# Geophone rotation analysis by polarity inversion

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## ABSTRACT

A technique is presented by which multicomponent geophone data are rotated such that the preferred geophone component is aligned with the direction of the incident wave form. Wave forms of interest are restricted to first-arrival P-waves, and S-waves polarized normal to the plane that contains both the source and the geophone. Rotation is based on that fact that the geophone orientation, and the apparent rotation that it imparts to the incident waveform, is equivalent to the application of a $3 \times 3$ unitary matrix, and that the inverse of the operator is also unitary.

Given a 3C recording, the inverse operator is deduced from a processed version of the recording through inversion by least-squares. This inverse operator is then applied to the raw recording to achieve the desired orientation. Decomposition of the inverse operator yields the dip and azimuth of the geophone orientation.

Synthetic examples are presented that demonstrate the performance of this inversion in the presence of noise. Inverted waveforms are compared to the idealized input waveforms, and dip and azimuth estimates are made. It is found that waveform comparisons compare very well qualitatively in the presence of noise, and that dip and azimuth estimates degrade with increased noise. Dip and azimuth estimates are found to improve to acceptable accuracy with judicious application of band-pass filters to the input.


## INTRODUCTION

When wells are deviated, and because the orientation of downhole logging tools can not be controlled reliably, the orientation of a downhole geophone is not known (DiSiena et al., 1984). Further, when a vertical seismic profile (VSP) is acquired (see, for example, Toksöz and Stewart (1984)), compressional-wave (P-wave) sources at the surface are recorded on all three geophone components (3C) as are shear-wave (S-wave) sources (DiSiena et al., 1984).

The geometry of a surface source and a deviated well is illustrated in Figure 1. A number of raypaths are indicated in Figure 1a, and an individual raypath from that Figure is indicated in Figure 1b. Vertical channel $Z$ points along the well trajectory, and horizontal components $S_1$ and $S_2$ are orthogonal to each other, and they lie in a plane normal to $Z$. None of $Z$, $S_1$, or $S_2$ align with a coordinate system determined at the recording surface. A further complexity arises when the source location is offset from the surface expression of the well as is illustrated in Figure 1. Orientation of $Z$, $S_1$, and $S_2$ are not aligned in any direction with the incident ray (black line).

Conventional seismic analysis proceeds with pure P- and S-modes, so 3C VSP recordings must be separated into these pure modes (Zeng and McMechan, 2006). With the orientation of geophones relative to the incident modes unknown, separation of the 3C recording into pure P- and S-modes through some procedure is required.

Conventional component-rotation schemes often seek to maximize energy on the desired component channel, and minimize energy on the other channels through analysis or inversion (DiSiena et al., 1984). This means that, numerically, the 3C recording is rotated such that the desired component points in the direction of the incident wave; Vertical channel $Z$ points in the direction of an incident P-wave, and $S_1$ $(S_2)$ points in the direction and incident S-wave. For the latter, the S-wave source is usually polarized in the direction normal to the plane that contains the source location and the 3C receiver. (An S-wave source polarized in that plane will be embedded in the P-wave coda, so it will be difficult to identify.)

The rotation technique developed here is based upon the unitary nature of rotation, and of anti-rotation. First arrival P-waves (or S-waves) are identified on the 3C recording, and amplitudes are extracted from a window that contains only the first arrivals. An inversion operator is determined in a least-squares sense that has the effect on the 3C data of maximizing the energy on the desired component. The rotation operator is unitary, so it's inverse, the anti-rotation operator, is also unitary. Based on this observation, the 3C recording is normalized such that the inversion operator is determined under the assumption that the desired waveform (pure P-wave on the $Z$ component for example) and the corresponding 3C component (the $Z$ recording) have the same polarity. The resulting operator is then applied to the 3C recording.

Following the theoretical development in this paper, a number of synthetic examples are presented. Pure P- and S-modes are rotated such that they represent 3C recordings on a geophone oriented arbitrarily, and noise is added. Anti-rotation operators are determined by inversion, and pure mode signals are returned and compared to the input modes. The operator is then interrogated for the geophone orientation, and these values are compared to the actual model parameters.

Ten percent random noise in the data is found to have less impact on the shape of the pure mode that is returned than it has on the estimates of the geophone orientation. The waveforms match quite well in a qualitative sense, but the orientation estimates have significant error. Simple, band-pass filters are found to significantly improve orientation estimates such that successful mode separation is achieved.

## THEORY

The rotation method presented here is a least-squares inversion. This inversion is based on a forward model in which a rotation matrix converts the multicomponent recording (the data) into a single channel output (the model). At the core of this method lies the single assumption that the polarity of each time sample in the model matches the polarity of each time sample in the corresponding channel in the data. This means that, for a P-wave incident on a multicomponent receiver in a well bore, the polarity of each sample in the vertical channel is the same as the polarity of the vertical channel if it were pointed in the direction of the incident P-wave. Here, the only the waveform of the P-wave first arrival is considered, so the multicomponent data are windowed accordingly to capture the waveform. A similar argument can be made for an incident S-wave so long as the direct S-wave arrival is isolated from other modes by windowing.

Rotation matrices are unitary, and this fact is used to modify the forward model such that only the rotation matrix is unknown, and the modified data matrix and modified model are deduced from the original data. The modified forward model is then inverted to estimate a rotation operator, and that operator is inverted for dip and azimuth. Formally, this procedure begins with a description of the forward model as follows.

## Forward model

Multicomponent data $V$ from a geophone are written here in matrix form as:

$$V = \begin{bmatrix} S_1 \\ S_2 \\ Z \end{bmatrix}, \tag{1}$$

where $S_1$, $S_2$, and $Z$ are in-line, x-line, and vertical components of the vector wavefield respectively. Each of $S_1$, $S_2$, and $Z$ are digital recordings of $N$ samples in time, so $V$ has dimension $3 \times N$. For ideal P-wave source $w$, located $\Delta x$ away from a well bore in a homogeneous, isotropic medium, a geophone oriented such that the $Z$ component points at the source records

$$W = \begin{bmatrix} 0 \\ 0 \\ w \end{bmatrix}, \tag{2}$$

where '0's here represent vectors of zeros and $W$ is $3 \times N$. For depth $\Delta z$ and for a known orientation of $S_1$ and $S_2$ relative to compass North, downhole recording $V$ is predicted from P-wave source $w$ in two stages: 1) dip and 2) rotation.

### *Dip $\phi$*

In a laterally invariant medium, angle (dip) $\phi$ of component $Z$ relative to an incident P-wave the source location is given by

$$\phi = \phi_Z + \arcsin{(\alpha\, p)}, \tag{3}$$

where $\alpha$ is P-wave velocity. Rayparameter $p$ characterizes the direction in which the P-wave is incident upon the downhole receiver, and it is determined relative to the normal to the surface. Angle $\phi_Z$ is the dip of component $Z$ relative to the incident ray in radians. For a convention, $\phi_Z$ positive indicates that the physical top of $Z$ pointing away from a source on the positive side of the origin. Note that the polarity of $Z$ and $w$ are the same for $\phi = 0$ ($Z$ points at the incident P-wave).

Given $\phi_Z$ and $W$, then, effective recording $V_\phi$ is computed

$$V_\phi = G_\phi W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} W. \tag{4}$$

Note that the $3 \times 3$ matrix in equation 4 is unitary.

*Azimuth $\theta$*

During downhole acquisition, it is difficult to control the orientation of horizontal components $S_1$ and $S_2$; the ability to orient one horizontal component towards into the plane that contains the source and the receiver is lost. Because of this difficulty, azimuthal angle $\theta$ between one of the horizontal components and the plane made with the source is usually non-zero, and it may change as the tool is hoisted through the well.

Similar to rotation of a multicomponent receiver from vertical, rotation $\theta$ about the vertical is computed

$$V_\theta = G_\theta \, W = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} W, \tag{5}$$

and, again, the rotation matrix above is a binary operator.

*Dip $\phi$ and azimuth $\theta$*

When there is an offset between the source and the downhole receiver, and when the ability to control the orientation is lost, both $\phi$ and $\theta$ are applied to the incident wavefield, and a single operator results according to

$$V = G_\theta \, G_\phi \, W = G_{\theta\,\phi} \, W = \begin{bmatrix} \cos\theta & \sin\theta\,\cos\phi & \sin\theta\,\sin\phi \\ -\sin\theta & \cos\theta\,\cos\phi & \cos\theta\,\sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} W, \tag{6}$$

where $G_\theta$ and $G_\phi$ come from equations 4 and 5 respectively. Binary operator $G_{\theta,\phi}$ is a $3 \times 3$ matrix that applies dip and azimuth to $W$ simultaneously. The result is multicomponent recording $V$.

## Least squares geophone orientation: inversion

When $G_{\theta\,\phi}$ is known, it is a simple matter to determine $W$ (the desired, single channel recording) from $V$ (the multicomponent data) according to

$$W = F\,V = G_{\theta\,\phi}^{-1}\,V, \tag{7}$$

where inverse $F$ of $G_{\theta\,\phi}$ exists for all $\theta$ and $\phi$ and is itself a binary operator.

When $G_{\theta\,\phi}$ is unknown, an estimate of $F$ is possible based only on $V$ (the geophone recording). First, assume that all three channels in $V$ are noise free, and that a window about the arrival of the wave mode of interest is employed on all three channels. In this idealized case, all time samples of $V\,(Z)$ within the window are identical in sign (i.e. $\pm$) to the ideal P-wave source within the same window. That is

$$\begin{aligned} \frac{V\,(Z)}{|V\,(Z)|} \equiv \frac{w}{|w|} &= 1,\; V\,(Z) > 0, \\ &= -1,\; V\,(Z) < 0, \\ &= 0,\; |V\,(Z)| = 0. \end{aligned} \tag{8}$$

Based on equation 8, rewrite inversion equation 7 as follows:

$$\bar{W} = \bar{F}\,\bar{V}, \tag{9}$$

where

$$\bar{W} = \begin{bmatrix} 0 \\ 0 \\ \frac{V(Z)}{|V(Z)|} \end{bmatrix}. \tag{10}$$

To understand $3 \times N$ matrix $\bar{V}$, note that $\bar{F}$ is a rotation operator, and so it must also be a unitary operator. Columns in $\bar{V}$, then, must be unit vectors

$$\bar{V} = \frac{V}{|V|}, \tag{11}$$

so that amplitude is conserved. Explicitly, $\bar{V}$ is given by

$$\bar{V} = \begin{bmatrix} \frac{H_{11}}{A_1} & \frac{H_{12}}{A_2} & \frac{H_{13}}{A_3} & \dots & \frac{H_{1n}}{A_n} \\ \frac{H_{21}}{A_1} & \frac{H_{22}}{A_2} & \frac{H_{23}}{A_3} & \dots & \frac{H_{2n}}{A_n} \\ \frac{Z_1}{A_1} & \frac{Z_2}{A_2} & \frac{Z_3}{A_3} & \dots & \frac{Z_n}{A_n} \end{bmatrix}, \tag{12}$$

where, for the $j^{th}$ time sample,

$$A_j = \sqrt{H_{1j}^2 + H_{2j}^2 + Zj^2}. \tag{13}$$

When $V$ is free of noise, $\bar{F}$ in equation 9 equals $F$ in equation 7 $\left(\bar{F} = F\right)$ so that one may substitute to get

$$W = \bar{F}\,V. \tag{14}$$

If, however, there is noise and leakage of non-P-wave modes into the geophone, then $\bar{F} \approx F$. Because P-waves are the fastest propagating modes, careful windowing helps eliminate undesirable modes. Noise, however, is always present and $\bar{F} \approx F$ is expected so that inversion equation 7 is an approximation

$$W \approx \bar{F}\,V. \tag{15}$$

The solution for $\bar{F}$ is obtained by least-squares inversion of equation 9, with both $\bar{W}$ and $\bar{V}$ computed from recorded data $V$ according to equations 10 and 11 respectively. The least-squares solution for $\bar{F}$ is

$$\bar{F} = \bar{W}\,\bar{V}^T \left[\bar{V}\,\bar{V}^T\right]^{-1}, \tag{16}$$

where superscript $T$ indicates matrix transpose. Inversion equation 16 computes $\bar{F}$ through inversion at a cost of two multiplications of $3 \times n$ operators plus the cost of computing a $3 \times 3$ inverse. To estimate $\phi$ and $\theta$, invert $\bar{F}$ so that

$$\bar{G}_{\theta\,\phi} = \bar{F}^T, \tag{17}$$

where for unitary $\bar{F}$, $\bar{F}^{-1} = \bar{F}^T$. Then, from equation 6, solve for $\theta$ and $\phi$ according to

$$\tan\theta \approx \frac{\left[\bar{G}_{\theta,\phi}\right]_{13}}{\left[\bar{G}_{\theta,\phi}\right]_{23}}, \tag{18}$$

where subscripts '13' and '23' indicate indexes (row '1' column '3', and row '2' column '3') within the $3 \times 3$ matrix in equation 6, and

$$\cos\phi \approx \left[\bar{G}_{\theta,\phi}\right]_{33}. \tag{19}$$

Estimates of $\phi$ and $\theta$ may now be used to rotate recorded data $V$ into three principle components.

## EXAMPLES

Synthetic examples are presented to demonstrate the accuracy of the rotation analysis of 3C-VSP recordings; they are based on a single component source and a 3C-geophone located in a well-bore that is offset a distance from the source. Figure 1a illustrates the geometrical relationships between the 3C geophone and the source at the surface for a deviated will bore. Red lines indicate raypaths from the source to a number of geophone levels (green triangles) in the well bore (solid green curve). A single raypath is highlighted in black, and a zoom-in of that raypath and it's associated geophone level is illustrated in Figure 1b. Horizontal components $S_1$ and $S_2$ of gimballed geophone are indicated with blue lines, and vertical component $Z$ lies along the well trajectory. Relative to the incident ray, then, the geophone is both tilted and rotated.

Figure 2 illustrates the 3C wavefield associated with a S-wave source. Here, the direct S-wave is the first arrival (no mode-conversion in the near surface). This source is oriented such that the shear motion is normal to the vertical plane that contains both the source and the geophone. The geophone dip $\phi = -27^o$ and azimuth $\theta = 20^o$, and the simulated environment imparts random noise to any signal that is recorded with a signal-to-noise-ratio of 10. The recorded shear wave for this geophone is given in Figure 3a - it is now a noisy recording with three components.

Inversion of the 3C recording in Figure 3a returns the expected S-wave (solid line) that overlies the source signal (dashed line). Red lines on Figure 3a indicate the analysis window designed to span the first arrival. Though the out put signal matches the expected signal, the estimated geophone orientation $\phi = -15^o$ and $\theta = 11^o$ are in error by $12^o$ and $9^o$ respectively. A simple band-pass filter applied prior to inversion Figure 4a, however, rectifies these errors as is shown in Figure 4b. Estimated $\phi$ is correct, and $\theta$ is just $1^o$ in error. The rotated output matches the desired output.

A P-wave source is illustrated in Figure 5. As for the synthetic S-wave example above, at the geophone, the P-wave source is rotated, noise is added, and here a trailing S-wave (mode-conversion in the near surface) is included as is indicate by the red arrow in Figure 6a. Data within the analysis window are used in the inversion, and the result is given in Figure 6b. Similar to the S-wave example, the returned waveform is nearly identical to the input P-wave, but error for $\phi$ and $\theta$ is $4^o$ and $5^o$ respectively. Again, band-pass filters applied prior to inversion improve $\phi$ and $\theta$ estimates with errors $2^o$ and $0^o$ respectively.

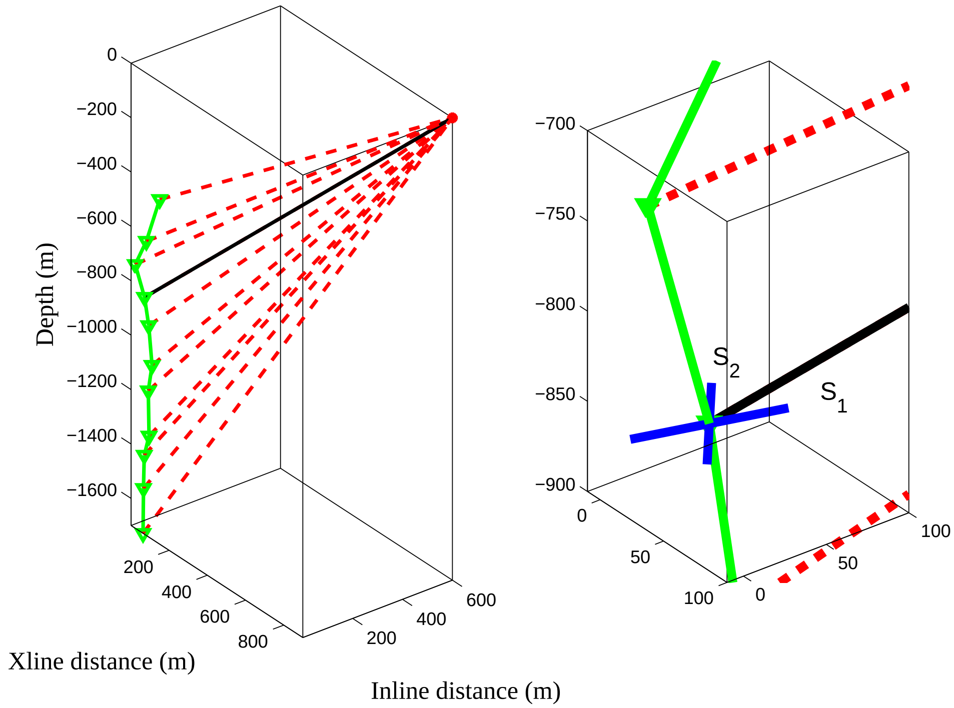


FIG. 1. An offset VSP into a deviated well. a) The deviated well is indicated by a green line, and geophones are indicated by green triangles. Source-receiver ray paths are indicated by dashed lines, and the source is indicated by a red asterisk. b) Close up of a tilted geophone. The orientation of the $S_1$, and $S_2$ components are indicated by the blue lines. The $Z$ component points along the well deviation.

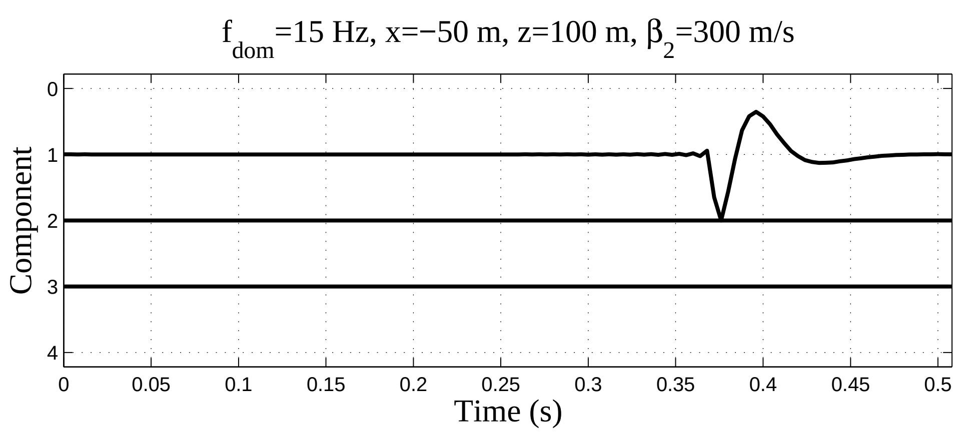


FIG. 2. Source waveform for an ideal S-wave source. Particle motion is in the X-line direction (component 1). No energy is present on the In-line and vertical components (components 2 and 3 respectively).

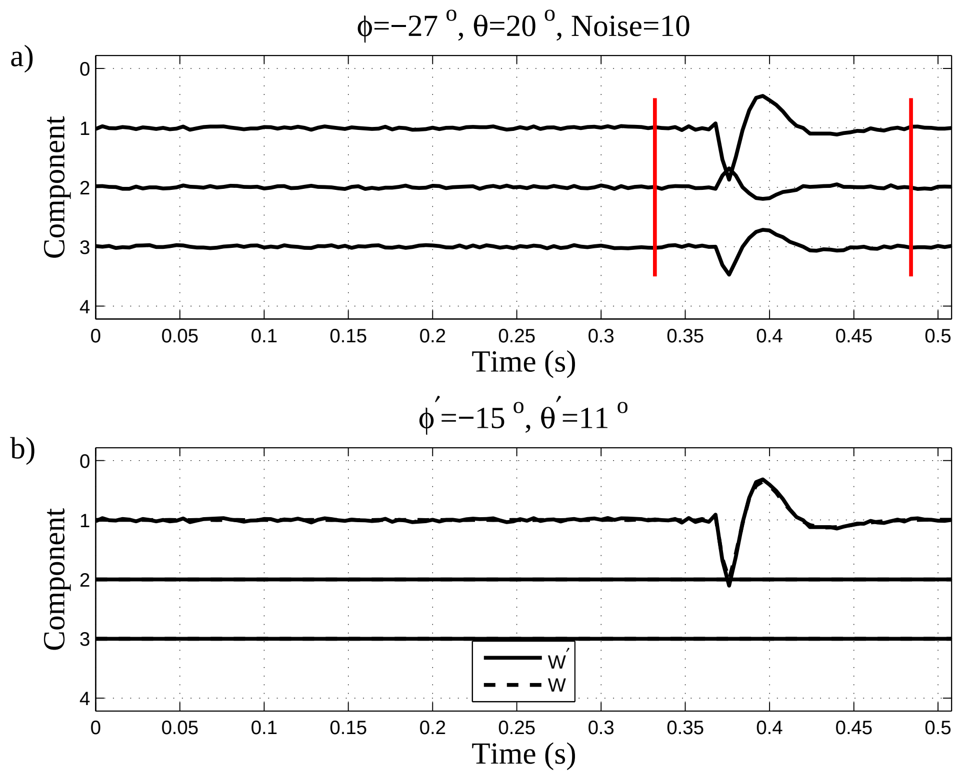


FIG. 3. Geophone recording corresponding to the X-line source (Figure 2) plus 10 % random noise. a) The Geophone orientation is $\phi = -27^o$ dip and $\theta = 20^o$ azimuth. The X-line source registers as a S-wave on all three channels. Red lines indicate the analysis window used for inversion. b) Inversion estimates $\phi = -15^o$ (dip) and $\theta = 11^o$ (azimuth) are $12^o$ and $-9^o$ in error respectively. The estimated waveforms (solid line) and actual waveforms (dashed line) are quite similar.

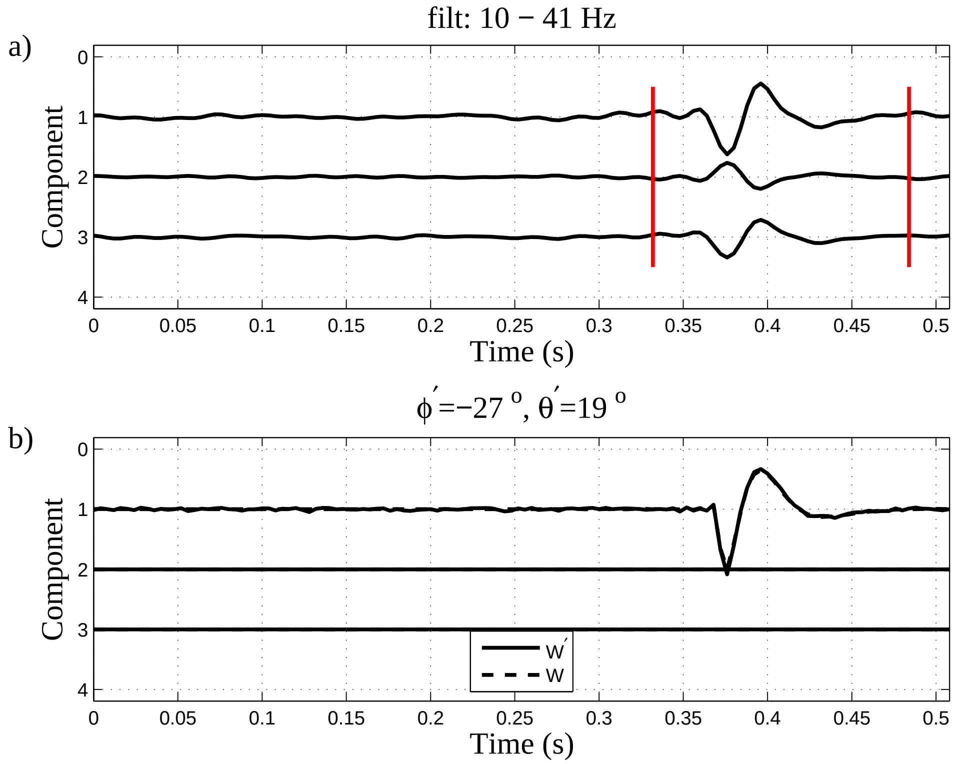


FIG. 4. Filtered version of the geophone recording in Figure 3a. a) 3C input. Red lines indicate the analysis window. b) Inversion estimates $\phi = -27^o$ dip and $\theta = 19^o$ ($0^o$ and $1^o$ error respectively) are much better than those from unfiltered inversion (Figure 3).

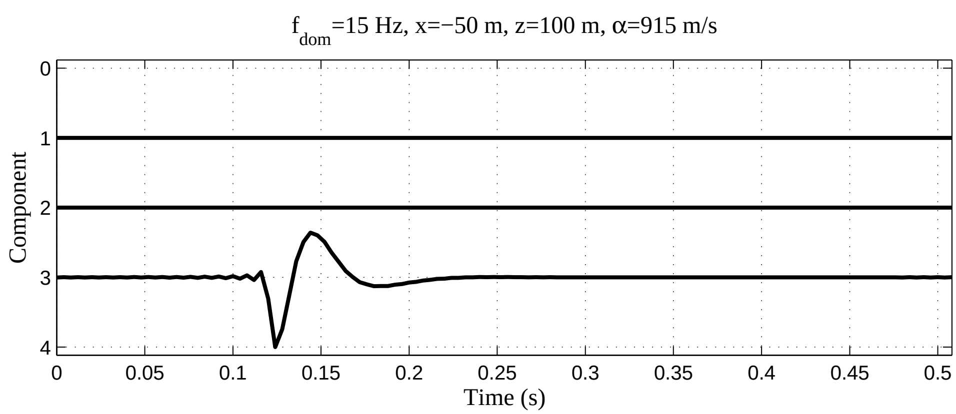


FIG. 5. P-wave source.

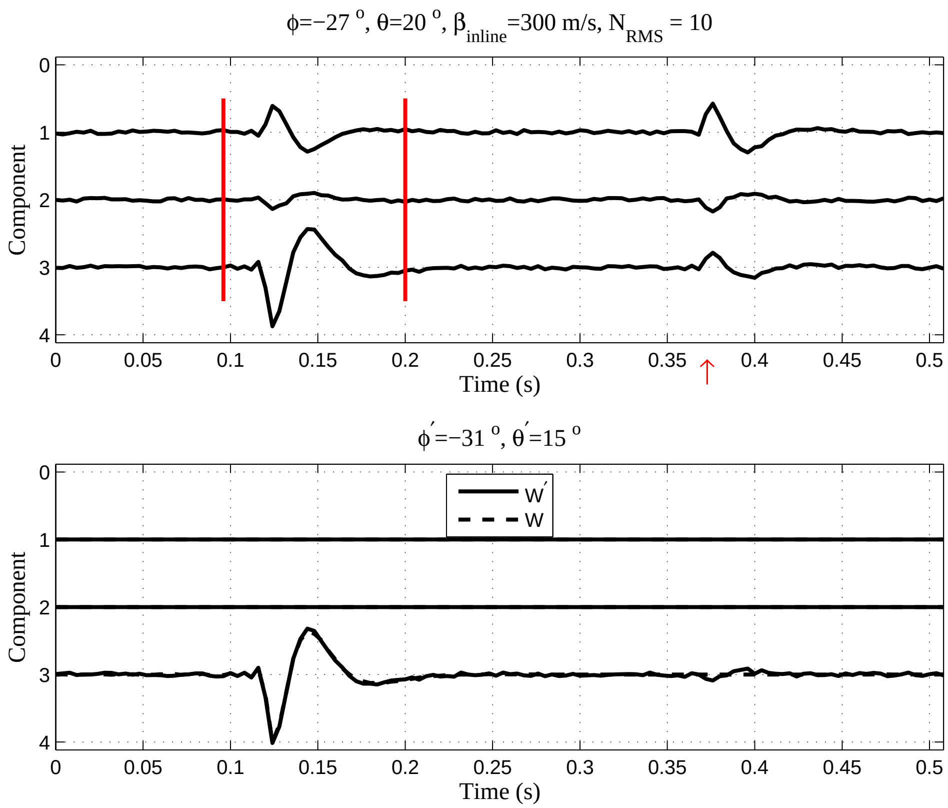


FIG. 6. Geophone recording corresponding to the P-wave source (Figure 5) plus random noise plus mode conversion. a) 3C input. Red lines indicate the analysis window, and the red arrow indicates the arrival time of the converted S-wave. b) The inversion estimate.

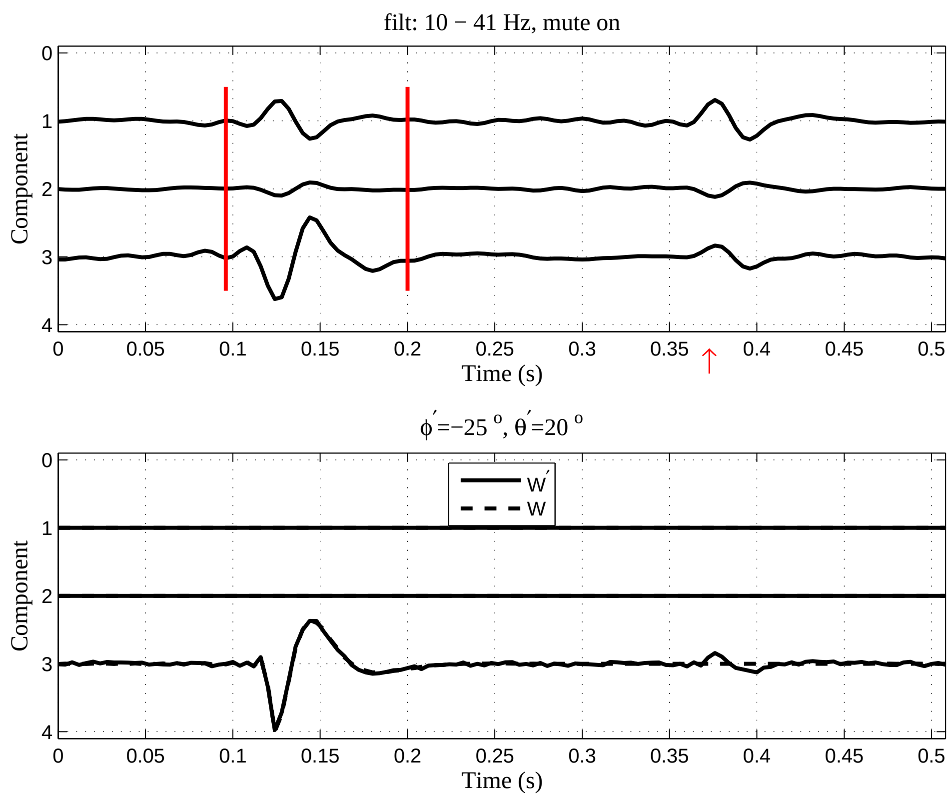


FIG. 7. Filtered version of the geophone recording in Figure 6a. a) 3C input. Red lines indicate the analysis window. b) The unfiltered inversion estimate.

## CONCLUSIONS

Data in 3C VSP recordings are rotated such that pure P- and S-modes are available for conventional interpretation. The scheme presented here rotates the 3C components to separate modes based on the unitary properties of rotation matrices. The underlying assumption of this method is that the polarity of the desired signal and that of the 3C component principally associated with that signal are the same.

In synthetic tests, even for 10 % addetive noise, the rotated signal from this method matches the desired output quite well, though $\phi$ and $\theta$ estimates of the geophone orientation degrade as noise increases. Band-pass filters improve error in these estimates, however, to an acceptable range as a simple pre-processing step.

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## REFERENCES

DiSiena, J. P., Gaiser, J. E., and Corrigan, D., 1984, Horizontal components and shear wave analysis of three-component VSP data, *in* Toksöz, M. N., and Stewart, R. R., Eds., Vertical seismic profiling, Part B: Advanced concepts, Geophysical Press.

Toksöz, M. N., and Stewart, R. R., Eds., 1984, Vertical seismic profiling: Geophysical Press.

Zeng, X., and McMechan, G. A., 2006, Two methods for determining geophone orientations from vsp data: Geophysics, **71**, No. 4, V87–V97.