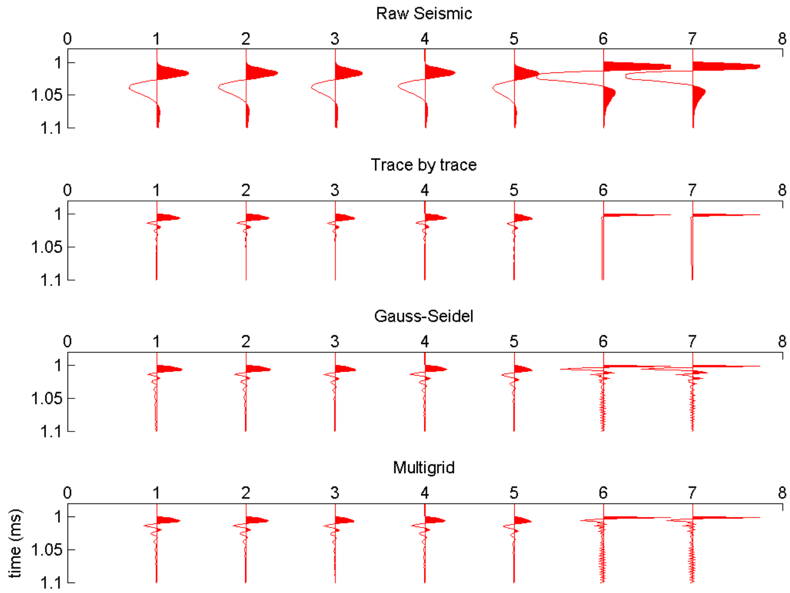

FIG. 11. A common midpoint gather near the receiver consistent anomaly, raw and deconvolved data. The midpoint station number is 445

deconvolution operators is minimal.

In this example, there is no coherent or random noise. In the absence of noise, a trace by trace estimation of the wavelet will provide the optimum deconvolution operator.

The midpoint stacked sections are shown in Figure 12. Judging by the stack section, even though the Gauss-Seidel receiver term seems unreasonable, the error in the shot and receiver terms seem to cancel effectively, and the results are fairly similar.

To see in more detail the difference in the stack sections of the various deconvolution operators, we inspect the stacked trace of a midpoint in Figure 13 . Midpoint station 445 is chosen, and is just to the right of the large receiver anomaly. The Fourier transform of these traces is found in Figure 14.

The improvement of the multigrid operators over the Gauss-Seidel opreators can be seen in many of the single traces of the midpoint gather, however the improvement is suitably small to show almost no difference in the either the quality of the stack or the whiteness of the post stack wavelet. The smearing effect seen in the statics solution is still present, but is not as important in the deconvolution problem, judging from our synthetic results.

The Gauss-Seidel and multigrid surface consistent operators both over-whiten the trace. This can be seen by the effect on the frequency domain. In Figure 14 the higher frequencies of the trace are raised too much relative to the low frequencies . This is specific to this trace. Other midpoint stacked traces produce mildy under-whiten traces as well as other small errors.

The trace by trace deconvolution is far superior to either surface consistent method.

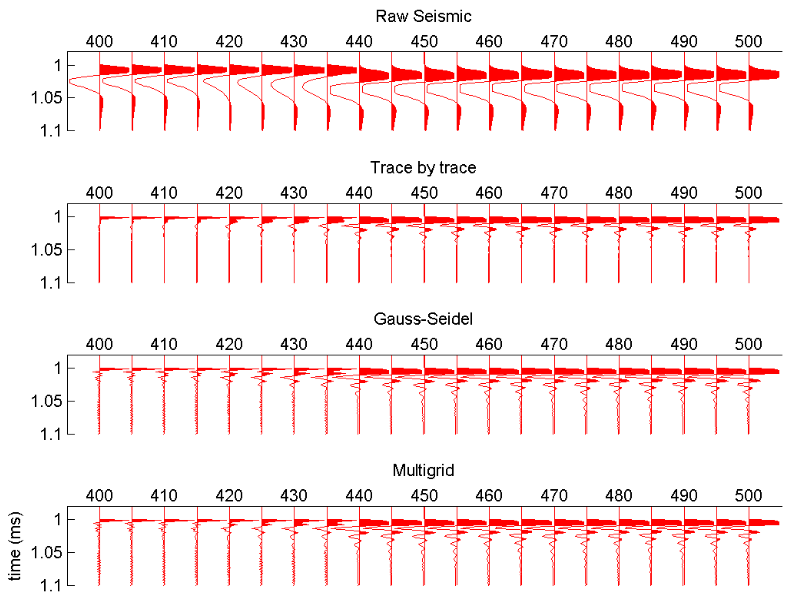

FIG. 12. A common midpoint stack section.

The trace by trace deconvolution does an excellent job of deconvolving all traces. In the absence of any noise the trace by trace deconvolution should be perfect, within the normal limits of resolution.

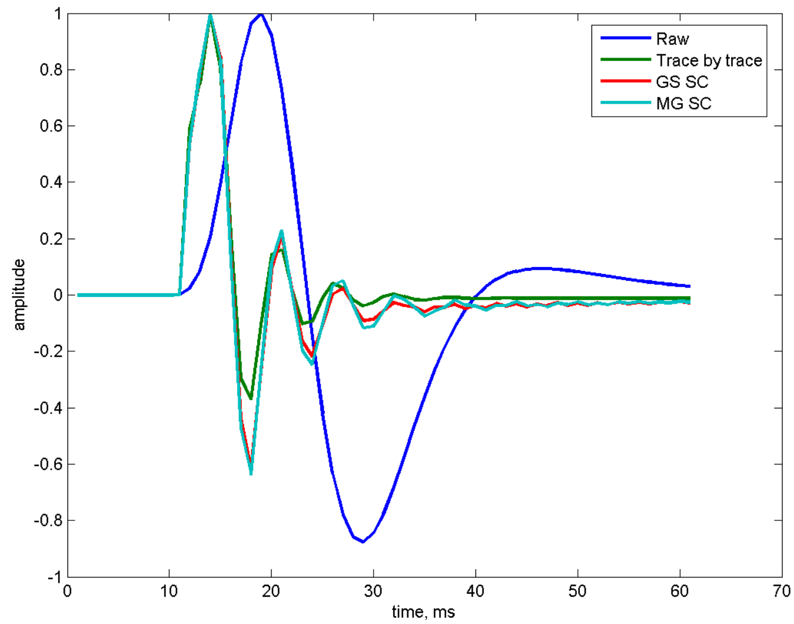

FIG. 13. A stacked midpoint gather, at midpoint station 445

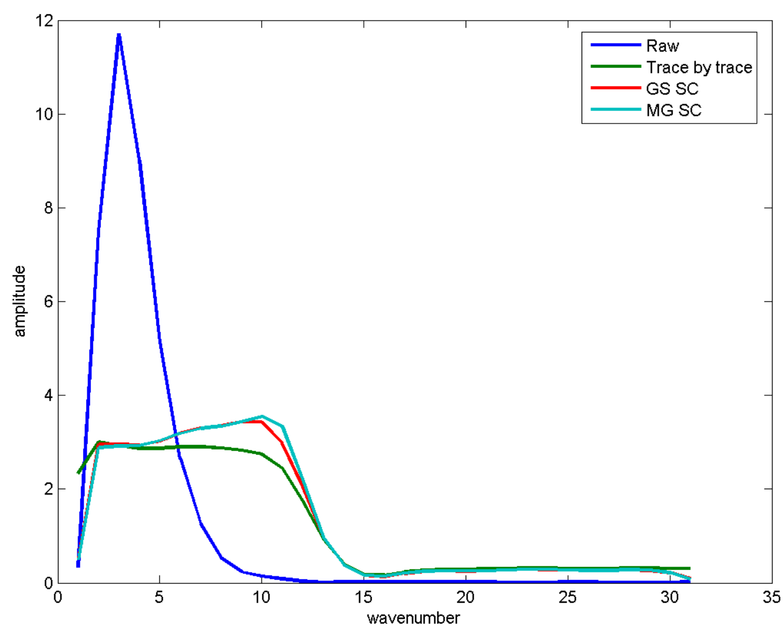

FIG. 14. Amplitude spectrum of the midpoint stacked trace at station 445. The spectrum of the trace by trace deconvolution is more flat than the surface consistent operators.

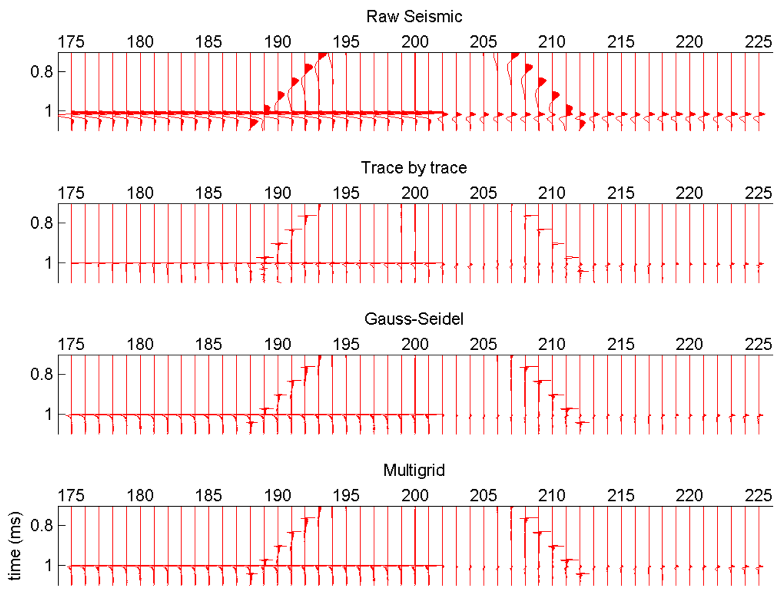

FIG. 15. A shot record, showing the effect of the 2 term surface consistent deconvolution, when ground roll is added to the raw traces.

ADDING GROUND ROLL TO THE MODEL

The advantage of using a surface consistent deconvolution is the ability of the operators to ignore the effects of coherent noise that varies with factors other than the receiver and shot location. Noise features in the data such as ground roll vary with offset.

To demonstrate this ability, a simple model to mimic the effects of ground roll on seismic deconvolution is used. The routine *tntamp.m* is used to calculate a 10 hertz minimum phase wavelet, which is added to each trace. The amplitude of the ground roll has a maximum at the center of the shot record, and decays linearly from 1 to 0 at the edge of the shot gather.

A shot gather including the ground roll model is shown in Figure 15.

In the near offsets, where the ground roll is large, the trace by trace deconvolution cannot distinguish the difference between the ground roll and the reflection. The trace by trace operator is designed partially on the ground roll, skewing the results and not concentrating the deconvolution on the reflector.

The effect that the ground roll has on the surface consistent operators is shown in Figures 16 and 17. The ground roll does degrade the quality of the surface consistent solutions in both the Gauss-Seidel and multigrid case, although the multigrid solutions still appear superior.

A midpoint gather including the ground roll is displayed in Figure 18. The near offsets, where the ground roll is strongest, suffers the most from the inability to properly estimate the wavelet.

a) b)

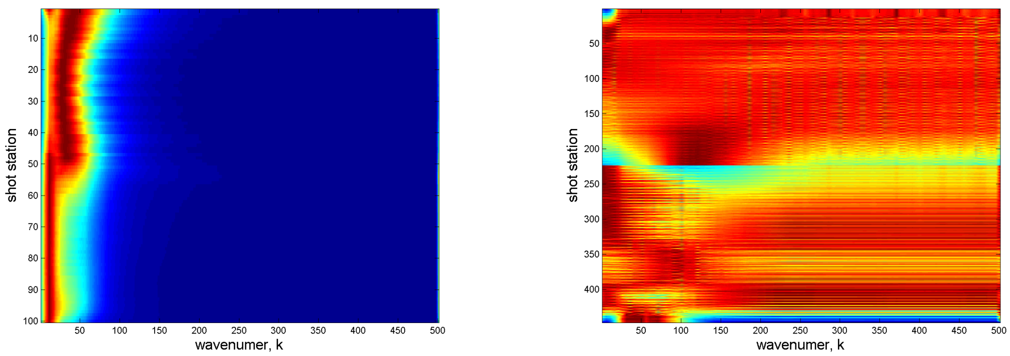

FIG. 16. The frequency spectrum of the shot consistent (a) and receiver consistent (b) solution calculated using the gauss-Seidel method, including ground roll

a) b)

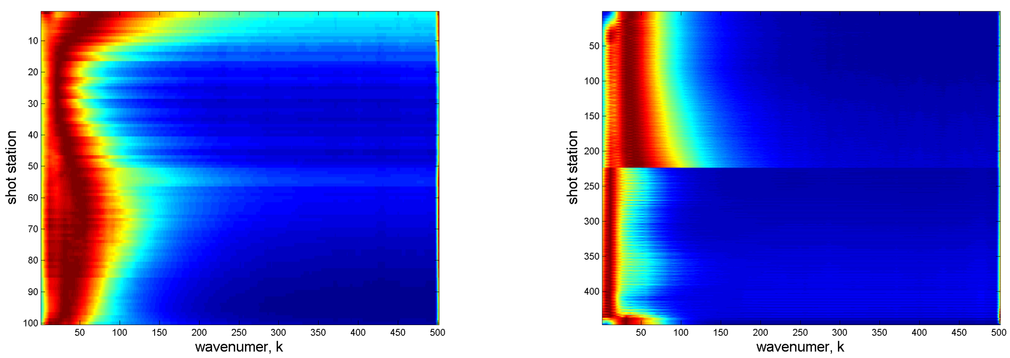

FIG. 17. The frequency spectrum of the shot consistent (a) and receiver consistent (b) solution calculated using the multigrid method, including ground roll

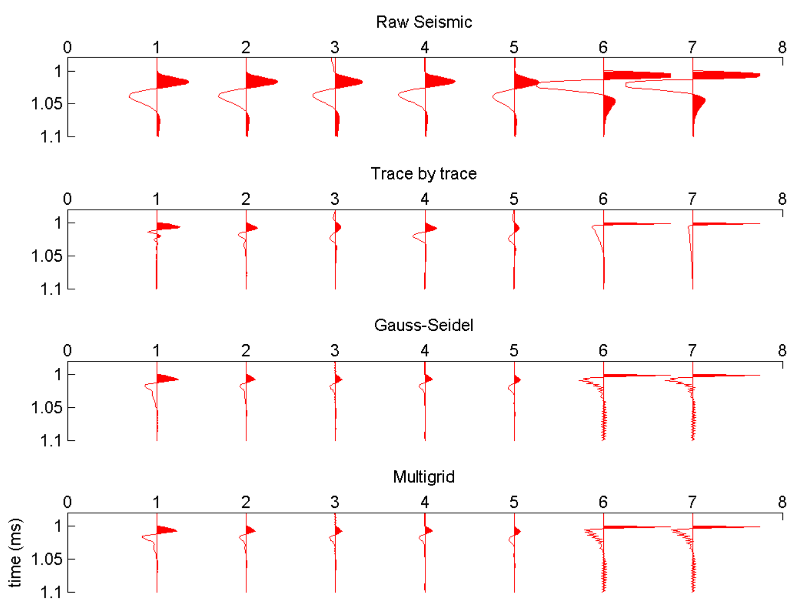

FIG. 18. Midpoint gather showing the results of our 2 term surface consistent deconvolution, when ground roll is added

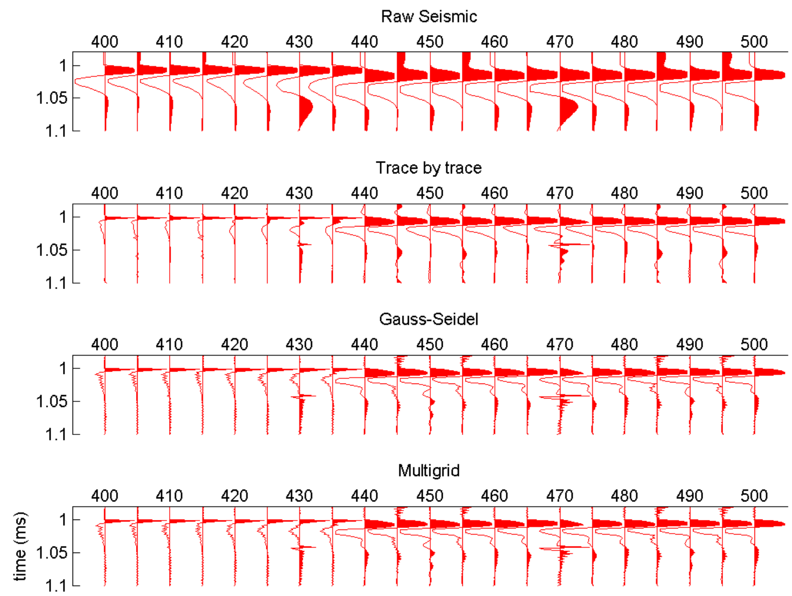

FIG. 19. Midpoint stack section of the example containing ground roll, and the deconvoluted sections using the 2 term surface consistent equations.

The stacked section shown in Figure 19 demonstrates how the ground roll, even though it is not near the reflector, can effect the quality of the stack section. Unlike the previous example, the surface consistent methods seem to mathc the trace by trace deconvolution. Subtle differences are present, but it is unlikely to be able to represent a significant improvement in the stack. The multigrid and Gauss-Seidel solutions appear similar in quality.

In Figure 21, both surface consistent methods are better able to whiten the spectrum of the reflector than the trace by trace deconvolution. Again there is very little appreciable difference between the deconvolution results of the Gauss-Seidel solution and the multigrid solution.

The effect of ground roll masks the spectrum of the reflector in all of the examples. It has the strongest negative effect on the trace by trace deconvolution. In the example without the ground roll, the trace by trace deconvolution was effectively perfect. However, when the ground roll is included in the trace design, it strongly influences the input spectrum of the operator.

The two term surface consistent deconvolution is able to partially ignore the ground roll in the solution. While the presence of ground roll contributed to the error, the negative effect was not as strong on the surface consistent operators as on the trace by trace operators. The spectrum displayed in Figure 21 shows the mild improvement in the whiteness of the spectrum of two term surface consistent operators over trace by trace operators, in the presence of our synthetic ground roll. The appearance of the multigrid operators shows them to have far less error than the Gauss-Seidel operators, However the effect of this on the stack section is limited.

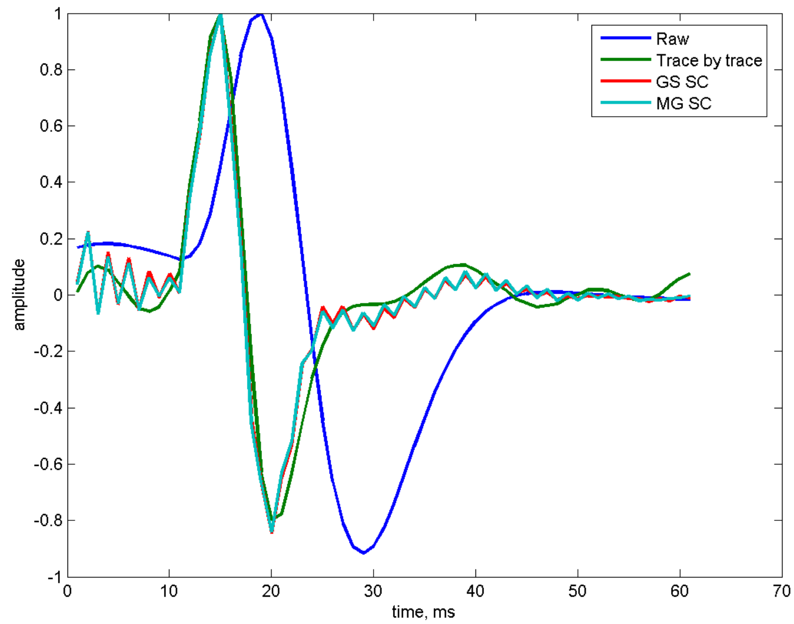

FIG. 20. A stacked trace of midpoint 445, when using a 2 term deconvolution on an example that contains ground roll

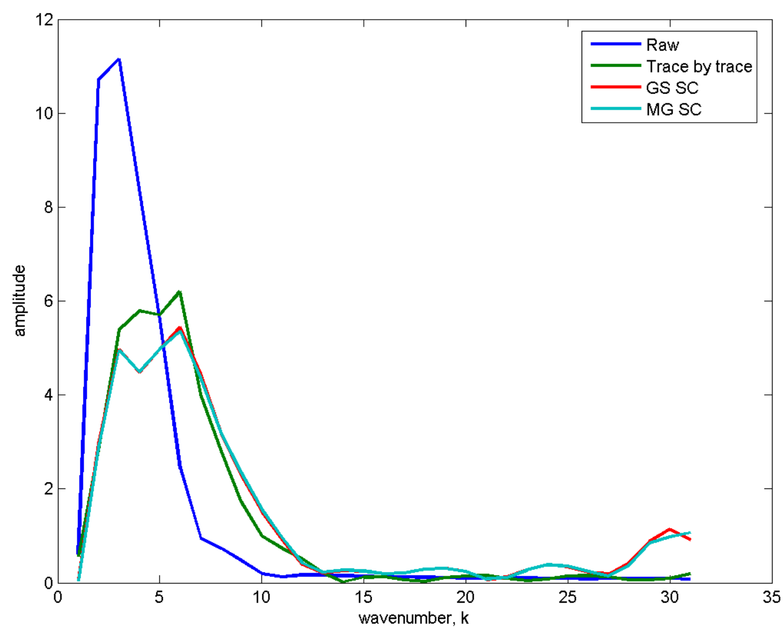

FIG. 21. The amplitude spectrum of the reflector from midpoint 445, including ground roll. The multigrid and Gauss-Seidel surface consistent operators provide a more white spectrum, boosting the frequency content near wavenumber 10 slightly.

FOUR TERM DECONVOLUTION OPERATORS

In the previous section, we saw that two term surface consistent operators were able to partially ignore the effect of the ground roll on the estimated spectra of the reflector wavelet. There was a mild improvement in the calculation of deconvolution operators, based on the whiteness of the spectrum of the stack section.

The effect of ground roll and surface noise is predicted to be offset consistent, meaning that the influence of the ground roll is dependant on the source receiver offset of the trace. By giving the equations an offset channel, it is thought that the ground roll will collect there, and reduce its influence on the shot and receiver deconvolution operators.

We can derive a system of equations that includes the possibility of an offset and midpoint consistent term. The amplitude A_w of the wavelet at frequency f is a product of four unknown quantities, the source A_s, receiver A_r, midpoint A_m and offset A_o consistent terms,

$$A_w(f) = A_s(f) \times A_r(f) \times A_m(f) \times A_o(f). \tag{9}$$

$$\log A_w(f) = \log A_s(f) + \log A_r(f) + \log A_m(f) + \log A_o(f). \tag{10}$$

This increases the number of unknowns in the problem by the number of midpoint stations plus the number of different offsets.

Introducing the extra two terms does come at a cost. The offset and midpoint coordinates are themselves calculated using the shot and receiver coordinates. While introducing a large number of new equations, a number of the new equations themselves can be derived using the existing shot and receiver equations. The new equations are therefore not all unique. Adding superfluous equations raises the number of singularities from 1 in the two term case, to about 10 percent of the total number of unknowns in the four term case (although this can change quite a lot with survey geometry). With the two term solution the one singularity manifests itself as an uncertainty by a constant. The four term matrix has many singularities that lead to an uncertainty by a polynomial (**?**). Each term in the solutions from a 4 term surface consistent inversion often has a quadratic or larger order polynomial the length of the survey superposed on it. To help mitigate this, a damping factor is added to the main diagonal of the matrix. However, the net result is that the very long wavelengths in the solution on the scale of the entire survey are not resolvable.

The additional terms and the new deconvolution operators are displayed in Figures 22 to 26 . Neither the multigrid nor the Gauss-Seidel methods provide an accurate depiction of the offset or midpoint consistent terms. The shot and receiver operators calculated by the multigrid method are still far more accurate than the Gauss-Seidel operators.

The four term surface consistent deconvolution is superior to the trace by trace deconvolution. The stacked trace at midpoint 445 in Figure 27 shows the multigrid solution to

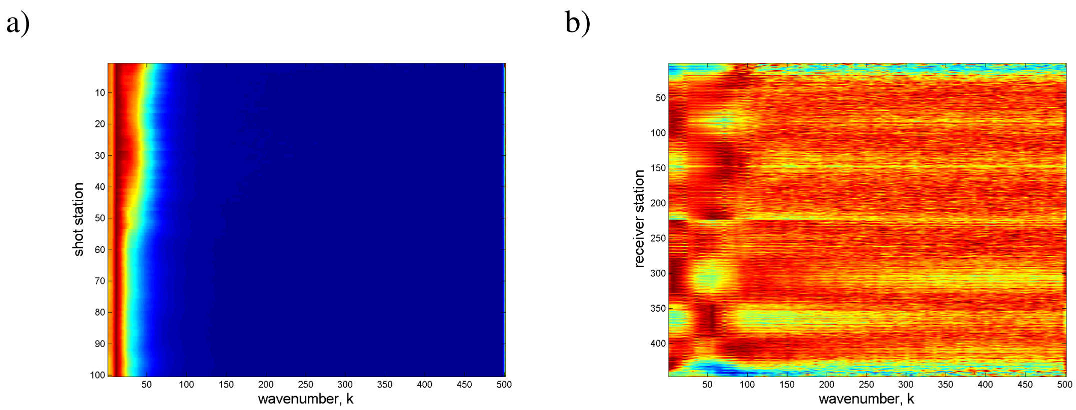

FIG. 22. The four term Gauss-Seidel surface consistent shot (a) and receiver (b) deconvolution operators

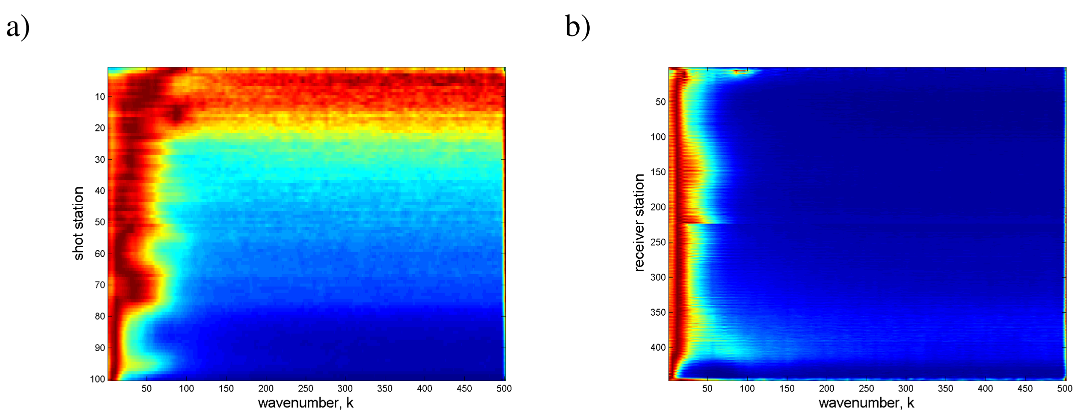

FIG. 23. The four term multigrid surface consistent shot (a) and receiver (b) deconvolution operators

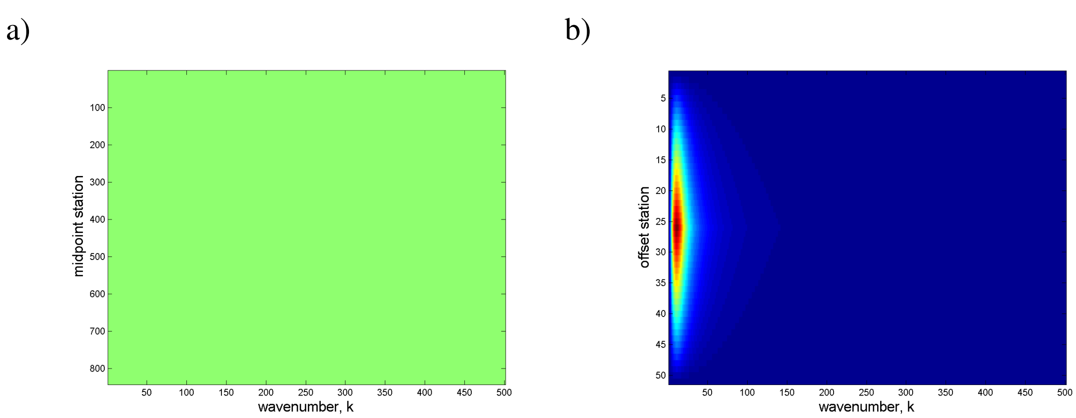

FIG. 24. The midpoint and offset terms used for the forward model of the synthetic seismic survey

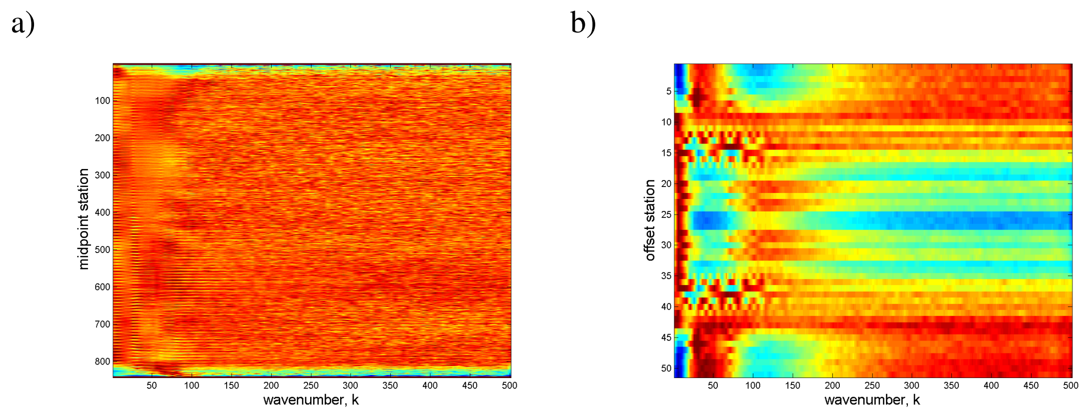

FIG. 25. The midpoint (a) and offset (b) consistent terms, calculated by the Gauss-Seidel method

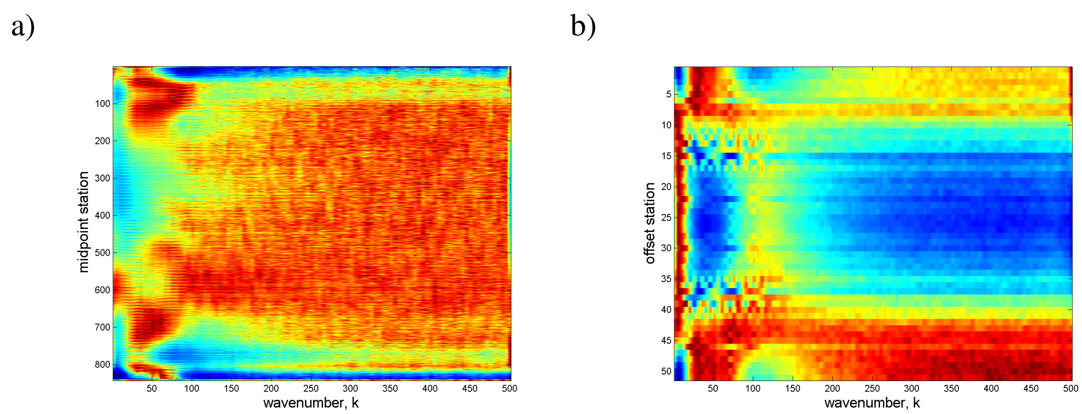

FIG. 26. The midpoint (a) and offset (b) consistent terms as calculated by the multigrid method

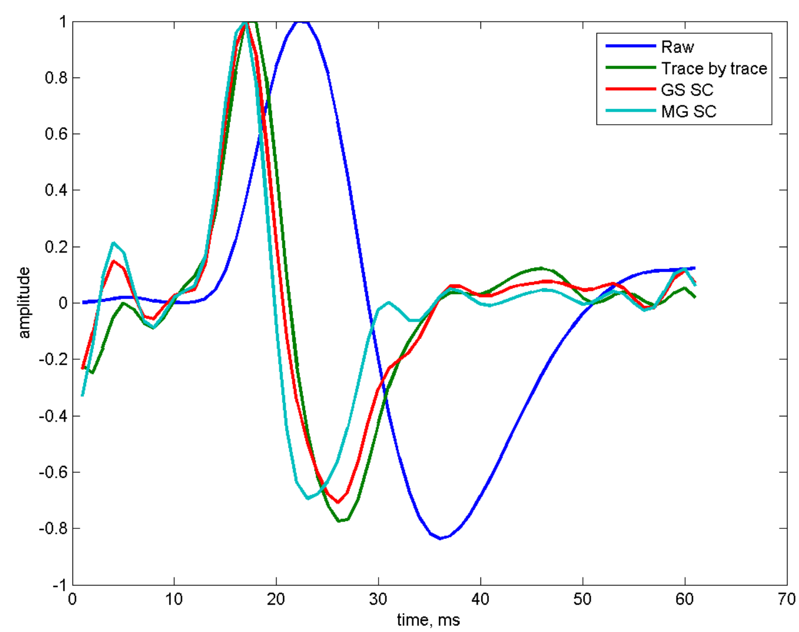

FIG. 27. A stacked trace from midpoint 445 from the synthetic seismic example, deconvolved using the 4 term surface consistent equations

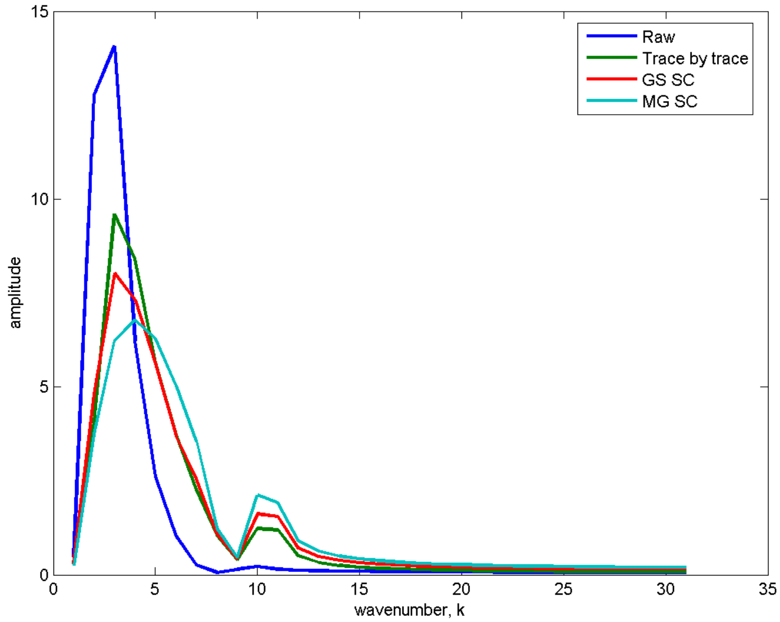

FIG. 28. The Fourier transform of the midpoint stacked trace, including the ground roll, random noise and using the four term surface consistent equations. The surface consistent operators whiten the reflector more than the trace by trace deconvolution, with the multigrid operators boosting frequencies near wavenumber 7 by more than the Gauss-Seidel operators

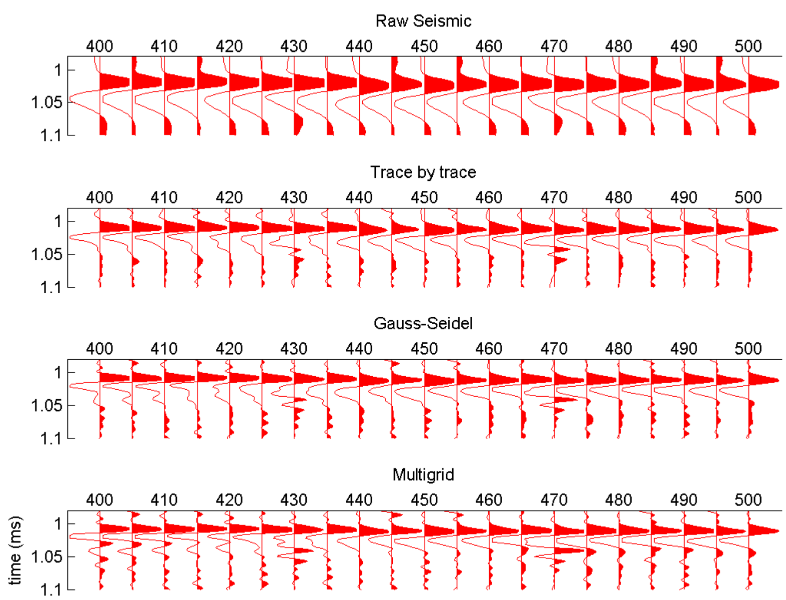

FIG. 29. A midpoint stack section, deconvolved using the four term surface consistent equations

produce a tighter output wavelet than either the trace by trace or the Gauss-Seidel surface consistent operators.

Figure 28 clearly shows the multigrid surface consistent operators consistently provide the reflector with a whiter spectrum more than the Gauss-Seidel operators.

CONCLUSIONS

The multigrid method provides superior solutions to the surface consistent equations when compared to Gauss-Seidel and conjugate gradient methods, at no significant cost in computer time. The intermediate wavelengths are only properly imaged by the multigrid method.

When deconvolution operators are calculated, the errors that plague the Gauss-Seidel method result in unrealistic receiver operators. Judging by the whiteness of the reflector in a stack section, even with the large error in the receiver terms, the Gauss-Seidel solutions do provide an improvement over the trace by trace deconvolution in the presence of our synthetic ground roll. The multigrid solutions provide more accurate deconvolution operators, but the influence on the whiteness of the stack section is minimal.

Near very sudden and persistent changes in the near surface, the errors in the Gauss-Seidel operators have a tendency to smear the estimated source and receiver wavelets. Reflectors below the change in the surface may be over or under deconvolved. The multigrid solutions help to remove some of this smearing that occurs, but the improvement does not appear significant.

The value of the improved multigrid solutions is realized with the inclusion of the midpoint and offset consistent terms. The added instabilities in the matrix due to the new terms are outweighed by the ability of the offset term to collect the ground roll. By absorbing the surface noise into a separate channel, the surface consistent source and receiver operators are calculated more accurately. This synthetic result here largely supports the findings of Cary and Lorentz (1993) and others, in justifying the use of four term surface consistent deconvolution for seismic data aquired on land.

The multigrid method, while not properly reproducing the true offset consistent solution in our synthetic model, appears to better allocate energy into the offset term, keeping it out of the design of the source and receiver deconvolution operators. Based on the whiteness of the stack sections amplitude spectrum, the deconvolution operators provided by the multigrid method improve the deconvolution operators as calculated by the Gauss-Seidel method.

A multigrid method can be easily substituted into the normal land processing flow, at little cost of computer time. Without doing anything significant to change the way data is processed, we can provide superior deconvolution results, which enhances resolution. When extended to surface consistent statics, more of the required static shift can be associated with the shot and receiver locations. This reduces the chance of smoothing over a fault in the subsurface using trim statics. It may also reduce the number of iterations required for the seismic processor to achieve a velocity and statics solution, saving valuable labor costs.

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