Complex spectral ratio method for Q estimation

Peng Cheng and Gary F. Margrave

ABSTRACT

Based on the constant-Q theory, this article presents a complex spectral ratio method for Q estimation. Testing on synthetic examples shows that the complex spectral ratio method can obtain better Q estimates than the conventional spectral ratio method in most cases. Further investigation should be conducted using real VSP data.

INTRODUCTION

As seismic waves propagate through the earth, they experience the absorption of energy and the consequent changes in transient waveform shapes, resulting from the irreversible anelastic behavior of rocks. This energy loss (attenuation) is usually characterized by the quality factor Q defined as the ratio between the energy stored and energy loss per frequency cycle due to anelasticity. The constant-Q model (e.g. Kjartansson, 1979) is a simple and robust description of attenuation, which is based on the assumption of linearity and causality of the material and can be completely specified by two parameters: phase velocity at an arbitrary reference frequency and the quality factor Q. Incorporation of attenuation into seismic trace models can be achieved with the help of constant Q theory. Margrave and Lamoureux (2002) presented a nonstationary convolutional model for attenuated seismic traces using the constant-Q theory and the theory of nonstationary linear filtering (Margrave, 1998).

The spectral ratio method (Haase, A. B., and Stewart, R. R., 2004, and Tonn, R., 1991) is commonly used to estimate Q from VSP data. However, this method only uses the amplitude spectrum of the downgoing wavelet. If we make full use of the complex spectrum, i.e. both amplitude spectrum and phase spectrum, an improved estimation of Q may be obtained.

The main subject of this article is to investigate a complex spectral ratio method for Q estimation. The first part of this article introduces the constant-Q model of seismic attenuation. The next section discusses the algorithm of the complex spectral ratio method. Following that, the performance of different spectral ratio methods is evaluated using synthetic examples. Finally, some basic conclusions are drawn.

CONSTANT-Q MODEL OF ATTENUATION

The constant-Q model (Kjartansson, 1979) predicts the amplitude decay given by

$$A(f,x) = A_0(f)\exp\left(\mathrm{e}\frac{\pi f x}{Qv}\right), \tag{1}$$

where $A(f,x)$ is the amplitude spectrum of the trace with a travelled distance x, $A_0(f)$ the initial amplitude spectrum, Q is the attenuation quality factor, f the frequency, and v the phase velocity. In the constant-Q theory, the earth can be model as a linear filter, which can be fully characterized by the corresponding impulse response. Kjartansson

(1979) gave the impulse response of the attenuating medium in frequency domain as following,

$$B(f) = \exp\,[\mathrm{e}\frac{\pi f x}{Qv(f)}]\exp\,[\mathrm{e}i\frac{2\pi f x}{v(f)}]. \qquad (2)$$

In addition, the dispersion relation between the phase velocity and frequency is an essential part for the constant-Q theory. We use the following dispersion relation (Kjartansson, 1979, Aki and Richards, 2002),

$$v(f) = \ v(f_0)\left[1 + \frac{1}{\pi Q} ln\left|\frac{f}{f_0}\right|\right], \qquad (3)$$

which gives the phase velocity $v(f)$ at any frequency in terms of the phase velocity $v(f_0)$ at an arbitrary reference frequency f_0. When all the frequencies of interest satisfy the condition

$$\frac{1}{\pi Q} ln\left|\frac{f}{f_0}\right| \ll 1, \qquad (4)$$

equation (2) can be approximated by the following formulation with sufficient precision,

$$B(f) \approx \exp\,[-\frac{\pi f x}{Qv(f_0)}]\exp\,[-i\frac{2\pi f x}{v(f_0)}\left(1 - \frac{1}{\pi Q} ln\left|\frac{f}{f_0}\right|\right)]. \qquad (5)$$

If we time-shift the attenuated impulse response to remove the linear phase delay, equation (5) becomes

$$B^{'}(f) \approx \exp\,[-\frac{\pi f x}{Qv(f_0)}]\exp\,[i\frac{2 f x}{Qv(f_0)} ln\left|\frac{f}{f_0}\right|]. \qquad (6)$$

Combining (2) with (3), a nonstationary convolution model for an attenuated seismic trace can be established by convolving the attenuated impulse response with a wavelet, and then a nonstationary convolution of the result with the reflectivity. Margrave and Lamoureux (2002) presented such a model in the frequency domain,

$$\hat{s}(f) = \widehat{w}(f)\int_{-\infty}^{\infty}\alpha_Q(\tau,f)r(\tau)e^{-i2\pi f\tau}d\tau, \qquad (7)$$

where $\hat{s}(f)$ and $\widehat{w}(f)$ are the Fourier spectra of the seismic trace and seismic wavelet respectively, $r(\tau)$ is the reflectivity, and $\alpha_Q(\tau,f)$ is the time-frequency attenuation function given by

$$\alpha_Q(\tau,f) = \exp\,[\mathrm{e}\frac{i\pi f\tau}{Q} + iH\left(\frac{\pi f\tau}{Q}\right), \qquad (8)$$

in which H denotes the Hilbert transform.

Corresponding to equation (6), a discrete formulation of the attenuated seismic trace can be expressed in time domain as

$$S = WQR, \qquad (9)$$

where S is the attenuated seismic trace, W is the Toeplitz-symmetric wavelet matrix, in which each column is a time-shift version of the source wavelet, Q is the attenuation matrix, in which each column is the attenuated impulse response (Q is the inverse DFT over f of the sampled version of equation (8) or equation (6)), and R is the time-domain reflectivity vector. For the examples in this article, we use equation (6) to create the attenuation matrix Q, then multiply it with W to get the attenuated downgoing wavelets W_d as

$$W_d = WQ. \tag{10}$$

Each column of matrix W_d corresponds to an attenuated wavelet with a particular travel-distance. We use equation (10) to create synthetic data for testing the Q estimation algorithms.

As long as the condition given by equation (4) holds, the created synthetic data will be consistent with the constant-Q model. Thus, the key point is to make our attenuation matrix match equation (6) precisely and this requires careful consideration of sampling effects. Let $b^{'}(t)$ be the impulse response that has the Fourier spectrum $B^{'}(f)$, and $b^{'}(n)$ be its discrete version calculated by sampling $B^{'}(f)$ and using IFFT. Since $B^{'}(f)$ is not strictly band-limited, there exists a wrap-around effect at the end of $b^{'}(n)$, i.e. the last part of the series $b^{'}(n)$ has nontrivial values which are not negligible. A problem would arise when we truncate $b^{'}(n)$ in practical processing, which makes the Fourier spectrum of truncated series obviously deviate from equation (6). To avoid this, we make a circular shift to $b^{'}(n)$ and then do the truncation. By doing so, we preserve the nontrivial end values, and only introduce a linear phase shift to $B^{'}(f)$, which can be easily removed by followed process.

COMPLEX SPECTRAL RATIO METHOD FOR Q ESTIMATION

Consider two attenuated wavelets with travel-distance x_1 and x_2 respectively. Their Fourier spectra can be expressed as

$$W_{d1}(f) = W(f)B_1^{'}(f) = W(f)\exp\left[-\frac{\pi f x_1}{Qv(f_0)}\right]\exp\left[i\frac{2fx_1}{Qv(f_0)} ln\left|\frac{f}{f_0}\right|\right], \tag{11}$$

and

$$W_{d2}(f) = W(f)B_2^{'}(f) = W(f)\exp\left[-\frac{\pi f x_2}{Qv(f_0)}\right]\exp\left[i\frac{2fx_2}{Qv(f_0)} ln\left|\frac{f}{f_0}\right|\right], \tag{12}$$

where $W(f)$ is the Fourier spectrum of source wavelet. Thus, the amplitude decay and phase variation can be measured using the (log) spectral ratio as following,

$$\ln\left[\frac{W_{d2}(f)}{W_{d1}(f)}\right] = -\frac{\pi f(x_2 - x_1)}{Qv(f_0)} + i\left[\frac{2f(x_2 - x_1)}{Qv(f_0)} ln\left|\frac{f}{f_0}\right|\right]. \tag{13}$$

The real part the (log) spectral ratio theoretically has a constant slope, which can be expressed as

$$k = \frac{1}{f}\mathrm{Re}\left(\ln\left[\frac{W_{d2}(f)}{W_{d1}(f)}\right]\right). \tag{14}$$

So, the Q value can be estimated as

$$Q = -\frac{\pi\tau}{k}, \tag{15}$$

in which τ is the time delay given by

$$\tau = (x_{2-}x_1)/v(f_0). \tag{16}$$

For the standard spectral ratio method, only the real part of spectral ratio is considered, the slope is estimated by fitting a straight line to the measured spectral ratio, in which either the least squares solution or the L_1 norm solution can be adopted. For the examples in this article, we use the least squares solution. Suppose that we get N spectral ratios for frequencies $f_1, f_2, \ldots, f_N$. Let R, L, G_1 and G_2 be the column vectors with N elements expressed as following

$$R = \left[Re\left(ln\left[\frac{W_{d2}(f_1)}{W_{d1}(f_1)}\right]\right), Re\left(ln\left[\frac{W_{d2}(f_2)}{W_{d1}(f_2)}\right]\right), \cdots, Re(ln\left[\frac{W_{d2}(f_N)}{W_{d1}(f_N)}\right])\right]^T,$$

$$I = \left[Im\left(ln\left[\frac{W_{d2}(f_1)}{W_{d1}(f_1)}\right]\right), Im\left(ln\left[\frac{W_{d2}(f_2)}{W_{d1}(f_2)}\right]\right), \cdots, Im(ln\left[\frac{W_{d2}(f_N)}{W_{d1}(f_N)}\right])\right]^T,$$

$$G_1 = [-\pi f_1\tau, -\pi f_2\tau, \cdots, -\pi f_N\tau]^T,$$

$$G_2 = \left[-2f_1\tau \cdot ln\left|\frac{f_1}{f_0}\right|, -2f_2\tau \cdot ln\left|\frac{f_2}{f_0}\right|, \cdots, -2f_{\mathrm{N}}\tau \cdot ln\left|\frac{f_N}{f_0}\right|\right]^T. \tag{17}$$

Then, equation (13) can be rewritten as

$$R + iI = G_1 m + \mathrm{i}G_2 m, \tag{18}$$

in which m is the reciprocal of Q, i.e

$$Q = 1/m. \tag{19}$$

For standard spectral ratio method, only the real part of equation (18) is used. The least square solution for Q estimation is as following

$$\mathrm{Q_{est1}} = 1/\left[({G_1}^{\mathrm{T}}G_1)^{-1}{G_1}^{\mathrm{T}}\mathrm{R}\right]. \tag{20}$$

Similarly, considering the imaginary part of spectral ratios only, the estimated Q value should be

$$\mathrm{Q_{est2}} = 1/\left[({G_2}^{\mathrm{T}}G_2)^{-1}{G_2}^{\mathrm{T}}\mathrm{R}\right]. \tag{21}$$

The methods above only use either amplitude or phase information of the attenuated wavelets. In this article, we refer to them as amplitude spectral method and phase spectral method respectively. Making full use of the spectral information, a complex spectral ratio method can be developed. Based on equation (18), the Q value can be estimated by solving the following matrix equation

$$\begin{bmatrix} G_1 \\ G_2 \end{bmatrix} m = \begin{bmatrix} R \\ I \end{bmatrix}. \tag{22}$$

Then, the estimated Q value for the complex spectral ratio method can be formulated as

$$Q_{est3} = 1/[(G^T G)^{-1} G^T D], \tag{23}$$

where the G and D are two vectors with 2N elements as following

$$G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}, D = \begin{bmatrix} R \\ I \end{bmatrix}. \tag{24}$$

For Q estimation in practical, an energy loss independent of frequency may be taken into account. So, equation (18) can be modifies as

$$R + iI = G_1 m + E_N b + iG_2 m, \tag{25}$$

in which E_N is a column vector with all ones, and b is a constant. For all the three method, Q estimation will reduce to estimate parameter m by solving the following forward model

$$\boldsymbol{L}\mathbf{m} = \mathbf{d}, \tag{26}$$

where $\boldsymbol{L}$ is the model operator that can be easily constructed from G_1 and G_2, $\mathbf{m}$ is the model vector that only contains parameter m and b, and $\mathbf{d}$ is the data vector that can be directly obtained from R and I. The least square solution for equation (26) can be formulated as

$$\hat{\mathbf{m}} = (\boldsymbol{L}^T \boldsymbol{L})^{-1} \boldsymbol{L}^T \mathbf{d}. \tag{27}$$

Sometimes, in order to improve the estimation, priori information or constraints about the model parameters may be added to the model. For this case, the model parameters are estimated by minimizing the following objective function

$$\mathbf{f}(\mathbf{m}) = \|\mathbf{d} - \mathbf{Lm}\|^2 + \lambda^2 \|\mathbf{Am} - \mathbf{Am}_{\mathrm{apr}}\|^2, \tag{28}$$

where λ is a scaling factor; $\mathbf{A}$ represents the imposed constraints in the form of linear operator; $\mathbf{m}_{\mathrm{apr}}$ represents priori information about the model. The solution can be formulated as

$$\hat{\mathbf{m}} = (\mathbf{L}^T\mathbf{L} + \lambda^2 \mathbf{A}^T\mathbf{A})^{-1}(\mathbf{L}^T\mathbf{d} + \lambda^2 \mathbf{A}^T\mathbf{Am}_{\mathrm{apr}}). \tag{29}$$

The choosen reference frequency f_0 and measuring frequency f should satisfy equation (4). In addition, f_0 should be properly chosen to make the calculation of division in equation (13) stable in case of noise.

EXAMPLE

For the examples in this article, the time sample rate is 0.002 s, reference frequency is the Nyquist frequency, the reference velocity is 2000 m/s, and the parameter Q is 50. The attenuated impulse response of the earth is calculated using equation (6). Figure 1 shows two truncated results of the original impulse response. One is truncated directly, and the other is circularly shifted 0.002 s (10 samples) and then truncated. Their corresponding slopes of amplitude (log) spectral ratio calculated by equation (14) are shown in Figure 2, which should be theoretically constant over all frequencies. We can see that the circular shift operation helps preserve all the information of original impulse response. Figure 3 shows two attenuated wavelets created by convolving the impulse response with a minimum phase wavelet. The progressive amplitude decay and waveform change caused by attenuation are apparent. Figure 4 shows the Q estimation results at different signal-to-noise levels, and Figure 5 shows the repeated Q estimations result at fixed signal-to-noise level. The phase spectral ratio method has comparable estimation results to the amplitude spectral ratio method, and the complex spectral ratio method, in most cases, gives a better result than the other methods.

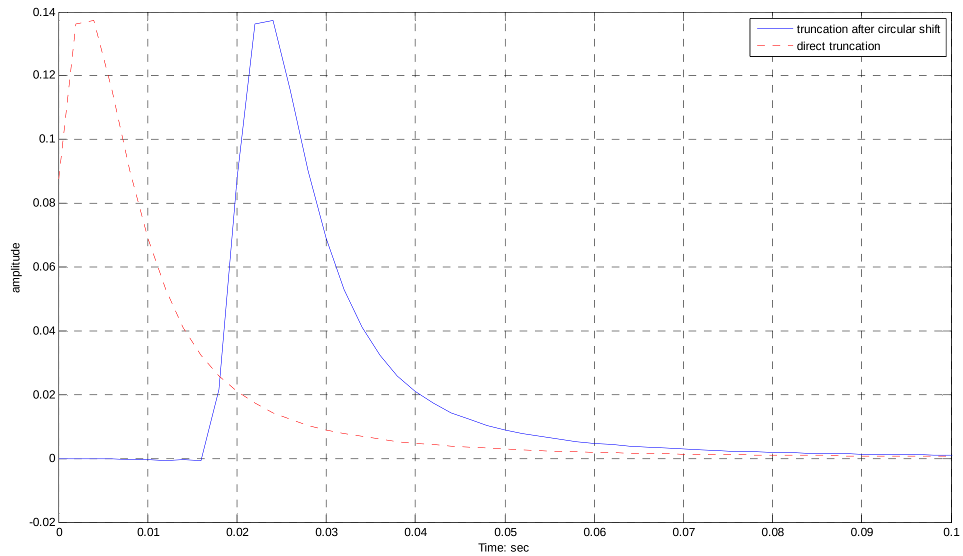

FIG. 1. Attenuated impulse response of earth.

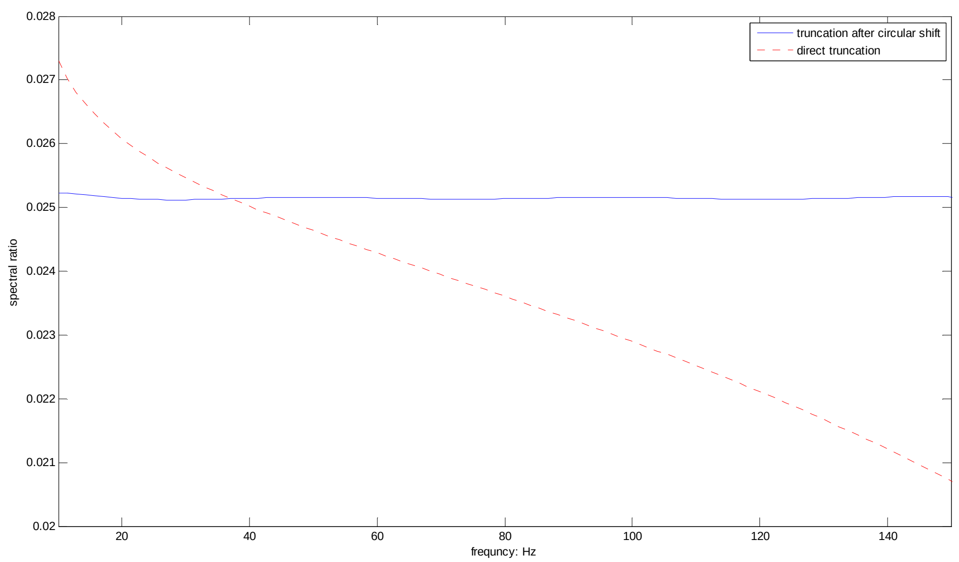

FIG. 2. Slope of amplitude (log) spectral ratio of attenuated impulse response.

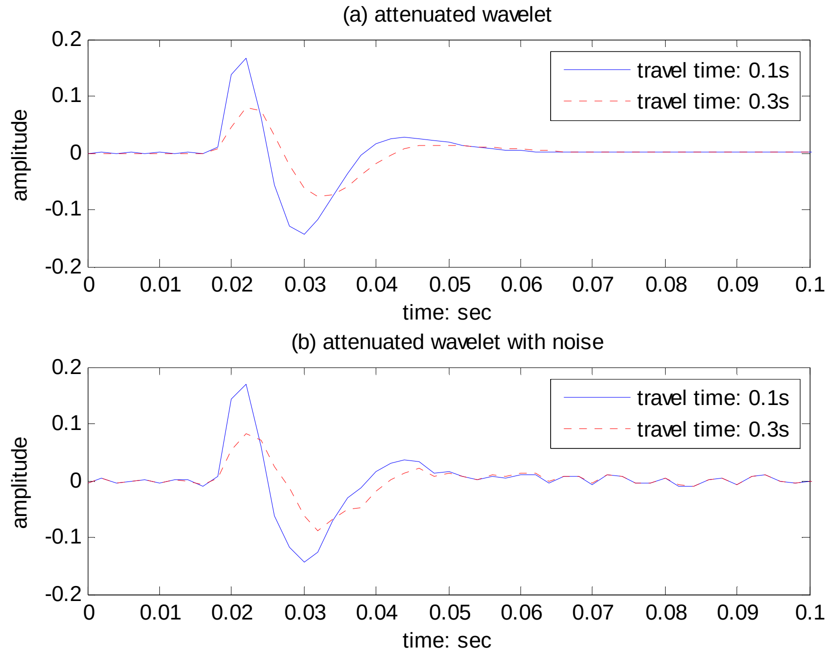

FIG. 3. Attenuated wavelet: (a) noise free, (b) with noise.

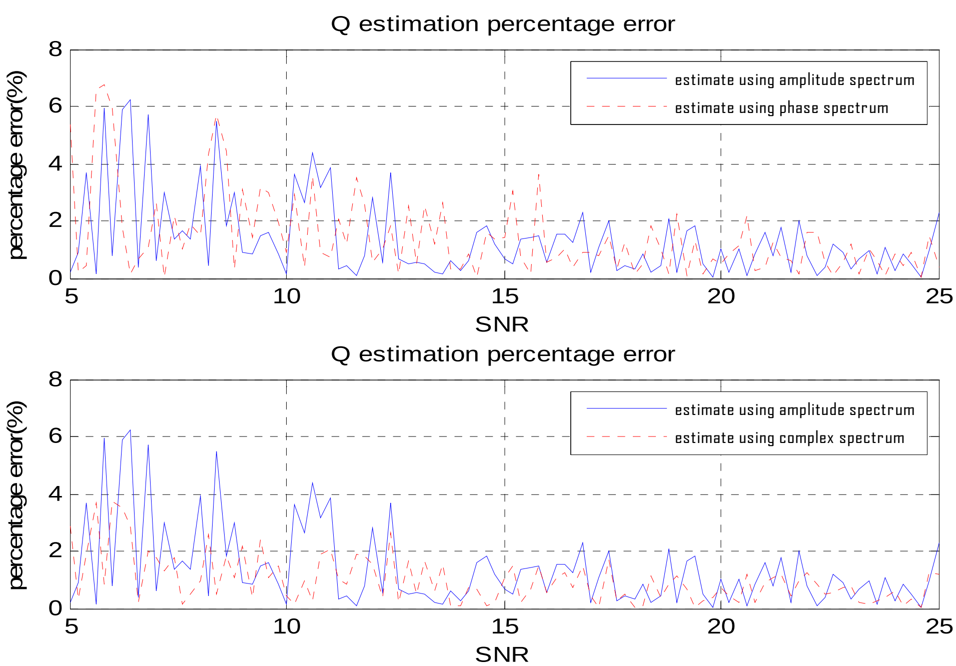

FIG. 4. Comparison of Q estimation at different SNR levels

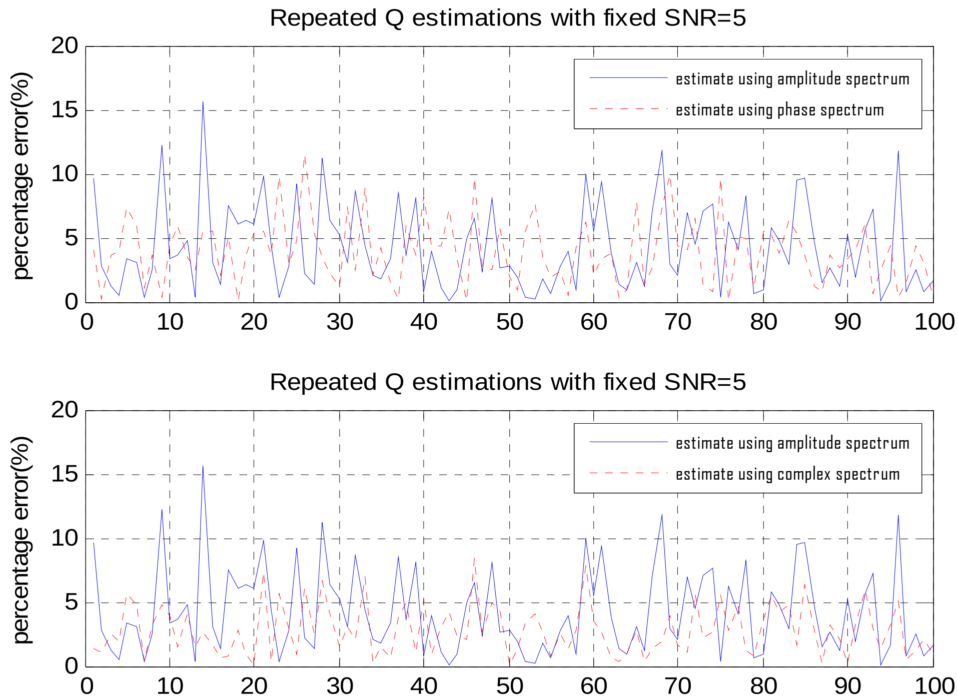

FIG. 5. Comparison of Q estimation at fixed SNR.

CONCLUSIONS

The circular shift operation helps to make the attenuated impulse response of earth more consistent with the constant-Q theory. The synthetic downgoing wavelet can be created by convolving the attenuated impulse response with source wavelet, which can be thought as the ideal version of VSP data.

The amplitude spectral ratio method, phase spectral ratio method and complex spectral ratio method for Q estimation are evaluated using synthetic data. Results show that the phase spectral method is comparable to the conventional amplitude spectral ratio method, and the complex spectral method can obtain the best estimation in most cases. Thus, the complex spectral ratio method may serve as an alternative to conventional spectral ratio method for Q estimation using VSP data.

The above conclusion is based on a simplified theoretical model. Actually, the key is that whether the phase spectral method can obtain similar estimation to the amplitude spectral method. In practice, the phase spectral method may be more sensitive to the difference between the real attenuation mechanism and constant-Q model, and to the noise in the real world. Therefore, further investigation should be conducted using the real VSP data.

ACKNOWLEGEMENTS

The authors would like to thank the sponsors of CREWES project, NSERC, POTSI and MITACS for their financial support.

REFERENCES

Aki, K., and Richards, P. G., 2002, Quantitative Seismology; Theory and methods: university Science Books.

Haase, A. B., and Stewart, R. R., 2004, Attenuation (Q) from VSP and log data: Ross Lake, Saskatchewan, 2004 CSEG National Convention.

Kjartansson, E., 1979, Constant-Q wave propagation and attenuation: J. Geophysics. Res., **84**, 4737-4748.

Margarve G. F., 1998, Theory of nonstationary linear filtering in the Fourier domain with application to time-variant filtering: Geophysics, **63**, 244-259.

Margrave, G. F., and Lamoureux, M. P., 2002, Gabor deconvolution: 2002 CSEG Annual Convention.

Tonn, R., 1991, The determination of seismic quality factor Q from VSP data: A comparison of different computational methods: Geophys. Prosp., **39**, 1-27.