P and S velocity approximations in a poroviscoelastic medium

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ABSTRACT

Biot's equations of particle motion for wave propagation in a fluid saturated poroviscoelastic medium are manipulated to obtain zero and first order approximations to the fast compressional (P) wave and shear (S) wave velocities. The expressions obtained are used in the numerical investigation of the effects on these velocities resulting from the variation of quantities defining the solid and the fluid, specifically porosity and permeability, as well as others, inherent in the theory. In addition, the first order velocity approximations are complex functions, in terms of the quality factors, Q_P and Q_S, which define the attenuation properties in a poroviscoelastic medium. There are numerous formulations of the equations of motion for this problem, together with differences in notation. A fairly standard definition has been chosen for use here, where the vector quantities indicate the particle displacement vector in the solid and the particle displacement vector of the fluid relative to that in the solid. Zero and first order expressions for the complex fast compressional wave velocity and the shear wave velocity are obtained and used within the context of viscoelasticity to obtain some initial insight into the more general poroviscoelastic problem.

INTRODUCTION

What is of interest in hydrocarbon exploration almost exclusively involve porous solids saturated by a fluid or fluids. To this end the study of wave propagation in fluid-saturated porous media is of considerable interest in the disciplines of seismology and petroleum reservoir engineering. Apart from the earlier work of Frenkel (1944) the basic theory of wave propagation in such a media type was developed by Biot (1956a, 1956b, 1962).

Biot described the coupled solid-fluid motion and showed that in an infinite fluid-saturated porous solids there exist two compressional waves traveling with two different velocities (the fast P_1 and slow P_2 waves, respectively) as well as a shear S wave. The fast P_1 compressional wave and the shear S wave correspond to similar modes in elastic theory. A diffusion-type process resulting from the viscous coupling between the solid and fluid phases produces the slow P_2 compressional wave. The existence of the slow P_2 compressional wave was confirmed experimentally by Plona (1980) and Coussy and Bourbie (1984).

Approximate expressions for velocities related to the fast P_1 compressional wave and the shear S wave are obtained here from the equations of displacement motion using a plane wave assumption and the theory of successive approximations. The formulae obtained are then plotted against a parameter, which are of interest in the seismic area of this general problem.

THEORY

The theory of wave propagation in a poroviscoelastic medium has been described by Frenkel (1944) and Biot (1956a, 1956b and 1962), which takes into consideration the dissipation of energy due to the relative motion between pore fluid and the solid matrix. The additional (secondary or slow) compressional *P*-wave, is predicted by the theoretical developments in the above works. The physical existence of this wave was confirmed by Plona (1980) and Coussy and Bourbie (1984)**.** Physical interpretations of the elastic constants in this theory may be found in the works of Biot and Willis (1957), Geertsma and Smit (1961) and Pride et al. (1992), among others.

A fluid-saturated porous medium is defined as an interacting two-phase elastic system, according to Biot's theory. The deformation of the saturated porous medium due to the impinging of seismic waves results in coupled solid-fluid motion. The flow of the viscous fluid with respect to the solid skeleton produces an energy loss.

In a poroviscoelastic medium that is fluid saturated, macroscopically isotropic and locally homogeneous, Biot's (1956a, 1956b, 1962) equations, the equations of motion in terms of displacement, which differ from those presented by Frenkel[1], have the following form

$$\rho\ddot{\mathbf{u}} + \rho_f\ddot{\mathbf{w}} - \left(\lambda_c + \mu\right)\nabla\nabla\cdot\mathbf{u} - \mu\nabla^2\mathbf{u} - \alpha M\nabla\nabla\cdot\mathbf{w} = 0 \qquad (1)$$

$$\rho_f\ddot{\mathbf{u}} + m\ddot{\mathbf{w}} + b\dot{\mathbf{w}} - \alpha M\nabla\nabla\cdot\mathbf{u} - M\nabla\nabla\cdot\mathbf{w} = 0 \qquad (2)$$

The problem is fully specified in an infinite isotropic space by including source terms together with initial conditions for $\mathbf{u}\big|_{t=0}$, $\dot{\mathbf{u}}\big|_{t=0}$, $\mathbf{w}\big|_{t=0}$ and $\dot{\mathbf{w}}\big|_{t=0}$. Here $\mathbf{u}$ is the displacement vector for the solid, $\mathbf{w}$ the displacement vector for the fluid relative to that for the solid, ρ the macroscopic density of the fluid saturated medium determined by $\rho = \phi\rho_f + \left(1-\phi\right)\rho_s$, ρ_f and ρ_s being the densities of the fluid and solid. The quantities $\gamma_f = \phi\rho_f$ and $\gamma_s = \left(1-\phi\right)\rho_s$ are often referred to as the effective densities of the fluid and solid where ϕ specifies the porosity. Lamé's parameter for the saturated matrix is denoted as λ_c $\left[\lambda_c = \lambda + \alpha^2 M\right]$2, and Lamé's shear modulus parameter for the dry porous matrix as μ. T [3] is an experimentally determined structure factor called the "Tortuosity". m [4] is the effective fluid density which describes the mass coupling and is

1 In Frenkel's formulation the two vector quantities are the displacement vector for the solid and the displacement vector for the fluid.

2 α, M = Biot's parameters accounting for compressibility in the two-phased material - (Philippacopoulos, 1988).

3 Tortuosity is an experimentally determined structure factor. often "estimated" as: $T = \left(1/2\right)\left(1 + 1/\phi\right)$. It will be assumed here to have that definition. Packing of spheres: $T = 1 - r\left(1 - 1/\phi\right)$. For the case of solid spherical particles in a fluid, $r = 1/2$.

4 The parameter m has the dimensions of density.

defined as $m = T\rho_f/\phi$. η is the viscosity of the fluid $(kg/m\,s)$. κ[5] the permeability of the porous medium (m^2). Viscous coupling due to the relative motion of pore fluid is accounted for by b, the mobility ratio of the fluid, defined as $b = \eta/\kappa$. The parameters K_s and K_f are the bulk moduli (inverse of compressibilities) of the solid and fluid and K_b is the bulk modulus of the dry porous matrix. α is the poroelastic coefficient of effective stress defined by $\alpha = 1 - K_b/K_s$ and M, the coupling parameter between the solid and the fluid given by $M = \left[\phi/K_f + (\alpha - \phi)/K_s\right]^{-1}$. In the high frequency case, where the assumption of Poiseuille flow6 is not valid, the viscosity may depend upon frequency and pore structure in a complicated manner, as discussed by Biot (1962). In most seismic situations, this is not what is being dealt with (Yang and Sato, 1998).

S - WAVE VELOCITY

Let $\mathbf{\Omega}_1 = \nabla \times \mathbf{u}$ and $\mathbf{\Omega}_2 = \nabla \times \mathbf{w}$ (the angular velocities of the solid matrix and the angular velocity of the liquid phase measured relative to the solid matrix angular velocity). Applying the operator "$\nabla\times$" to equations (1) and (2) results in

$$\begin{aligned}\rho(\nabla\times\ddot{\mathbf{u}}) + \rho_f(\nabla\times\ddot{\mathbf{w}}) - (\lambda_c + \mu)(\nabla\times\nabla\nabla\cdot\mathbf{u}) - \mu(\nabla\times\nabla^2\mathbf{u}) \\ -\alpha M(\nabla\times\nabla\nabla\cdot\mathbf{w}) = 0\end{aligned} \tag{3}$$

and

$$\begin{aligned}\rho_f(\nabla\times\ddot{\mathbf{u}}) + m(\nabla\times\ddot{\mathbf{w}}) + b(\nabla\times\dot{\mathbf{w}}) - \alpha M(\nabla\times\nabla\nabla\cdot\mathbf{u}) - \\ M(\nabla\times\nabla\nabla\cdot\mathbf{w}) = 0\end{aligned} \tag{4}$$

Employing vector identities yields

$$\rho\ddot{\Omega}_1 + \rho_f\ddot{\Omega}_2 - \mu\nabla^2\Omega_1 = 0 \tag{5}$$

$$\rho_f\ddot{\Omega}_1 + m\ddot{\Omega}_2 + b\dot{\Omega}_2 = 0 \tag{6}$$

Assume plane wave solutions for $\mathbf{\Omega}_1$ and $\mathbf{\Omega}_2$ of the form

$$\mathbf{\Omega}_i = \mathbf{\Omega}_i e^{-i\omega t + i\omega q x} \quad (i = 1, 2). \tag{7}$$

The direction x is arbitrary as the poroviscoelastic medium has been specified as being isotropic, so that the propagation velocity is independent of direction. As a consequence, equations (5) and (6) become

5 There are the absolute permeability, k, and effective permeability, $\kappa = k/\phi$, which is assumed here.
6 Poiseuille flow implies laminar flow indicating there is no turbulence.

$$\rho\mathbf{\Omega}_1 + \rho_f\mathbf{\Omega}_2 - \mu q^2\mathbf{\Omega}_1 = 0 \tag{8}$$

$$\rho_f\mathbf{\Omega}_1 + m\mathbf{\Omega}_2 + \frac{b}{i\omega}\mathbf{\Omega}_2 = 0 \tag{9}$$

where both of the above equations have been divided by $(i\omega)^2$.

Let $\xi = q^2 = V_S^{-2}$, with V_S being the shear wave velocity in the poroviscoelastic medium. Simple algebraic methods can be employed to isolate the quantities $\mathbf{\Omega}_1$ and $\mathbf{\Omega}_2$ in (8) and (9) to yield

$$\begin{bmatrix} \mu\xi - \rho & \rho_f \\ \rho_f & m + b/i\omega \end{bmatrix} \begin{bmatrix} \mathbf{\Omega}_1 \\ \mathbf{\Omega}_2 \end{bmatrix} = 0 \tag{10}$$

For (10) to have a solution, the determinant must equal zero, resulting in

$$\left[m + \frac{b}{i\omega}\right]\left[\mu\xi - \rho\right] - \rho_f^2 = 0 \tag{11}$$

The solution of equation (11) for ξ, employing the definition of V_S in terms of ξ, is

$$\frac{1}{V_S^2} = \frac{\rho}{\mu}\left[1 + \frac{\omega^2\rho_f^2 m}{\rho b^2\left[1 + \left(\omega m/b\right)^2\right]}\right] + \frac{i\omega\rho_f^2}{\mu b\left[1 + \left(\omega m/b\right)^2\right]} \tag{12}$$

If it is assumed that the dimensionless quantity $\omega m/b$ is such that $\omega m/b << 1$ equation (12) reduces to the approximate form

$$\frac{1}{V_S^2} = \frac{\rho}{\mu}\left[1 + \frac{\omega^2\rho_f^2 m}{\rho b^2}\right] + \frac{i\omega\rho_f^2}{\mu b} \tag{13}$$

or

$$\frac{1}{V_S^2} \approx \frac{\rho}{\mu}\left[1 + \frac{i}{\left(\rho b/\omega\rho_f^2\right)}\right] = \frac{1}{V_{S_0}^2}\left[1 + \frac{i}{\left(\rho b/\omega\rho_f^2\right)}\right]. \tag{14}$$

Comparing the above approximate equation for the shear wave velocity in a poroviscoelastic medium with that for an viscoelastic media, the generally complex velocity for an viscoelastic medium where the quality factor Q is such that $Q \gg 1$ may be written as

$$\frac{1}{v^2} = \frac{1}{V^2}\left(1 + \frac{i}{Q}\right) \quad \leftarrow \quad \textit{Anelastic case} \tag{15}$$

where V is some real phase velocity and v is the complex viscoelastic velocity characterizing the medium.

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Comparing equations (14) and (15) results in

$$Q_S^{-1} \approx \frac{\omega \rho_f^2}{\rho b} = \frac{1}{\left[\rho b / \omega \rho_f^2\right]}, \tag{16}$$

a real dimensionless quantity, as required. Consequently for a poroviscoelastic medium the complex shear wave velocity has the approximate form

$$\frac{1}{V_S^2} \approx \frac{\rho}{\mu}\left[1 + \frac{i}{Q_S}\right] = \frac{1}{V_{S_0}^2}\left[1 + \frac{i}{Q_S}\right] \tag{17}$$

with Q_S given by equation (16) and $1/V_{S_0}^2 = \rho/\mu$, where both ρ and μ have been previously defined.

An alternate manner of obtaining an approximation for the shear wave velocity from the determinant, equation (10) is to use the method of successive approximations. As this manner of solution will be required for the determination of the *fast* – *P* – wave velocity requires it will initially be introduced here to treat a much simpler problem. Assume a solution of equation (11) of the form

$$\xi = \xi_0 + i\varsigma\xi_1 + \cdots \tag{18}$$

where $\varsigma = \omega m/b \ll 1$, ς being a dimensionless variable. Starting with the determinant (equation (11)) and substituting equation (18) leads to

$$\mu\xi_0 + i\varsigma\mu\xi_1 - \rho + \frac{\mu\xi_0}{i\varsigma} + \mu\xi_1 - \frac{\rho}{i\varsigma} = \frac{\rho_f^2}{m} \tag{19}$$

Extracting terms in $(i\varsigma)^{-1}$ yields

$$\xi_0 = \frac{\rho}{\mu} \tag{20}$$

so that

$$\frac{1}{V_{S_0}^2} = \frac{\rho}{\mu}. \tag{21}$$

Continuing with terms in $(i\varsigma)^0$ has

$$\mu\xi_0 - \rho + \mu\xi_1 = \frac{\rho_f^2}{m}$$
or (22)
$$\xi_1 = \frac{\rho_f^2}{\mu m}$$

Introducing equations (20) and (22) into equation (18) has

$$\xi = \left(\frac{\rho}{\mu}\right) + i\varsigma\left(\frac{\rho_f^2}{\mu m}\right) = \frac{\rho}{\mu}\left[1 + i\varsigma\left(\frac{\rho_f^2}{\rho m}\right)\right] = \frac{\rho}{\mu}\left[1 + i\left(\frac{\omega}{b}\right)\left(\frac{\rho_f^2}{\rho}\right)\right] \tag{23}$$

resulting in

$$\frac{1}{V_S^2} = \frac{1}{V_{S_0}^2}\left[1 + \frac{i}{\rho b/\omega\rho_f^2}\right], \tag{24}$$

which is the same as that derived in the previous instance (equation (14))

P – WAVE VELOCITY[7]

For determining the *fast – P –* wave velocity, let $\theta = \nabla\cdot\mathbf{u}$ and $\phi = \nabla\cdot\mathbf{w}$ [8], and apply the divergence operator to equations (1) and (2) and utilize the vector identities given in the footnote to yield

$$\rho\ddot{\theta} + \rho_f\ddot{\phi} - (\lambda_c + 2\mu)\nabla^2\theta - \alpha M\nabla^2\phi = 0 \tag{25}$$

and

7 Vector Identities:

$$\nabla\times\nabla\times\mathbf{v} = \nabla(\nabla\cdot\mathbf{v}) - \nabla^2\mathbf{v}$$

The divergence of the curl of any vector field is always zero: $\nabla\cdot(\nabla\times\mathbf{v}) = 0$ so that $\nabla\cdot(\nabla\times\nabla\times\mathbf{v}) = 0$.

Thus

$$\nabla\cdot(\nabla\times\nabla\times\mathbf{v}) = \nabla\cdot\nabla(\nabla\cdot\mathbf{v}) - \nabla\cdot\nabla^2\mathbf{v}$$

or

$$\nabla\cdot\nabla^2\mathbf{v} = \nabla\cdot\nabla(\nabla\cdot\mathbf{v}) = \nabla^2(\nabla\cdot\mathbf{v}).$$

8 Dilation.

$$\rho_f \ddot{\theta} + m\ddot{\phi} + b\dot{\phi} - \left(\alpha M + \mu\right)\nabla^2\theta - M\nabla^2\phi = 0 \qquad (26)$$

Assume plane wave solutions $\theta = \theta e^{-i\omega t + i\omega qx}$ and $\phi = \phi e^{-i\omega t + i\omega qx}$. As in the shear case addressed in the previous section, equations (25) and (26) may then be written in the form

$$\left(\rho - \left(\lambda_c + 2\mu\right)q^2\right)\theta + \left(\rho_f - \alpha M q^2\right)\phi = 0 \qquad (27)$$

and

$$\left(\rho_f - \left(\alpha M + \mu\right)q^2\right)\theta + \left(m + ib/\omega - Mq^2\right)\phi = 0 \qquad (28)$$

so that with $A = \left(\lambda_c + 2\mu\right)/\rho$ and $B = \left(\alpha M + \mu\right)/m$ the following problem results

$$\begin{bmatrix} 1 - Aq^2 & \rho_f/\rho - \alpha M/\rho\, q^2 \\ \rho_f/m - Bq^2 & 1 + b/i\omega m - M/m\, q^2 \end{bmatrix} \begin{bmatrix} \theta \\ \phi \end{bmatrix} = 0\,. \qquad (29)$$

As is standard practice, for these two equations to have a solution, the determinant of the coefficient matrix must be equal to zero, i.e.,

$$\left[1 - A\xi\right]\left[1 + b/i\omega m - M/m\,\xi\right] - \left[\rho_f/m - B\xi\right]\left[\rho_f/\rho - \alpha M/\rho\,\xi\right] = 0 \qquad (30)$$

from which the following sequence of steps are employed to obtain an approximate analytical result.

Let $\xi = q^2$ .The approximate determination of the value of ξ, corresponds to the square of the velocity of the compressional wave of the first kind (*fast* – P_1 – wave). Represent ξ in the form of a series in powers of the small parameter $i\omega m/b = i\varsigma$ [9], so that

$$\xi = \xi_0 + i\varsigma\xi_1 + \ldots \qquad (31)$$

Substituting this series into equation (30) and equate the coefficients of the various powers of $\left(i\varsigma\right)$, starting with $\left(i\varsigma\right)^{-1}$ produces

[9] In the shear wave case, ς is a dimensionless quantity. After looking at equation (*), either it must be modified by a factor of “"$\left(\text{density}\right)^{-1}$"” or ς must have the dimensions of "$\left(\text{density}\right)^{-1}$".

$$\begin{aligned}
&\left[(AM/m)-(\alpha M/\rho)B\right]\xi_0^2 + \\
&\left[(AM/m)-(\alpha M/\rho)B\right]2i\varsigma\xi_0\xi_1 - \\
&\left[(AM/m)-(\alpha M/\rho)B\right]\varsigma^2\xi_1^2 + \\
&\left[-(M/m)-A+(\rho_f/m)(\alpha M/\rho)+(\rho_f/\rho)B\right]\xi_0 - A(1/i\varsigma)\xi_0 + \\
&\left[-(M/m)-A+(\rho_f/m)(\alpha M/\rho)+(\rho_f/\rho)B\right]i\varsigma\xi_1 - A\xi_1 + \\
&1+1/i\varsigma-\left(\rho_f^2/\rho m\right)=0
\end{aligned} \tag{32}$$

For the $(i\varsigma)^{-1}$ term, with $\xi_0 = 1/V_{P_0}^2$, the above equation yields

$$\xi_0 = \frac{\rho}{\lambda_c + 2\mu} \quad \text{so that} \quad \frac{1}{V_{P_0}^2} = \frac{\rho}{\lambda_c + 2\mu} \tag{33}$$

where as previously, $\rho = \phi\rho_f + (1-\phi)\rho_s$. Solve the recursive system for ξ_1 in terms of ξ_0 and other quantities by equating powers of $(i\varsigma)^0$ has

$$\xi_1 = \left[-A+(\rho_f/m)(\alpha M/\rho)-(\alpha M/\rho)(B/A)+(\rho_f/\rho)B\right]\xi_0^2 + \left[1-\rho_f^2/(\rho m)\right]\xi_0 \tag{34}$$

Remembering that $\xi = \xi_0 + i\varsigma\xi_1 + \ldots$, $(\varsigma = \omega m/b)$, the following holds

$$\xi = \xi_0\left(1 + i\varsigma\frac{\xi_1}{\xi_0}\right) = \xi_0\left(1 + \frac{i\omega m\xi_1}{b\xi_0}\right) \tag{35}$$

so that

$$\frac{1}{V_P^2} = \frac{1}{V_{P_0}^2}\left(1 + i\varsigma\frac{\xi_1}{\xi_0}\right) = \frac{1}{V_{P_0}^2}\left(1 + \frac{iV_{P_0}^2\omega m\xi_1}{b}\right) = \frac{1}{V_{P_0}^2}\left(1 + \frac{i}{\left(b/V_{P_0}^2\omega m\xi_1\right)}\right) \tag{36}$$

It may be determined from the above equation that

$$Q_P^{-1} = \left(\frac{\omega m\xi_1}{b\xi_0}\right) \tag{37}$$

NUMERICAL RESULTS

As may be seen in the previous section, the number of independent variables required to specify $P-$wave propagation in a poroviscoelastic medium is fairly large. For this reason the shear wave case has been chosen for use in obtaining preliminary results for graphical presentation. The dependent variable in the plots displayed here is the mobility ratio, b, defined as $b = \eta/\kappa$, $\eta-$the viscosity of the fluid and $\kappa-$the permeability of the porous medium. In Figure 1 the approximate and exact values of Q are plotted versus

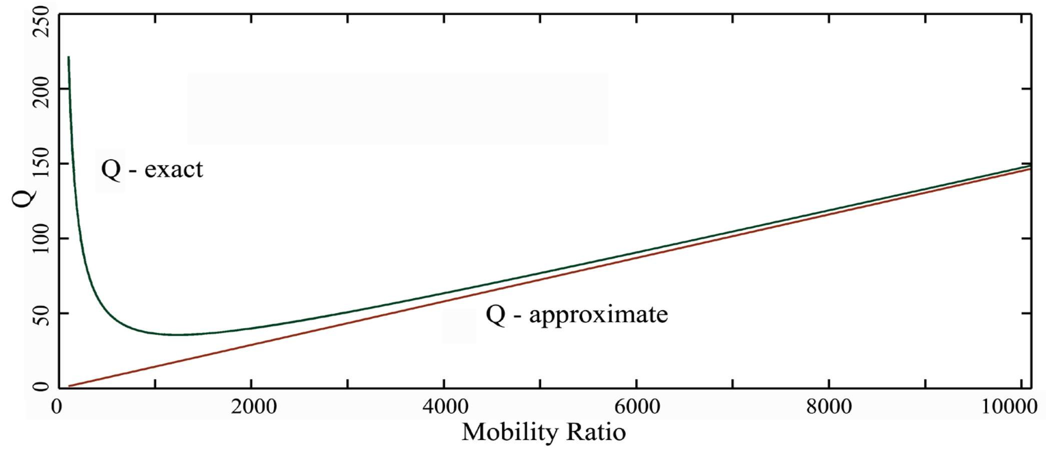

Fig. 1 Approximate and exact values of *Q* plotted versus the mobility ratio.

the mobility ratio while in Figure 2 the approximate and exact values of the real part of the shear wave velocity are plotted for b varying. The required input parameters are:

$\mu -$ shear modulus parameter for the dry porous matrix ($5100\, dyne/cm^2$).

$\rho_s -$ density of the solid ($2.65\, gm/cm^2$).

$\rho_f -$ density of the fluid ($0.88\, gm/cm^2$ 0.88 gm/cm3).

$\phi -$ porosity (0.3).

$\omega -$ frequency in Hz ($30 Hz$).

Using the above values all other parameters required in the computations may be obtained using formulae presented earlier in this report.

CONCLUSIONS

Both exact and approximate expressions for the fast $P-$wave and the shear wave in a poroviscoelastic medium. In this process it becomes evident that numerous parameters are required to be known, especially in the $P-$wave case, for these computations. For this reason, only some simple plots for the shear wave were presented. The large number of parameters required to describe a poroviscoelastic medium make it an unlikely candidate for the initial processing of seismic data and for preliminary modeling. The use of this theory, in the seismic sense, would be more applicable to the investigation of the properties of mature hydrocarbon structures where many of the required parameters have been obtained from core samples, logs of various types and production history. What is suggested is the implementation of this theory in an incremental manner; that is, upgrading the basic elastodynamic description of seismic wave propagation towards seismic wave propagation in a poroviscoelastic medium, as more information regarding the medium parameters becomes available. The use of the more advanced theory would provide a more realistic basis for reservoir development and exploitation decisions than an elastodynamic based evaluation.

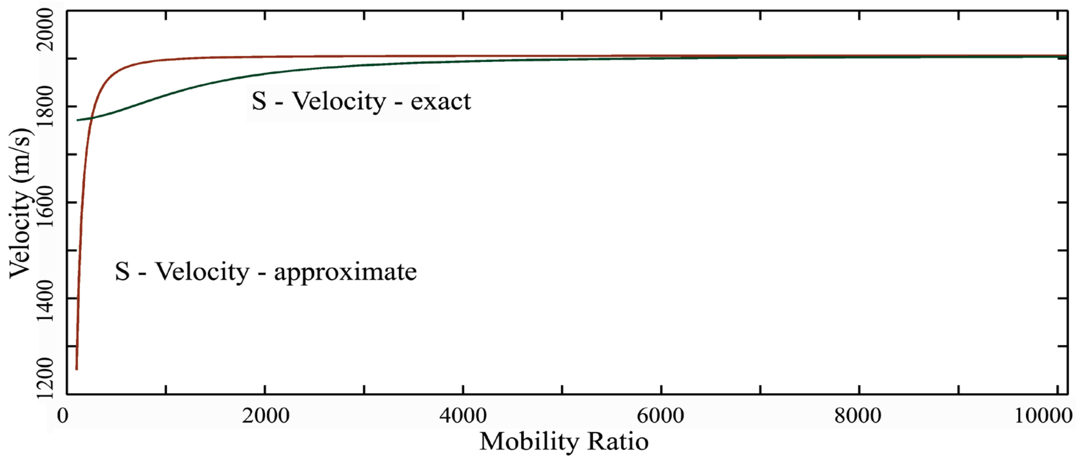

Fig. 2 Approximate and exact values of *S* – velocity (real) plotted versus the mobility ratio.

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