EOM hyperbolae

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ABSTRACT

The equivalent offset method of migration is a prestack migration that spreads energy from one input time sample to all the neighbouring gathers. In a time migration the energy is spread along a hyperbolic path in a constant time plane defined by the input sample. This path is referred to as the EOM hyperbola. Moveout correction of this hyperbola creates the prestack migration ellipse.

Properties of the EOM hyperbola are displayed relative to the prestack migration ellipse to evaluate the extent of the EO hyperbola, and to establish an ad hoc method for applying amplitude scaling.

Comparisons of EOM with conventional prestack time migration are provided.

INTRODUCTION

Equivalent offset migration was introduced in the mid 1990's (Bancroft et al. 1994, 1998) and is based on the Kirchhoff algorithm. It can be either a prestack depth migration or a prestack time migration. The advantages of an anisotropic prestack depth migration were discussed by Bancroft and Vestrum in 1999. The prestack depth migration required the conventional raytracing or wavefront computations, and its main advantage was in focusing the prestack migration gathers.

The equivalent offset (EO) prestack time migration was based on the assumptions associated with RMS velocities in which traveltimes could be computed with a hyperbolic equation. The traveltime of a source raypath t_s (source to a scatter point) and the traveltime of a receiver raypath t_r (scatterpoint to a receiver) are summed ($t = t_s + t_r$) to produce the double-square-root (DSR) equation

$$t = t_s + t_r = \sqrt{\frac{t_0^2}{4} + \frac{(x+h)^2}{v^2}} + \sqrt{\frac{t_0^2}{4} + \frac{(x-h)^2}{v^2}} , \qquad (1)$$

where h is half the source - receiver offset, t_0 is the vertical two-way traveltime from the scatterpoint to the surface, and v the velocity. We assume the migrated trace to be located at the CMP location x_{mig}, and the midpoint of the input trace at the CMP location x_{mp}. The distance from the migrated trace to the midpoint of the input trace is x, hence $x = x_{mig} - x_{mp}$.

EOM equates the DSR equation with a single hyperbolic equation that assumes a colocated source and receiver at the equivalent offset h_e. The two-way time to the equivalent offset T is maintained to give

$$2\sqrt{\frac{t_0^2}{4}+\frac{h_e^2}{v^2}}=\sqrt{\frac{t_0^2}{4}+\frac{\left(x+h\right)^2}{v^2}}+\sqrt{\frac{t_0^2}{4}+\frac{\left(x-h\right)^2}{v^2}}\,. \tag{2}$$

Solving for the equivalent offset h_e gives

$$h_e^2 = x^2 + h^2 - \frac{4x^2h^2}{t^2v^2}\,. \tag{3}$$

This single offset enables all the prestack data to be stacked into a single gather where the reflection energy lies on a hyperbolic path. The high fold of his gather allows for an accurate velocity analysis which is then used to apply moveout correction, then stacking, to produce a prestack migrated trace. Gathers are formed at all CMP locations to create the prestack migrated section or volume (Bancroft et al. 1998).

A main feature of the method is that the prestack migration gathers are formed with no time shifting of the input data. A sample from an arbitrary input trace is summed into the neighbouring gathers at the origonal time t and at the equivalent offset h_e.

Consider one sample on the prestack migration gather at an equivalent offset h_{e1} and time t_1. Energy from all input traces with the same h_{e1} will sum into the gather. The path of this summation is defined by the time intersection of a Cheops pyramid, illustrated in Figure 1 that shows the lower part of the prestack volume. It assumes a continuum of prestack traces with CMP location x and offset h.

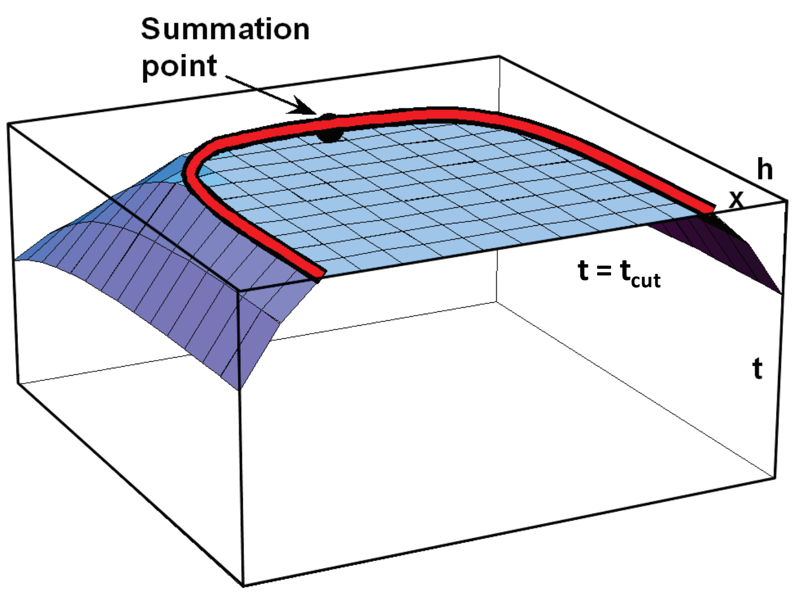

FIG. 1 The prestack volume showing the summation path for the equivalent offset h_e in red.

The EO hyperbola

Now consider <u>one input sample</u> at time t_A, offset h_A, with a midpoint location x_{mp} of zero, i.e. $x_{mp} = 0$ and the displacement to the migrated trace as $\tilde{x}$. The gathering process will spread the energy of this sample to all neighbouring prestack migration gathers to equivalent offsets h_e. We can describe this path by rearranging equation (3) as

$$h_e^2 = \tilde{x}^2 \left(1 - \frac{4h_A^2}{t_A^2 v^2}\right) + h_A^2, \tag{4}$$

where the bracketed term is the constant c, giving the hyperbolic form of the equation as

$$\boxed{h_e^2 - c\tilde{x}^2 = h_A^2}. \tag{5}$$

The EO hyperbola is displayed in Figure 2 as the green curve in the prestack volume. Energy at the input sample (t_A , x=0, h_A) is spread to neighbouring prestack migration gathers. How far should this curve be extended, and should there be any amplitude scaling along this curve?

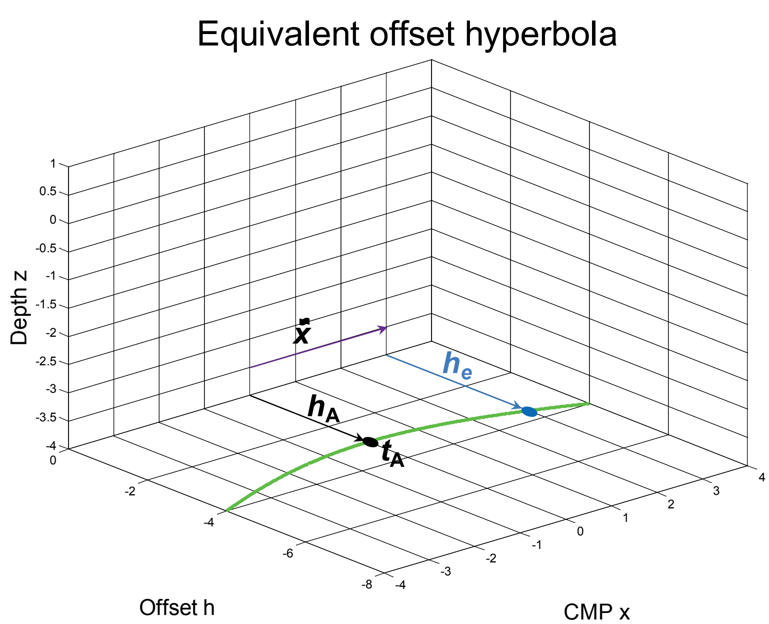

FIG. 2 The prestack volume showing an input time sample (t_A , x=0, h_A) and the distribution of its energy along the EO hyperbola (green) to the neighbouring prestack migration.

After all the prestack migration gathers have been formed, they are similar to CMP gathers that are ready for velocity analysis, normal moveout (NMO) correction, and stacking. CMP gathers will produce stacking velocities for a stacked section. However, prestack migration gathers produce a more accurate velocity analysis because of the high fold, and because the estimated RMS velocities are independent of dip and azimuth.

Consider the prestack migration gather at $\tilde{x} = 0$ that is shaded in blue in Figure 3. It contains and the input sample. It also contains on another hyperbolic curve in the vertical plane that I refer to it as the moveout hyperbola. Energy along this moveout hyperbola will sum to produce the prestack migrated sample at zero offset. The energy of this input sample at (t_A , x=0, h_A) lies on this moveout hyperbola and will stack into the prestack migrated sample.

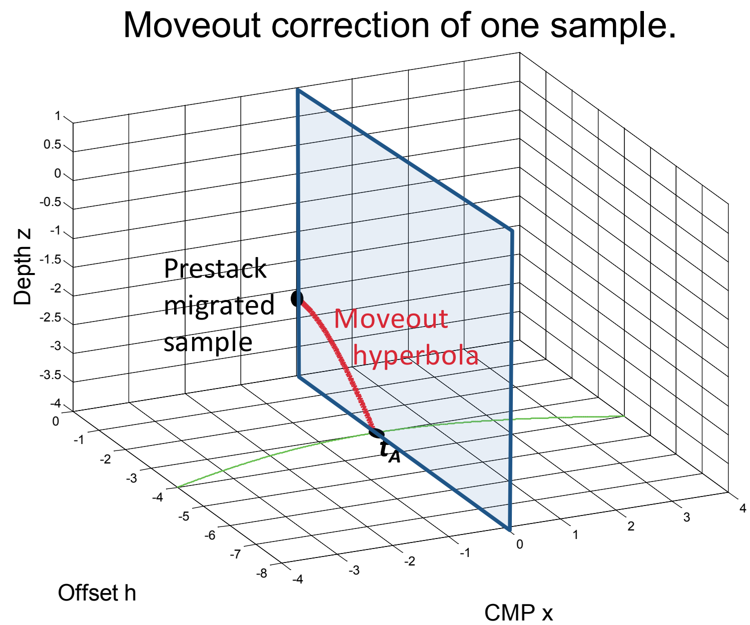

FIG. 3 Prestack volume showing the moveout hyperbola on a prestack migration gather. The input sample at t_A lies on the red hyperbolic curve.

The energy along the EO hyperbola will also be moveout corrected to zero offset. As the offset on this hyperbola increases, the migrated sample will move up to a smaller time and form a new curve. This energy at zero offset is part of the prestack migration ellipse, as illustrated in Figure 4. The movement of one sample of input energy to the prestack migration ellipse is discussed in Appendix A. Applying MO correction and moving all the energy on the EO hyperbola to zero offset (see insert) defines the complete prestack migration ellipse. The EO method moves the energy from an input sample (t_A , x=0, h_A) in two steps to the prestack migration ellipse: spreading the energy on the EO hyperbola and moving this energy to zero offset.

The movement of input energy to a prestack migration ellipse is limited to a constant velocity environment. The time t_0 is defined along the ellipse and time t at the bottom of the ellipse. RMS velocities are defined at t_0, and they would have to remain constant to describe an ellipse.

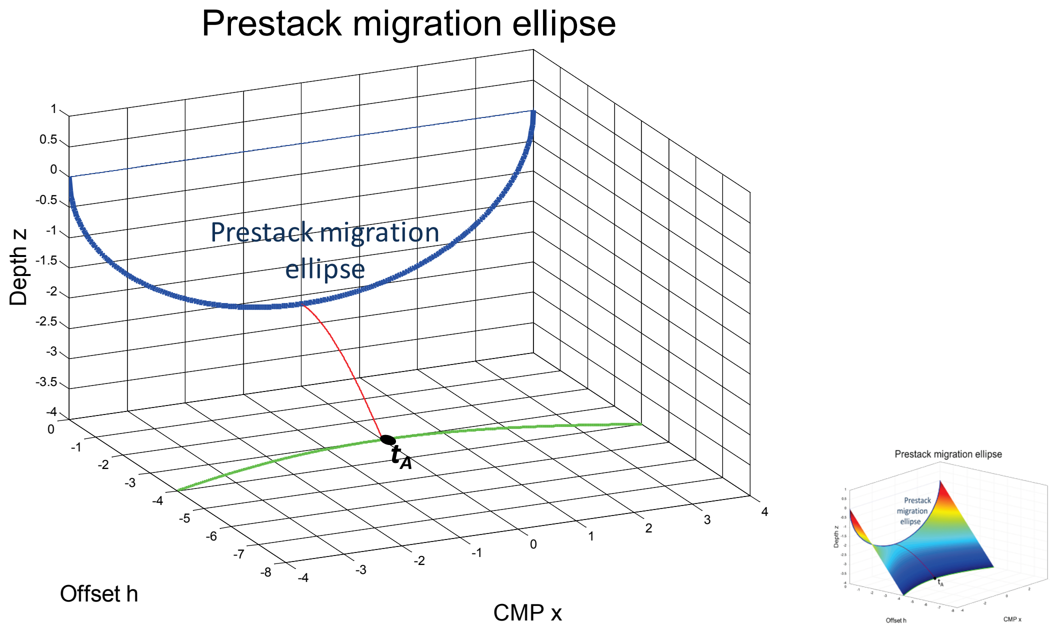

FIG. 4 Prestack volume showing energy moving from the EO hyperbola to the prestack migration ellipse.

With this new information, we can answer the previous questions: how far should this curve be extended, and should there be any amplitude scaling along this curve?

The answer to the first question comes from Figure 5 which shows five input samples from the same CMP gather at equal offsets.

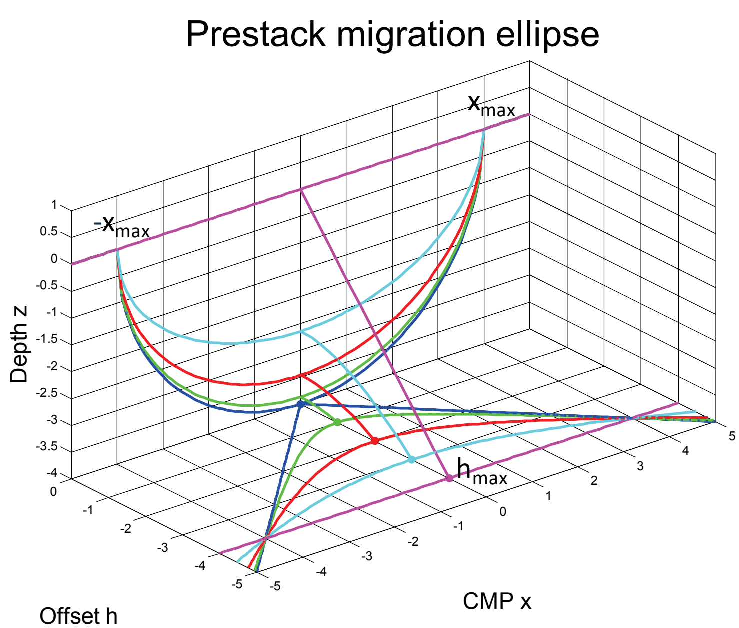

FIG. 5 Prestack volume showing a five input samples, their EO hyperbolae, the MO hyperbolae for each sample, and the corresponding prestack migration ellipse.

Figure 5 also includes the EO hyperbolae, the MO hyperbolae for one sample, and the prestack migration ellipse. The first sample is at zero offset and its EO hyperbola consists of two 45 degree lines that map energy to a circle at zero offset. The fifth sample is at the maximum offset for this time (or depth) and maps energy to the surface (z = 0) at zero offset. The other three samples map energy to the prestack migration ellipse. All the EO hyperbolae intersect at the maximum offset h_{max} and at a maximum displacement $\tilde{x}_{\max}$. From the geometry of the figure we can observe that $h_{\max} = \tilde{x}_{\max} = z_A = v_{rms} t_A / 2$. Energy from the input samples at a time t_A should not be extended beyond $\tilde{x}_{\max}$ or h_{max}. These parameters are defined for the given time t_A.

Amplitude scaling

In an ad hock manner, we can guess at an amplitude for scaling the energy along the EO hyperbola. The zero-offset sample spreads energy to the prestack migration circle. The amplitudes along the circle are assumed to be defined by t_0/t, where t_0 is the time on the circle, and t is the time at the bottom of the circle. (In Kirchhoff migration, t_0 is the time at the apex of the diffraction, and t is the time on the flank of the diffraction.) Using convenience as the motivator, we use this same relationship for all the amplitudes of the other ellipse, and transfer this amplitude to the EO hyperbola.

The displacement from the input sample to the prestack migration gather was defined to be $\tilde{x}$, and the maximum displacement of the prestack migration ellipse is $\tilde{x}_{\max}$. Using these parameters, we can define the amplitude on the EO hyperbola $A\left(\tilde{x}, t_A\right)$ to be

$$A\left(\tilde{x}, t_A\right) = \sqrt{1 - \frac{\tilde{x}^2}{\tilde{x}_{\max}^2}} = \sqrt{1 - \frac{4\tilde{x}^2}{v_{rms}^2 t_A^2}} . \qquad (6)$$

We can use the same arguments for forming the prestack migration gathers when summing the input traces into one prestack migration gather, as indicated in Figure 1. We can scale the summed energy with equation (6) by replacing $\tilde{x}$ with x.

Comments and conclusion

Equivalent offset prestack time migration is usually accomplished by summing all prestack traces into each prestack migration gather. The same prestack migration gathers could also be formed by spreading the energy from one input trace to all the neighbouring prestack migration gathers along curves that are referred to as equivalent offset (EO) hyperbolae. The latter approach is useful for evaluating the spatial extent of this energy, and for deriving an ad hoc concept to define amplitudes of this distributed energy. These amplitudes may then be used in forming the prestack migration gathers by summing the input traces into the prestack migration gathers.

Appendix B contains two plots: the first is a time-slice of Cheops pyramids, and the second is a plot of the EO hyperbolae. Both are created for the same depth and can be slid across each other to map energy from one to the other.

SOFTWARE

The figures created in this report were created with MATLAB.

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2009-Matlab\EOM software\
EOMFamilyOfHyperbolaToEllipse.m
EOMhyperbolicSurfaceToEllipse.m
EOMcspGathres.m
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ACKNOWLEDGEMENTS

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REFERENCES

Bancroft, J.C. and Vestrum, R., 1999, Anisotropic prestack depth migration using the equivalent offset method, CREWES Research Report, v. 51, Ch 18, p 699-704.

Bancroft, J. C., Geiger, H. D. and Margrave, G. F., 1998, The equivalent offset method of prestack time migration: Geophysics, Vol.63, N0. 6, P. 2042-2053.

Bancroft, J.C., Geiger, H.D., Foltinek, D.S. and Wang, S., 1994, Pre-stack migration by equivalent offsets and CSP gathers, CREWES Project Research Report

APPENDIX

Appendix A

I derive the equation of the prestack migration ellipse by starting with the familiar form of the double-square-root (DSR) equation. The equation defines the total or two-way traveltime t of the raypaths from the source to the reflection point with time t_s, and from the reflection point to the receiver with time t_r, i.e.,

$$t = t_s + t_r = \sqrt{\frac{t_0^2}{4} + \frac{h_s^2}{v^2}} + \sqrt{\frac{t_0^2}{4} + \frac{h_r^2}{v^2}}\ , \tag{7}$$

where h_s and h_r are the horizontal distances from the source and receiver to the refection point, and t_0 is the vertical two-way time to the point of reflection. Assuming the location of the reflection point to be at (x, z), with the midpoint MP located at x = 0, and denoting half the source-receiver offset as h, we can write this equation as

$$vt = \sqrt{z^2 + \left(x+h\right)^2} + \sqrt{z^2 + \left(x-h\right)^2}\ . \tag{8}$$

Squaring both sides gives

$$\left(vt\right)^2 = \left[z^2 + \left(x+h\right)^2\right] + \left[z^2 + \left(x-h\right)^2\right] + 2\sqrt{\left[z^2 + \left(x+h\right)^2\right] \times \left[z^2 + \left(x-h\right)^2\right]}\ , \tag{9}$$

then

$$\frac{v^2t^2}{2} - \left(z^2 + x^2 + h^2\right) = \sqrt{z^4 + 2z^2\left(x^2 + h^2\right) + \left(x^2 - h^2\right)^2}\ , \tag{10}$$

which is squared again, to get

$$\frac{v^4t^4}{4} - 2\frac{v^2t^2}{2}\left(z^2 + x^2 + h^2\right) = z^4 + 2z^2\left(x^2 + h^2\right) + \left(x^2 - h^2\right)^2 - \left(z^2 + x^2 + h^2\right)^2 . \tag{11}$$

Continued squaring and cancelling terms gives

$$\frac{v^4t^4}{4} - v^2t^2\left(z^2 + x^2 + h^2\right) = -4x^2h^2\ , \tag{12}$$

which can be expressed as

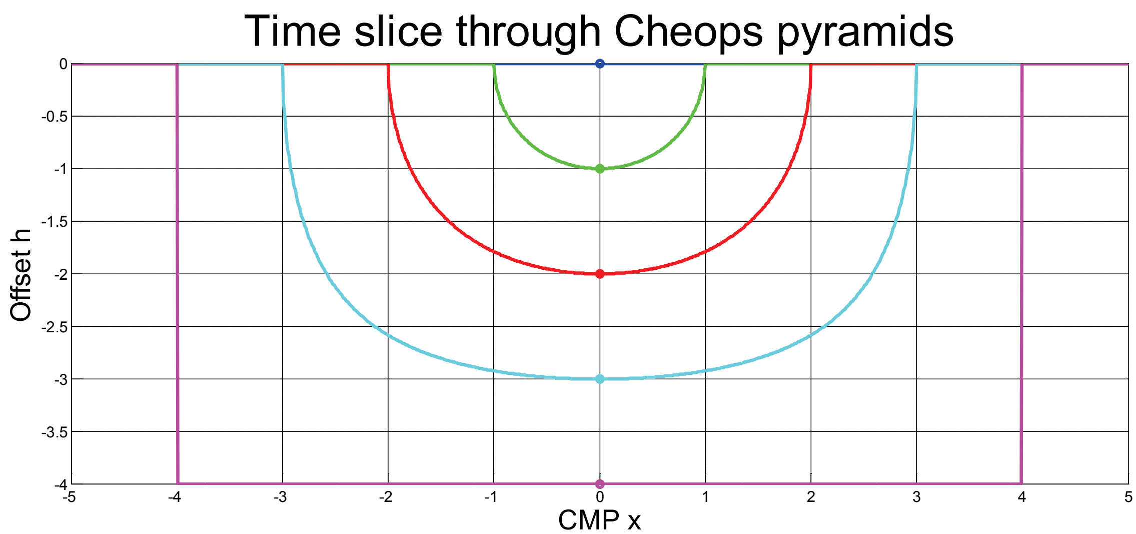
$$\frac{x^2}{\left(\frac{vt}{2}\right)^2} + \frac{z^2}{\left(\frac{vt}{2}\right)^2 - h^2} = \frac{x^2}{\left(\frac{vt}{2}\right)^2} + \frac{z^2}{\left(\frac{vt_0}{2}\right)^2} = \frac{x^2}{a^2} + \frac{z^2}{b^2} = 1 \quad . \tag{13}$$

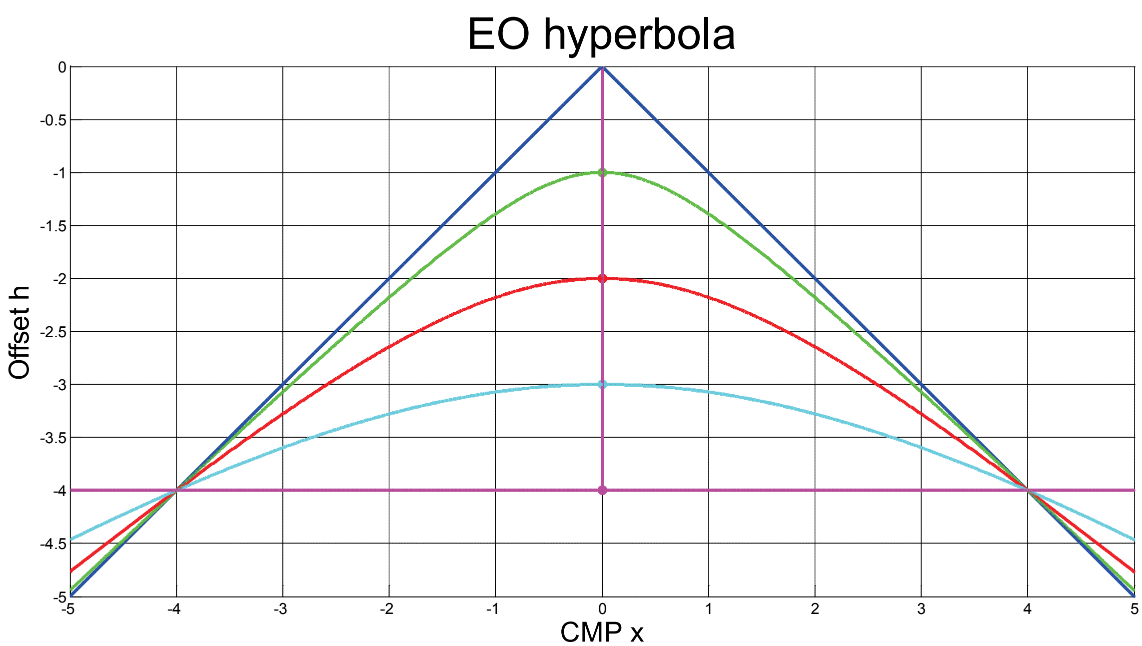
This equation is in the form of an ellipse where the semi-major axis is *a* and the semi-minor axis is *b*. The semi-major axis *a* is equal to the depth of the original time *t*/2 and would be the depth of a horizontal reflector with zero-offset time equal to *t*/2. The semi-minor axis is the moveout corrected time $t_0/2$.

All offset samples with a traveltime *t* will produce ellipse that have the same semi-major axis *a*.

Appendix B

Below are two plots: the first is a time-slice of Cheops pyramids and the second is a plot of the EO hyperbolae. Both are created for the same depth and can be slid across each other to map energy from one to the other. The time-slice of Cheops pyramids are defined at a prestack migration location, and the contours are the summation paths for energy on CMP gathers. The EO hyperbolae represent the spreading of energy from a CMP gather to the neighbouring prestack migration traces. Try making copies and sliding them across one another to verify these processes.





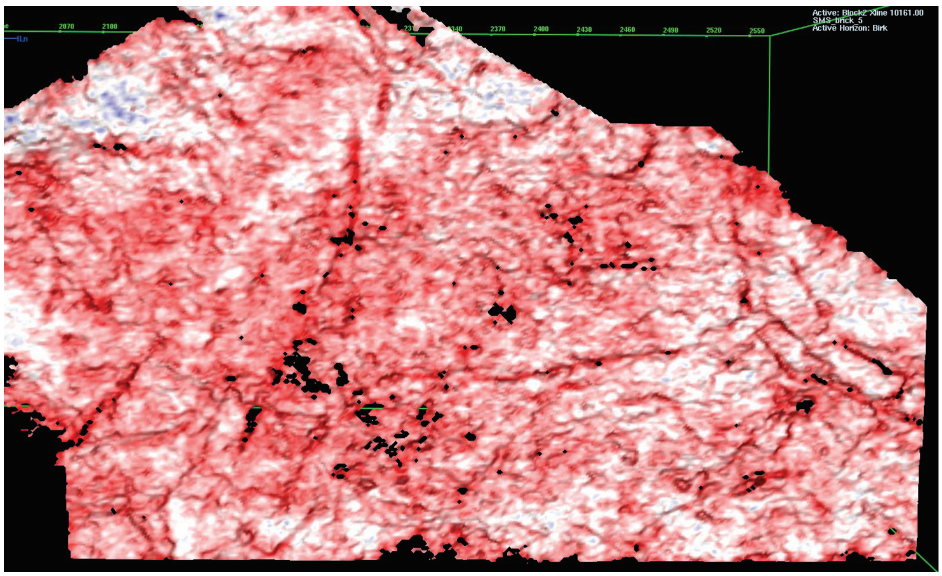
Appendix C

The following examples were produced and provided by Santos to compare EOM with other migrations.

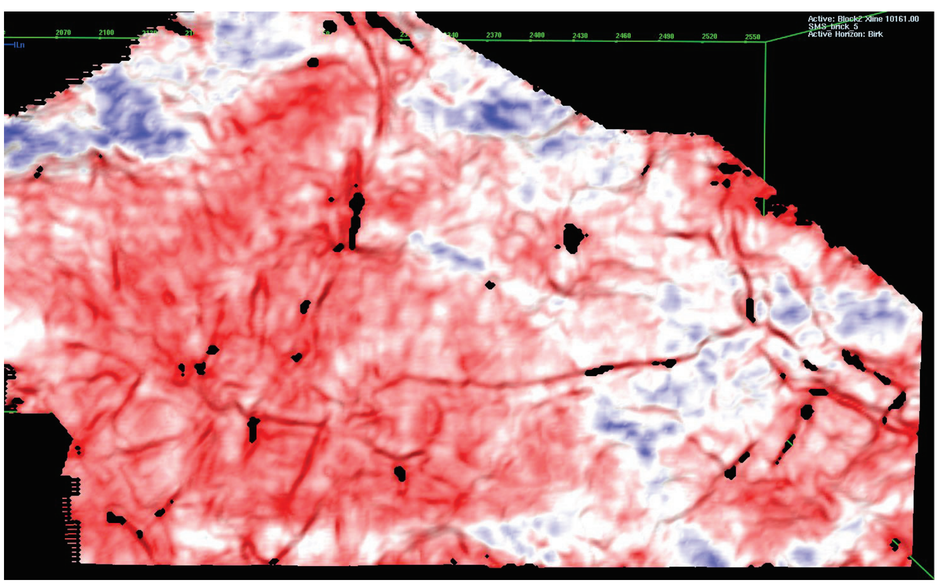
Santos is an Australian oil and gas exploration and production company with high quality assets and projects throughout Australia and the Asia-Pacific region.

http://www.santos.com/company-profile.aspx

Amplitude maps from "Cooper" showing a poststack migration followed by EOM prestack migration.

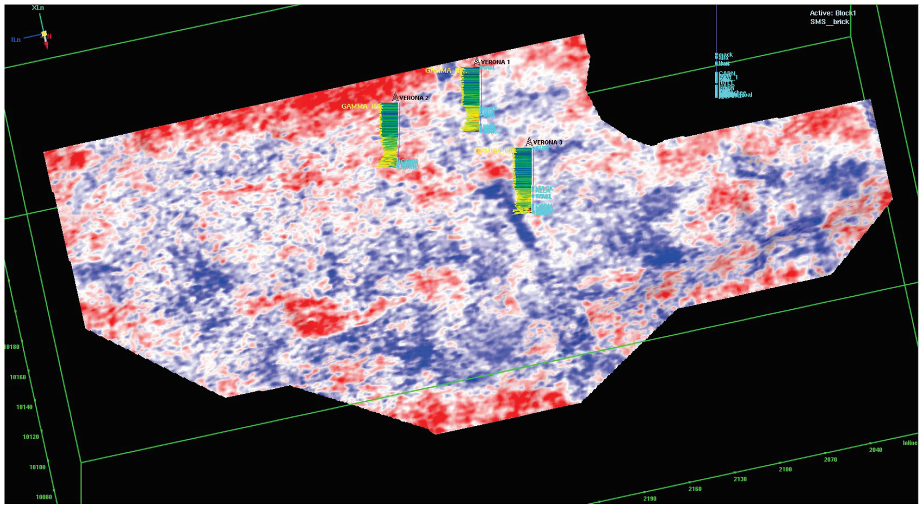


PostSTM

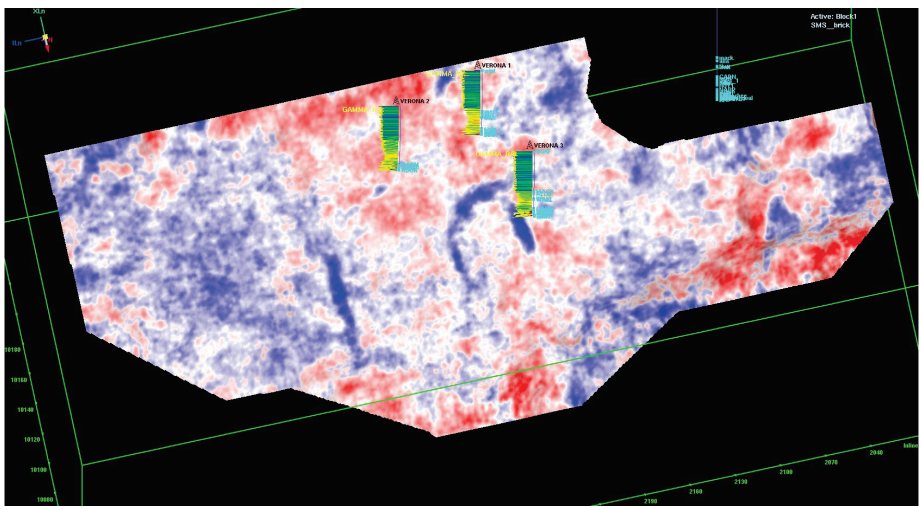


EOM

A comparison of a prestack time migration with EOM at "Birkhead".

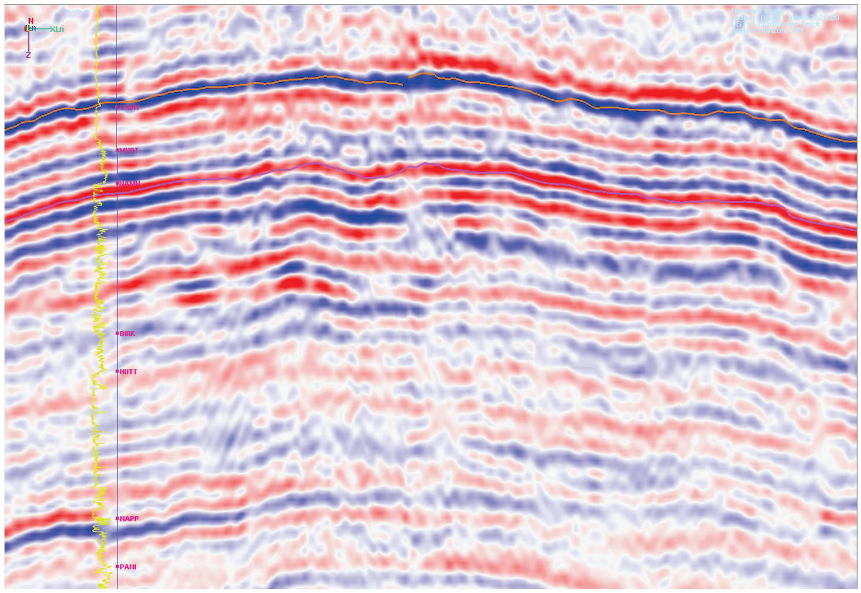


PSTM



EOM

Comparison of a prestack time migration with EOM at “Cooper Basin”.



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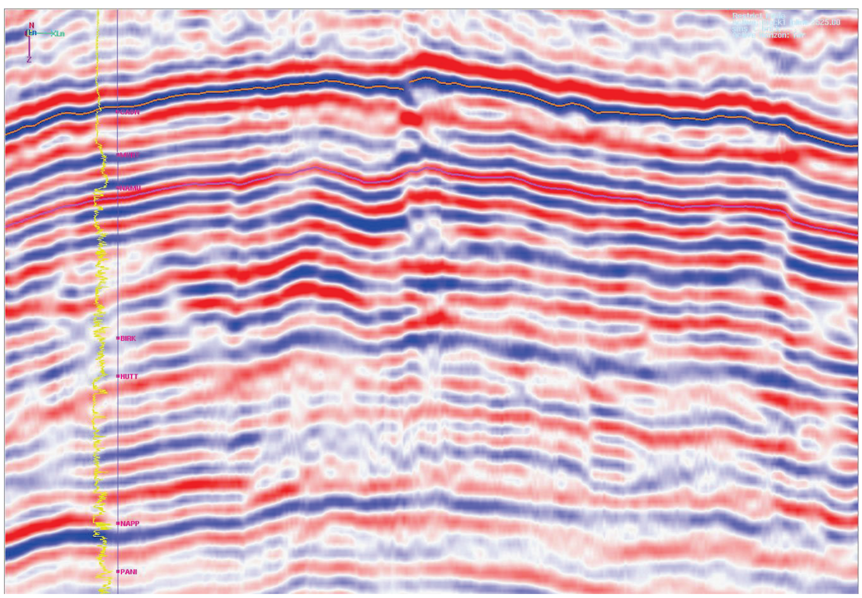
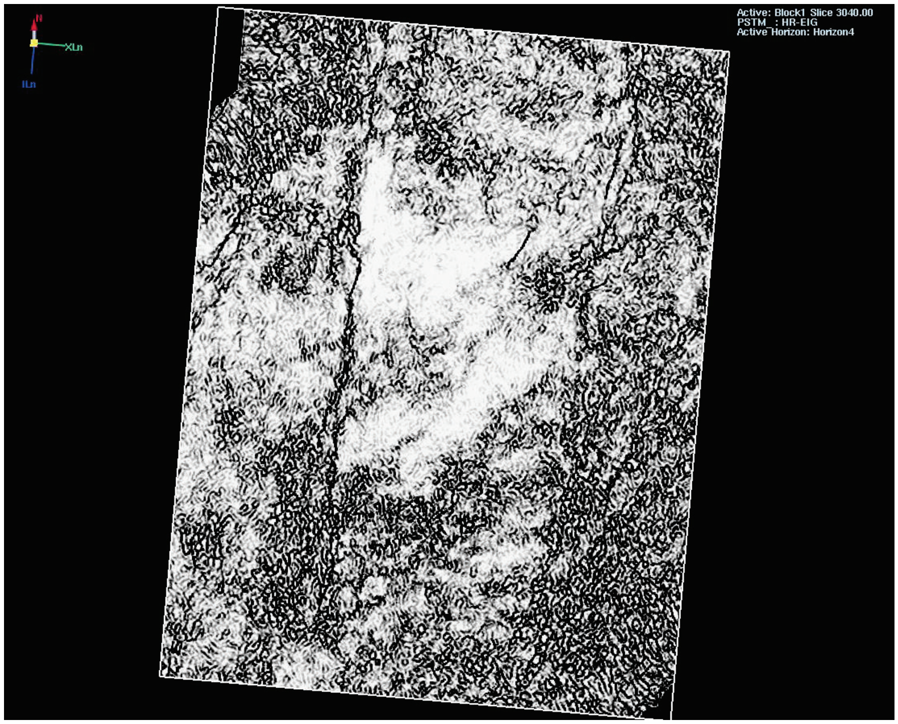


EOM

Comparison of a prestack time migration with EOM showing semblance at “Cooper Basin”.



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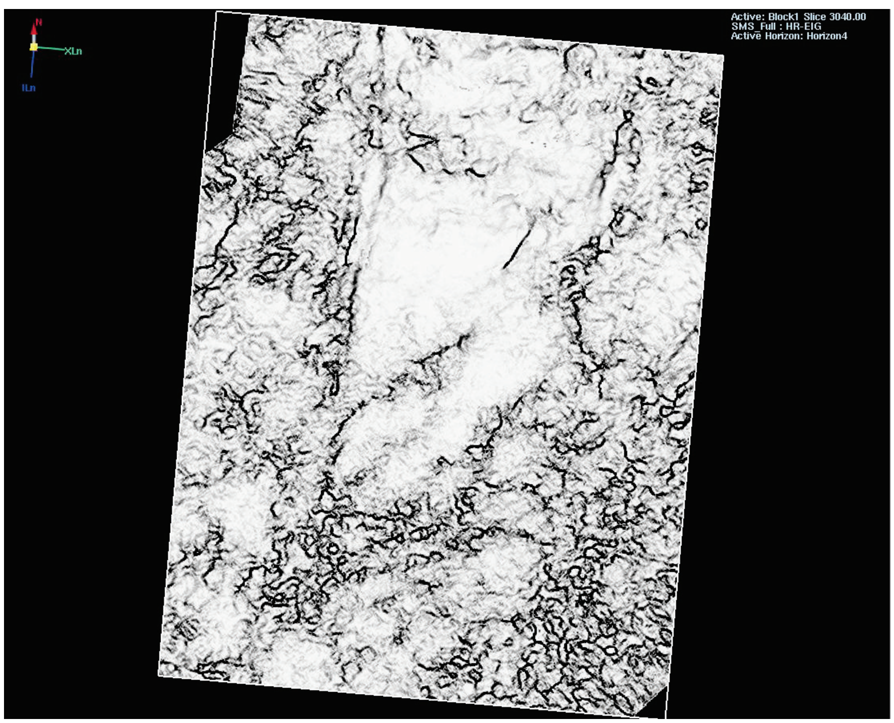
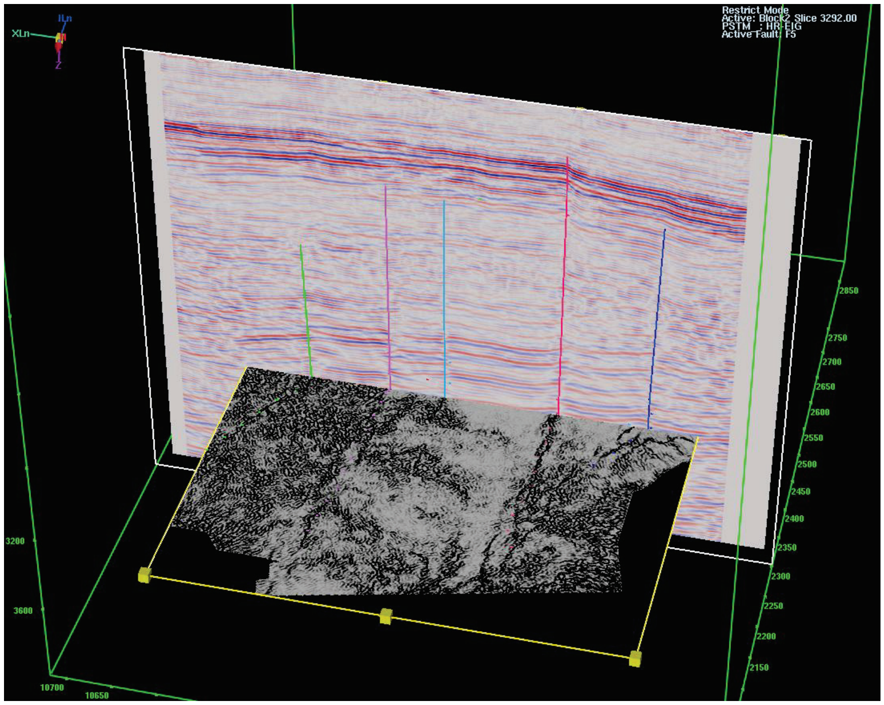
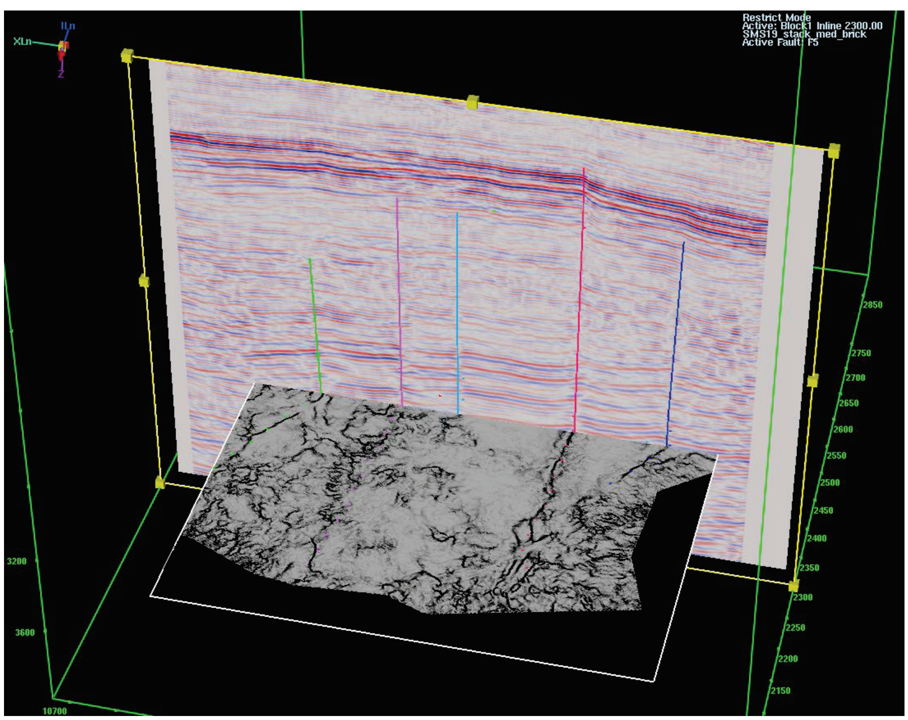


EOM

A comparison of prestack time migration with EOM coherence slices at "Cooper Basin".



PSTM



EOM