Q analysis using synthetic viscoacoustic seismic data

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ABSTRACT

The spectral ratio method is used to conduct Q analysis with synthetic VSP and reflection data. Testing results show that the spectral method can deal with frequency independent energy loss including geometric spreading and transmission loss, and can also be used to derive the layered Q structure of subsurface. For real data, the notches in spectrum, due to reflectivity, are a severe problem for spectral ratio calculation. The adaptive multitaper method for spectral estimation is shown to give a smooth estimate with little evidence of notching, and is superior to both DFT and Burg spectral estimates.

INTRODUCTION

The spectral ratio method (Tonn, R., 1991 and Haase, A. B., and Stewart, R. R., 2004) is commonly used for Q estimation from VSP data, which is based on the frequency dependence of the amplitude decay of the traveling wavelet. However, the traveling wavelet also experiences energy loss due to other factors such as geometric spreading, transmission and reflection loss, which can be regarded as frequency independent energy loss and should be taken into account in the spectral ratio method for Q analysis.

For real data, there are usually notches in the amplitude spectrum of seismic wavelet, which will cause problems for the spectral division. To apply the spectral ratio method, it is necessary to obtain an appropriately smoothed spectrum for each individual wavelet. Thomson (1982) proposed a multitaper method used to produce a smooth, high resolution spectral estimation, which has been shown to provide low variance estimation with less spectral leakage when applied to seismic data (Park et al., 1987; Neep et al., 1996).

The purpose of our work is to investigate the capability of the spectral method to deal with frequency independent energy loss and layered Q structure, and the multitaper method to estimate a smooth spectrum from a short time series. This article is organized as follows: the first part introduces the spectral ratio method. Then, the adaptive multitaper method for spectrum estimation is discussed. Following the above theoretical material, numerical examples will be used to evaluate the spectral ratio method for Q analysis, and the multitaper method for spectrum smoothing. Finally, some basic conclusions are drawn from results of the examples.

SPECTRAL RATIO METHOD FOR Q ESTIMATION

Based on the constant Q model (e.g. Aki and Richards, 1980) of seismic attenuation, the amplitude spectrum of the seismic wave with a travel-time t can be formulated as

$$A(f) = A_0(f)\exp\left(\frac{-\pi f t}{Q}\right), \quad (1)$$

where f is the frequency, $A(f)$ is the amplitude spectrum of the traveling wavelet, $A_0(f)$ is the amplitude spectrum of source wavelet or a reference wavelet, and Q is the attenuation quality factor. Taking geometric spreading into account, equation (1) can be modified as

$$A(f,x) = \mathrm{G(x)}A_0(f)\exp\left(\frac{-\pi f t}{Q}\right), \quad (2)$$

where $\mathrm{G(x)}$ is the frequency-independent geometric spreading factor. Taking the natural logarithm of both sides, equation (2) can be rearranged as

$$\ln\left(\frac{\mathrm{A(x,f)}}{A_0(f)}\right) = \ln\big(\mathrm{G(x)}\big) - \frac{\pi f t}{\mathrm{Q}}\,. \quad (3)$$

Therefore, the Q factor and be estimated from fitting a straight line to the spectral ratio over a finite frequency range, whose intercept on the coordinate is a measure of energy loss due to geometric spreading. More generally, the $\mathrm{G(x)}$ term in equation (3) can represent all the frequency independent energy loss in total, including spherical divergence, reflection and transmission loss. Then, the relation between Q and the slope k of the straight line can formulated as

$$\mathrm{Q} = -\frac{\pi f t}{k}\,. \quad (4)$$

The subsurface is usually supposed to be a layered medium. Accordingly, the Q factor varies with layers or depth. The Q factor estimated by equation (4) is the average Q of a specific time/depth range which corresponds to a single layer or a few layers. The relation between average Q and interval Q can be expressed as following

$$\frac{\mathrm{T_n}}{\mathrm{Q_a^n}} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{\Delta \mathrm{t_i}}{\mathrm{Q_i}}. \quad (5)$$

Where $\mathrm{Q_a^n}$ is the average Q value for n layers, $\mathrm{T_n}$ is the total travel time for n layers, $\Delta\mathrm{t_i}$ and $\mathrm{Q_i}$ are the travel-time and interval Q corresponding layer i respectively. From equation (5), the interval Q value can be expressed using average Q value as

$$\frac{1}{\mathrm{Q_n}} = \frac{\frac{\mathrm{T_n}}{\mathrm{Q_a^n}} - \frac{\mathrm{T_{n-1}}}{\mathrm{Q_a^{n-1}}}}{\mathrm{T_n} - \mathrm{T_{n-1}}}\,. \quad (6)$$

For VSP data, the interval Q can be directly obtained by the spectral ratio method. For surface reflection data, the estimation of average Q is more robust than the interval Q estimation (Wang, 2008), then, the average Q values can be converted to interval Q values based on equation (6).

ADAPTIVE MULTITAPER METHOD FOR SPECTRAL ESTIMATION

When applying the Fourier transform to a finite-length signal or a finite-length segment of infinite signal, it appears that some energy has leaked out of the original spectrum into neighbouring frequencies. Such an effect is called spectral leakage. The general idea of mutitaper technique is to weight the data by several spectral leakage resistant tapers, then combine the spectra of tapered data to form a single spectral estimation. The details can be found in the original work by Thomson (1982). We introduce the multitaper method following the outline given by Park et at (1987).

The aim is to estimate the amplitude spectrum S(f) of the finite time series $\{ s_{i;}\, i = 0,1,2,\dots,N-1\}$. Suppose the taper series is $\{ w_{i;}\, i = 0,1,2,\dots,N-1\}$. So, the DFT of the taper is

$$W(f) = \sum_{j=0}^{N-1} w_j \, e^{i2\pi fj} \, . \quad (7)$$

Here, we assume the interval time between two successive samples is unit 1. The frequency can be defined on the principal domain (-1/2, 1/2]. The spectral leakage property of the taper can be deduced from its DFT. Suppose the taper is designed to minimize the bias at a given frequency caused by the spectral leakage from outside the frequency band $|f - f_0| \leq F$. This is equivalent to maximizing the fraction of energy within the 2F frequency band formulated as

$$\lambda(N,F) = \frac{\int_{-F}^{F} |W(f)|^2 df}{\int_{-1/2}^{1/2} |W(f)|^2 df}. \quad (8)$$

Substitute equation (7) into equation (8). The $\lambda(N,F)$ can be expressed in a matrix form

$$\lambda(N,F) = \frac{\boldsymbol{w}\cdot\mathbf{C}\cdot\boldsymbol{w}}{\boldsymbol{w}\cdot\boldsymbol{w}}, \quad (9)$$

where $\boldsymbol{w}$ is the taper vector , and matrix $\mathbf{C}$ has following components

$$\mathbf{C}_{kl} = \frac{\sin[2\pi F(k-l)]}{\pi(k-l)}; \;\; k,l = 0,1,2,\dots,N-1 \, . \quad (10)$$

Then the equation (9) becomes an eigenvalue problem

$$\mathbf{C}\cdot\boldsymbol{w} = \lambda\boldsymbol{w}, \quad (11)$$

which has a solution with N eigenvalues $1 > \lambda_0 > \lambda_1 > \cdots > \lambda_{N-1} > 0$ and associated normalized eigenvectors $\boldsymbol{w}^k$, k=0,1,2,….N-1. The $\boldsymbol{w}^k$ are so-called discrete prolate spheroidal sequence (Slepian, 1978) and also referred as prolate eigentapers (Park et al, 1987). A prolate eigentaper with a time bandwidth product of P=NF is called a Pπ prolate taper, which concentrate the spectral energy within a bandwidth of 2F. Note that the matrix C is Hermitian. Then, the N eigentapers are orthogonal. So, each of them provide an orthogonal sample of the original wavelets. The tapers are constructed so that different parts of wavelet are recovered by different tapers without interference while optimizing the resistance to spectral leakage.

To obtain a multitaper spectral estimation, the spectra of tapered data is calculated first as

$$S^k(f) = \sum_{j=0}^{N-1} w_j^k s_j \, e^{i2\pi fj} \, . \quad (12)$$

It is conventional to employ only several lowest order eigentapers in equation (12), because the resistant to spectral leakage becomes poor with the increase of the order of eigentaper. Then, an estimation can be made from weighted sum of the spectra $S^k(f)$ as following

$$\tilde{S}(f) = \frac{1}{M}\sum_{k=0}^{M-1}\frac{|S^k(f)|^2}{\lambda_k}. \tag{13}$$

A better estimation can be obtained by an adaptive multitaper method formulated as

$$\bar{S}(f) = \frac{\sum_{k=0}^{M-1}|d^k(f)S^k(f)|^2}{\sum_{k=0}^{M-1}|S^k(f)|^2}, \tag{14}$$

where the frequency dependent weight function $d^k(f)$ is given by

$$d^k(f) = \frac{\sqrt{\lambda_k}S(f)}{\lambda_k S(f)+E\{L_k(f)\}}. \tag{15}$$

$S(f)$ is the true unknown spectrum and $L_k(f)$ is the spectral leakage at frequency f , which can be approximated as

$$E\{L_k(f) = (1-\lambda_k)\sum_{i=1}^{N-1}s_i^2. \tag{16}$$

The true $S(f)$ in equation (15) can be replaced by its estimation. Then, the adaptive estimation $\bar{S}(f)$ can be obtained through iterative calculation.

EXAMPLES

The synthetic VSP and reflection data in this article are produced using the Tiger software developed by the SINTEF research group. To incorporate the Q attenuation into the seismic modeling, we used viscoacoustic method to which The Tiger software gives time-domain finite difference solution in 3D. The source wavelet used for seismic modeling is a ricker wavelet with the peak at 0.1s and a maximum frequency of 70Hz.

Figure 1 shows a simple two layer velocity model and the zero offset VSP data with a constant Q of 50. It is clear that the downgoing wavelets decay with depth. To make a comparison between the two cases of constant Q attenuation and no Q attenuation, we repeat the VSP modeling without Q attenuation. Figure 2 shows the VSP records at depths of 150m and 450 for these two cases. We can see that the downgoing wavelet experiences geometric spreading, transmission loss and Q attenuation, and the former two factors are the main source of energy loss. So, the spectral ratio method for Q estimation should take the frequency independent energy into account. In addition, we should use some taper window to retrieve the first break from the VSP records for calculating the spectrum. For the wavelet shown in figure 2, a box-car window is sufficient. Figure 3 shows the spectral ratio calculated using equation (3) from the wavelets shown in figure 2. The estimated Q values for the two cases are 51.6 and 874.8, which is consistent with the theoretical Q values. The results show that the spectral ratio method can deal with frequency independent energy loss, and the Tiger software can produce synthetic data for viscoacoustic media with sufficient accuracy.

Using the velocity model in figure 1, but assigning Q1=50 and Q2=100 to the two layer respectively gives a more interesting test. After repeating the VSP modeling and obtaining the VSP records at depth 150m and 450m, the two traces are shown in figure 4, and the corresponding spectral ratio is shown in figure 5. Choosing a frequency band 10-60Hz, the estimated Q value is 62.3. Based on equation (5), the theoretical average Q value from depth 150m to depth 450m is 60. Therefore, the spectral ratio method gives an answer consistent with the expected average Q estimation and can work under layered Q subsurface.

Figure 6 displays a three layered velocity and Q model. The corresponding synthetic shot record gather is shown in figure 7. And the zero offset trace after first break removal is shown in figure 8. For this case, we can retrieve the two reflection events by windowing the seismic trace, and then conduct the Q estimation using spectral ratio method. The calculated spectral ratio between the two reflection events are shown in figure 9. The estimated Q value is 70.2, which is still a close estimation to the true Q value 80.

A more realistic velocity model is calculated from well log as shown in figure 10. Running the zero offset VSP modeling using the true velocity model with a constant Q of 50, gives the two records at depth 200m, 700m, as shown in figure 11. Appling the box-car window to retrieve the first breaks of dowgoing wavelets, and then calculating the spectrum leads to the spectral ratio shown in Figure 12. Choosing a frequency band-with of 10-70Hz, the estimated Q is 56.1. Using the same velocity and Q model as above, a shot record gather is obtained. The zero offset shot record is shown in figure 13. Two pieces of the shot record are chosen to conduct a Q analysis. One is from 200ms to 350ms, and the other is from 350ms to 500ms. Three methods are employed to calculate the spectrum for these two record pieces, which are direct FFT, adaptive multitaper method, and the Burg method. For the multitaper method, we use the five lowest order 4π prolate tapers shown in figure 14. The calculated spectra are demonstrated in figure 15. We can see that the mutitaper method has the best smoothing effect. The corresponding spectral ratios are shown in figure 15. A frequency band 10-70 Hz is chosen to do the Q estimation, the estimated Q values are 137.1, 34.2 and 38.4 in order for the three method. The estimate result is highly dependent on which pieces of the shot record are chosen. We only can get a rough estimation of the Q attenuation. From this test, we can see that the adaptive multitaper method is effective to estimate a smooth spectrum for a short time series.

From the above tests, we can see that the VSP and reflection date is consistent with the velocity and Q model very well, which demonstrates that the Tiger tool gives accurate seismic modeling for viscoacoustic media.

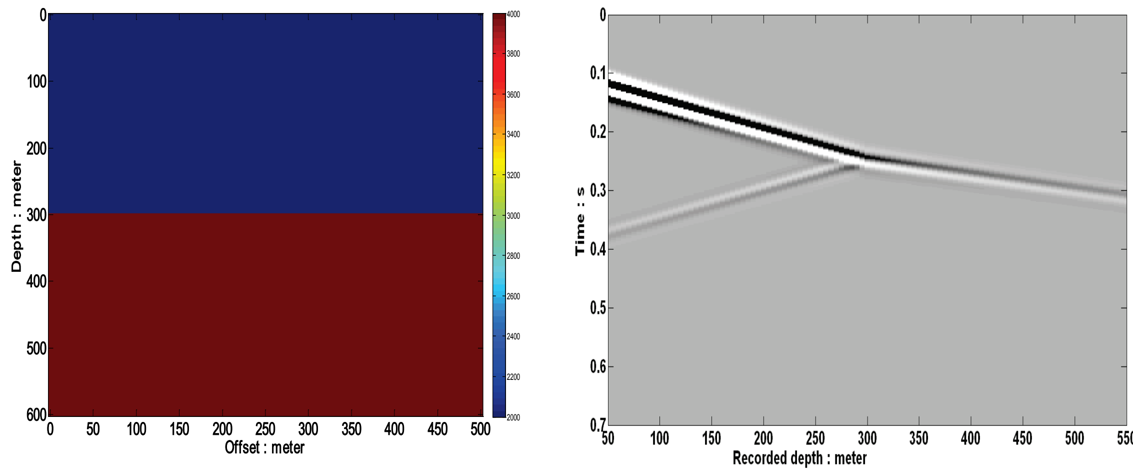

Figure 1. (left) a two layer velocity model; (right) the VSP data recorded from 50-550m using the left velocity model and Q=50.

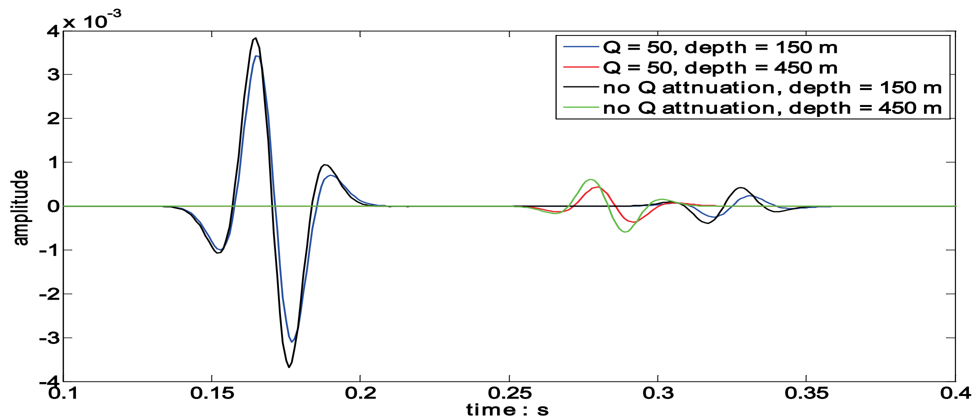

Figure 2. Comparison of VSP trace with/without constant Q attenuation

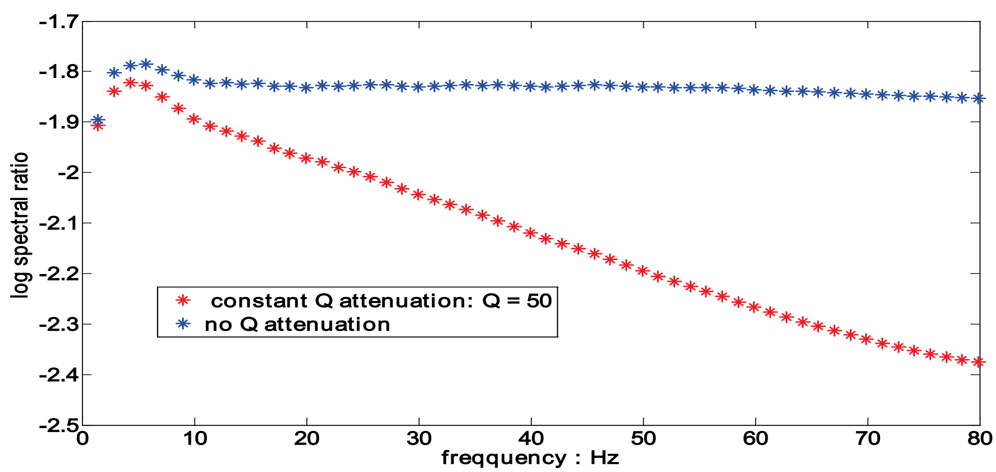

Figure 3. The spectral ratio calculated from the wavelets shown in figure 2: the estimated Q is 51.6 for the constant Q=50 attenuation case, and 874.8 for the no Q attenuation case(box-car window applied before spectrum calculation).

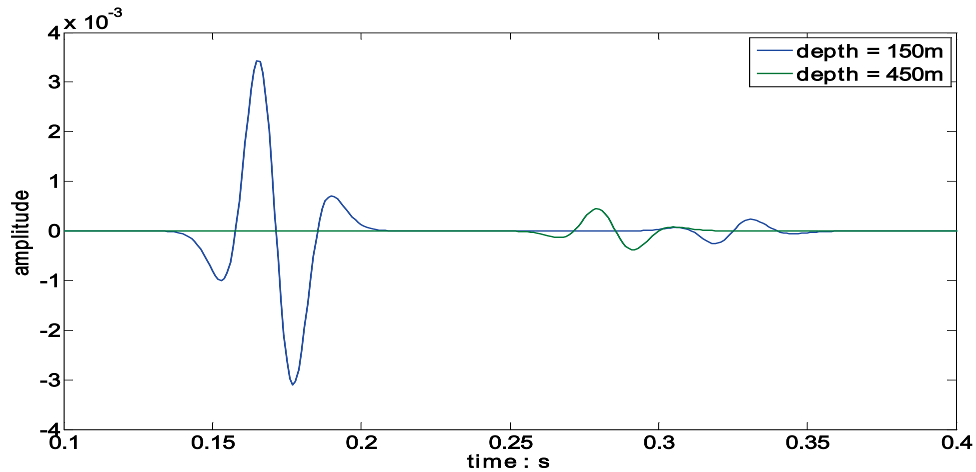

Figure 4. Two VSP records at depth of 150m and 50m for two layer model with (v1=2000m/s,Q1=50) and (v2=3000m/s, Q2=100).

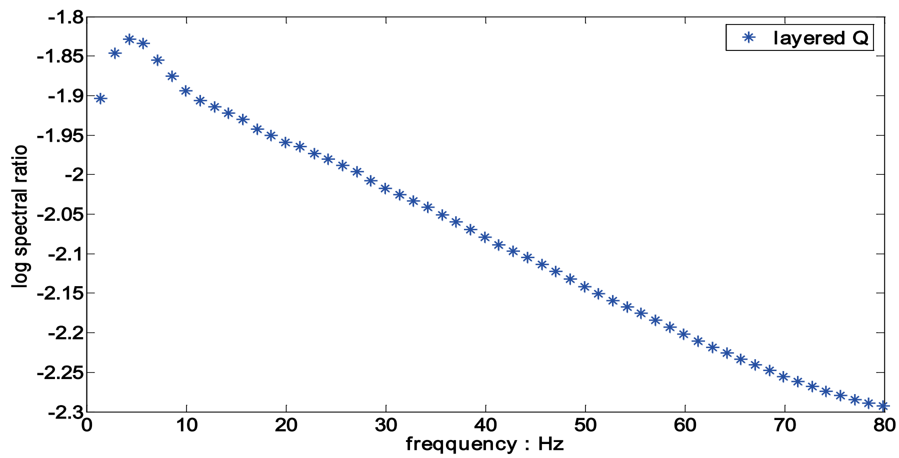

Figure 5. The spectral ratio calculated from the wavelets shown in figure 4 (box-car window applied before spectrum calculation).

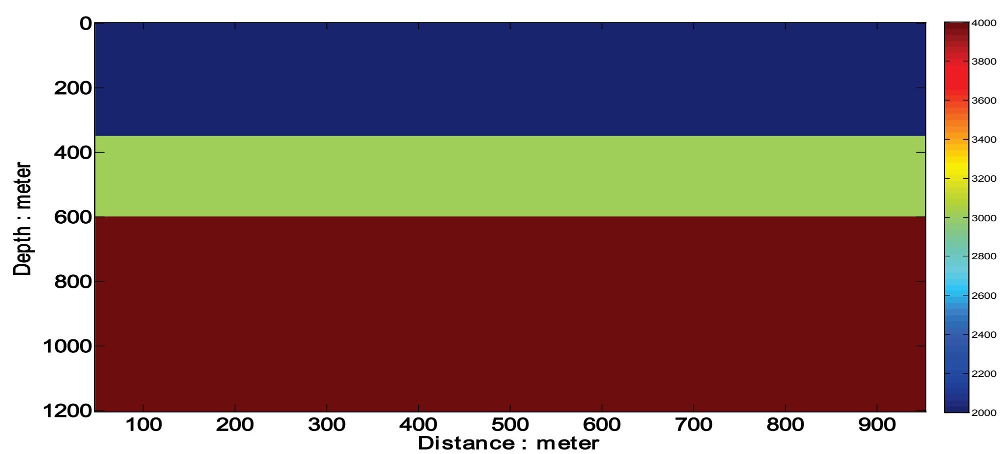

Figure 6. A three layer velocity and Q model with (v1=2000m/s, v2=3000m/s, v3=4000m/s) and (Q1=50, Q2=80, Q3=100).

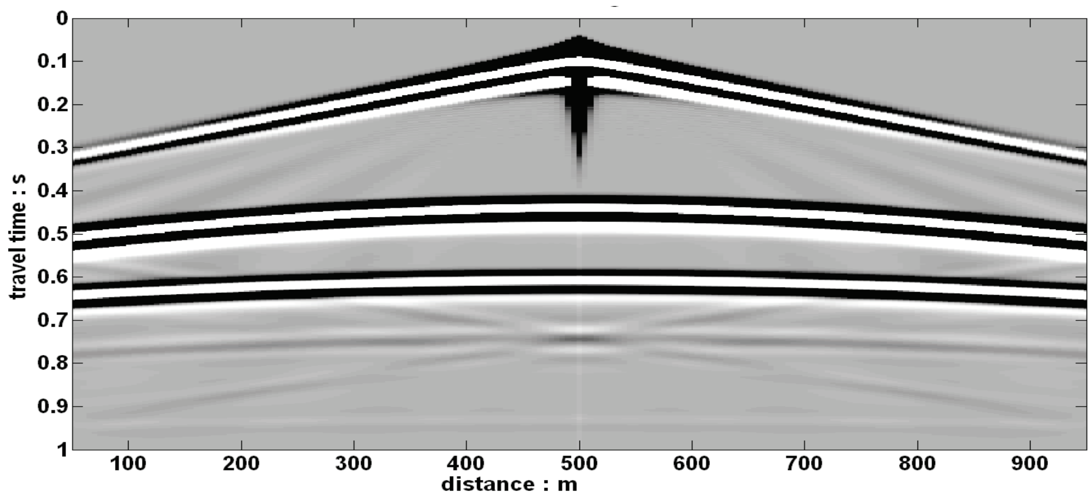

Figure 7. Shot record using the velocity and Q model shown in figure 6.

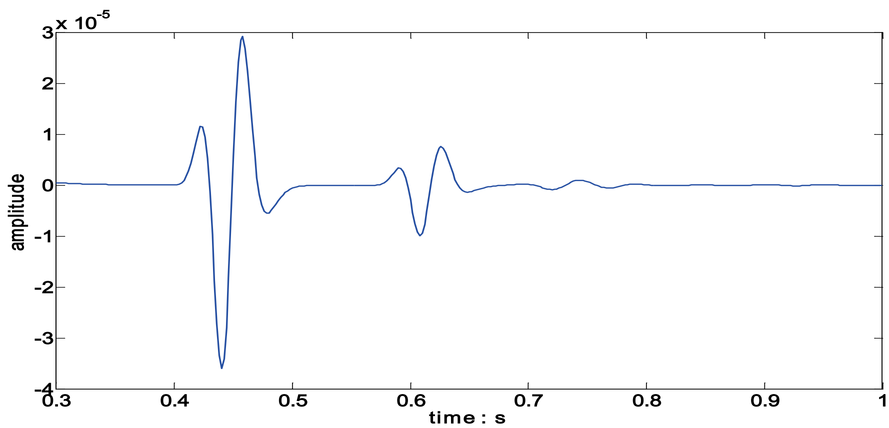

Figure 8. The zero offset shot record shown in figure 7 (first break removed).

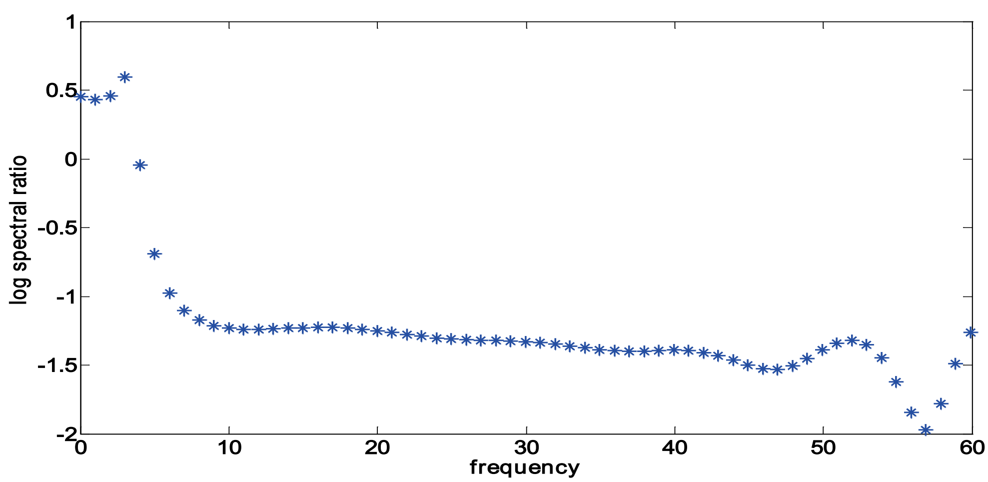

Figure 9. The spectral ratio for the two reflection events in figure 8 (box-car window used to retrieve separate events before spectrum calculation).

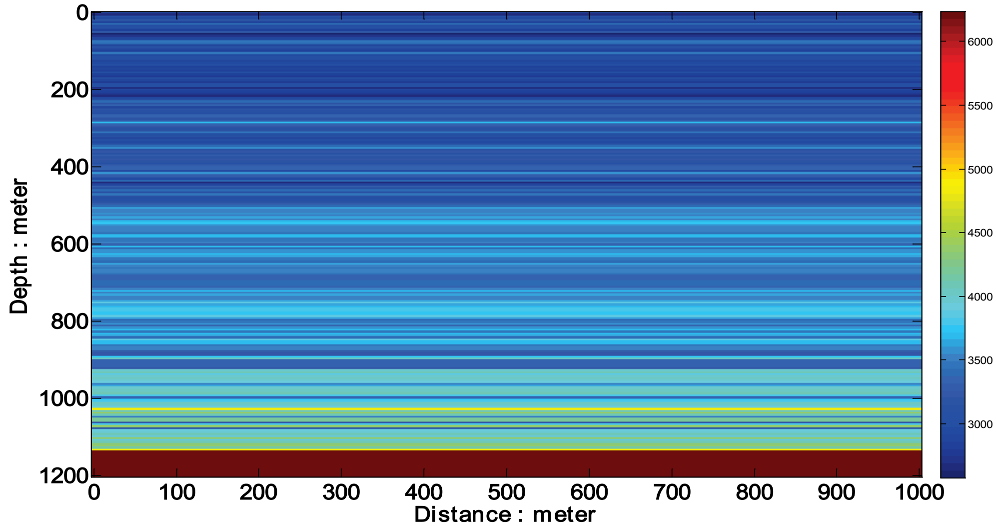

Figure 10. velocity model calculated from a well log 0/14-09-023-23W4 in Alberta

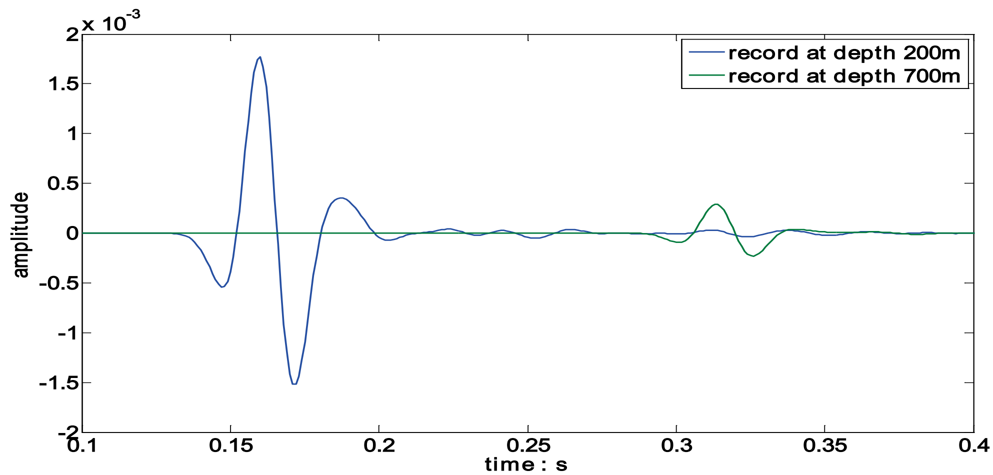

Figure 11. Two VSP records at depth 200m and 700m using the velocity model in figure 11 and a constant Q=50.

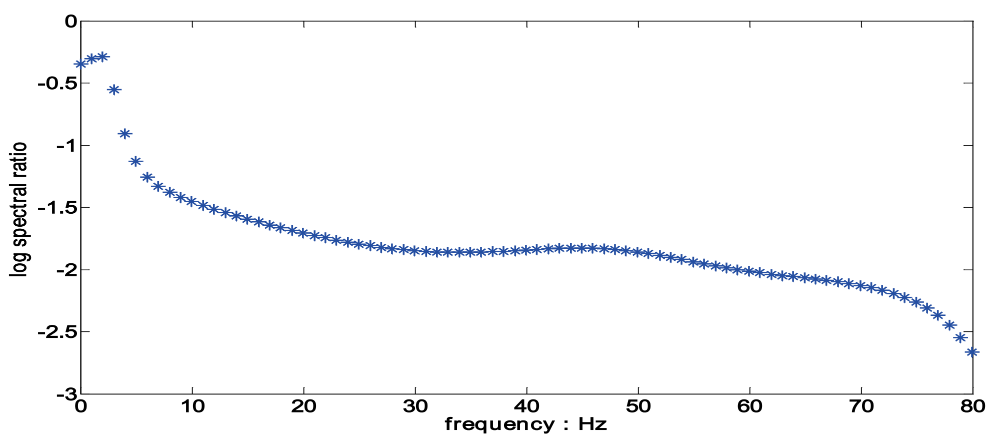

Figure 12. the spectral ratio calculated from the wavelets shown in figure 11 (box-car window applied to retrieve first breaks before spectrum calculation).

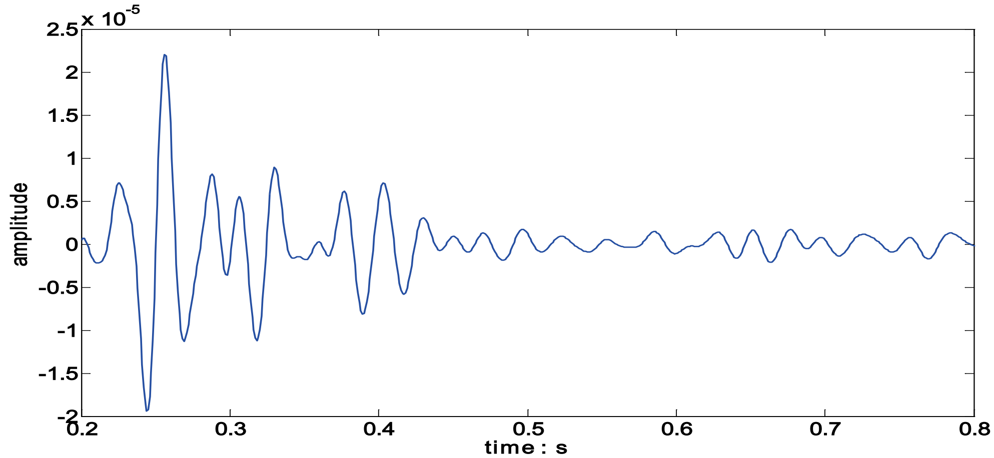

Figure 13. A zero offset shot record using the velocity model shown in figure and a constant Q of 50 (first break removed).

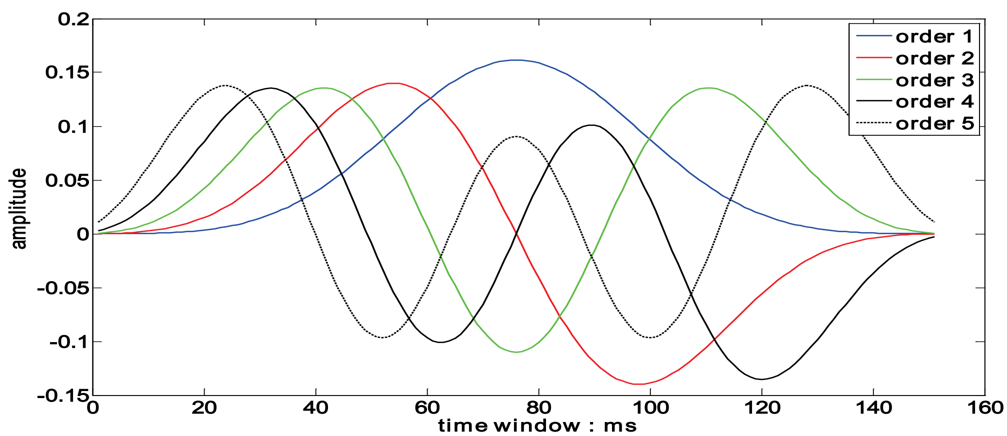

Figure 14. The five lowest order 4π prolate tapers with 151 points.

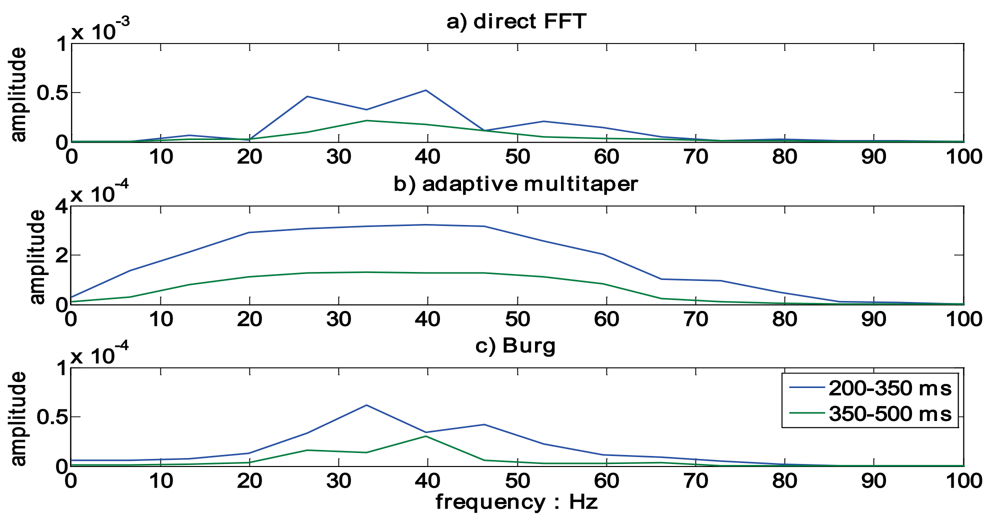

Figure 15. The amplitude spectra of the 200-350ms and 350-500ms pieces of the shot record in figure 13.

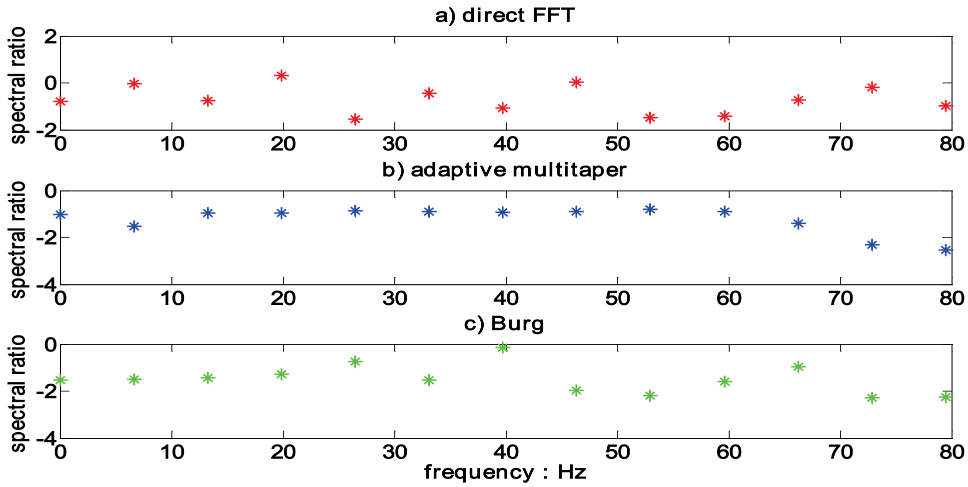

Figure 16. The Spectral ratio calculated from the spectra shown in figure 15.

CONCLUSION

The spectral ratio method is a classic method for Q estimation. Theoretically, the spectral ratio method can deal with various frequency independent energy loss such as geometric spreading, transmission and reflection loss, and can give accurate average Q estimation in case of layered Q structure. In addition, the spectral method can give a rough Q estimation using reflection data.

To deal with real data, it is necessary to obtain smooth spectra estimation for calculating spectral ratio. The adaptive multitaper method is shown to be effective for estimating smooth spectrum from a short time series.

In addition, testing shows that the Tiger software gives very realistic results for viscoacoustic modeling.

ACKNOWLEDGEMENTS

The authors would like to thank Faranak Mahmoudian for her help in using the Tiger software. We also acknowledge the sponsors of CREWES project, NSERC, POTSI and MITACS for their financial support.

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