

## **Radial filtering on steroids: the latest algorithm**

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### **ABSTRACT**

Coherent noise attenuation is an ever-evolving technology, and radial trace domain filtering algorithms continue to change and advance, as well. Our previous version of the ProMAX radial filtering module provides several ways in which to attenuate coherent noise, including the recommended one of estimating coherent noise with a low-pass filter in the radial trace domain, then subtracting the noise estimate from the original trace gather in the X-T domain. We now have a new option in which the X-T noise subtraction is done by a least-squares subtraction algorithm. We demonstrate here that the new method provides significantly better noise attenuation, both with single filter applications and with sets of cascaded filters.

### **INTRODUCTION**

Estimation and subtraction of coherent noise from seismic trace gathers using the radial trace domain has been established for several years as an effective means for reducing coherent noise interference both pre-stack and post-stack (Henley, 2003a). We have believed for some time, however, that the subtraction step could be further optimized, since we often observe residual noise on processed data gathers, even after subtracting seemingly high-fidelity estimates of the noise. Previous experience has further shown us that multiplying the noise estimate by a coefficient slightly larger than unity before subtraction can sometimes remove more noise (Henley 2003b). Likewise, simply repeating a radial filter pass, using the same parameters, can sometimes leave less residual noise than a single pass.

One well-known technique that we decided to try in order to reduce the residual noise is least-squares subtraction, in which a noise trace is scaled by a gain factor, whose value is determined by requiring that the sum of squared differences between raw trace and noise trace be minimized. This constraint leads to a gain factor consisting of the product of the raw trace and noise trace, summed over some window, divided by the square of the noise trace, summed over the same window. This is the same as the zero-lag cross-correlation of raw and noise traces divided by the zero-lag autocorrelation of the noise. See the Appendix for the short derivation.

In order to add some flexibility to the least-squares subtraction, we allow the user to choose the summation window length; so that instead of a single gain factor for each noise trace, the result is a gain function whose values change with the incremental movement of the summation window (boxcar smoother) over the raw trace and noise estimate. Thus, when the window length equals that of the trace, a single gain factor is computed. In the other extreme, with window length of 1 sample, the gain function closely replicates the input raw trace, and the subtraction leaves mostly random noise.

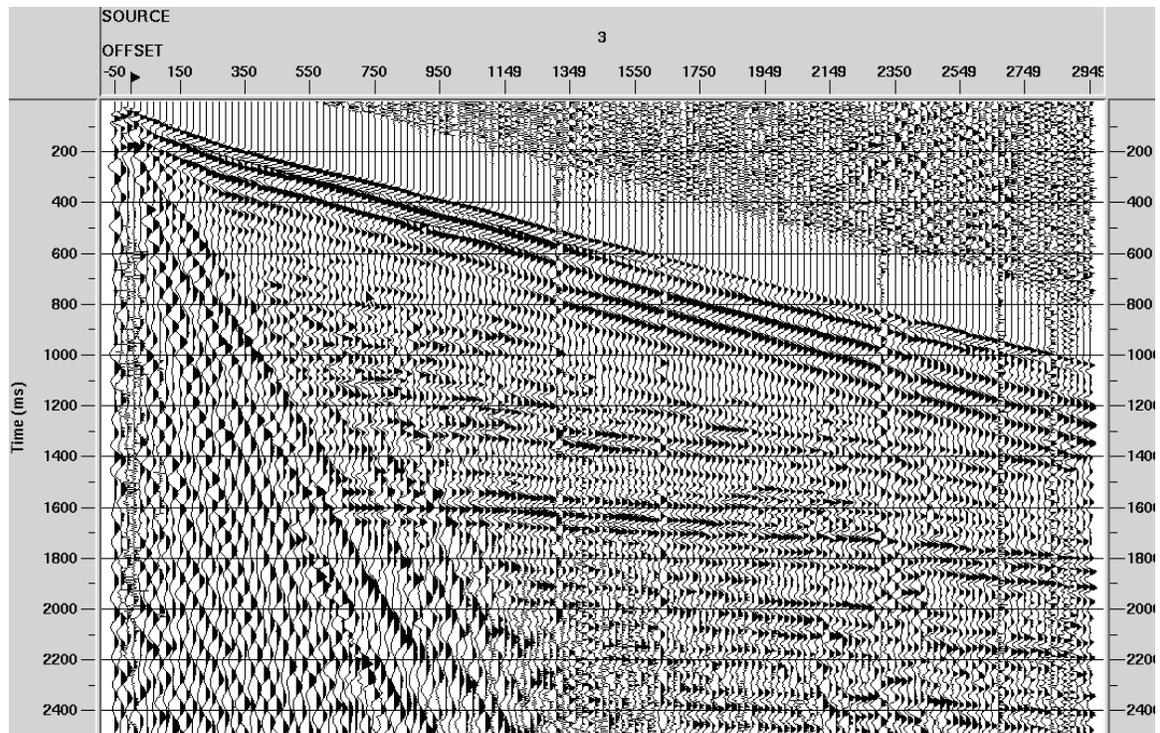
## ALGORITHM DETAILS

The new least-squares noise subtraction option has been incorporated into the ProMAX module, Radfilt, where the only visible difference is in the menu, in the ‘filter type’ selection parameter. The least-squares subtraction option is now available as a ‘filter type’ choice. If that option is selected, then the parameter which immediately follows in the menu, instead of being a scalar to multiply a noise vector before subtraction from the raw input trace, becomes the length of the smoothing window for the gain function to be applied to the noise vector before subtraction. The default value (1.0) works well. If the least-squares subtraction option is not chosen, Radfilt operates exactly as in previous releases.

## EXAMPLES

### Blackfoot shot gathers

To demonstrate the improved noise attenuation achieved with one pass of a standard radial trace filter, we select three different shot gathers from the Blackfoot 2D survey. In Figure 1 we show the unprocessed gather for shot 3 on this line. There are very strong first arrivals and repeats (guided waves), as well as considerable ground roll and air blast in the centre of the record.



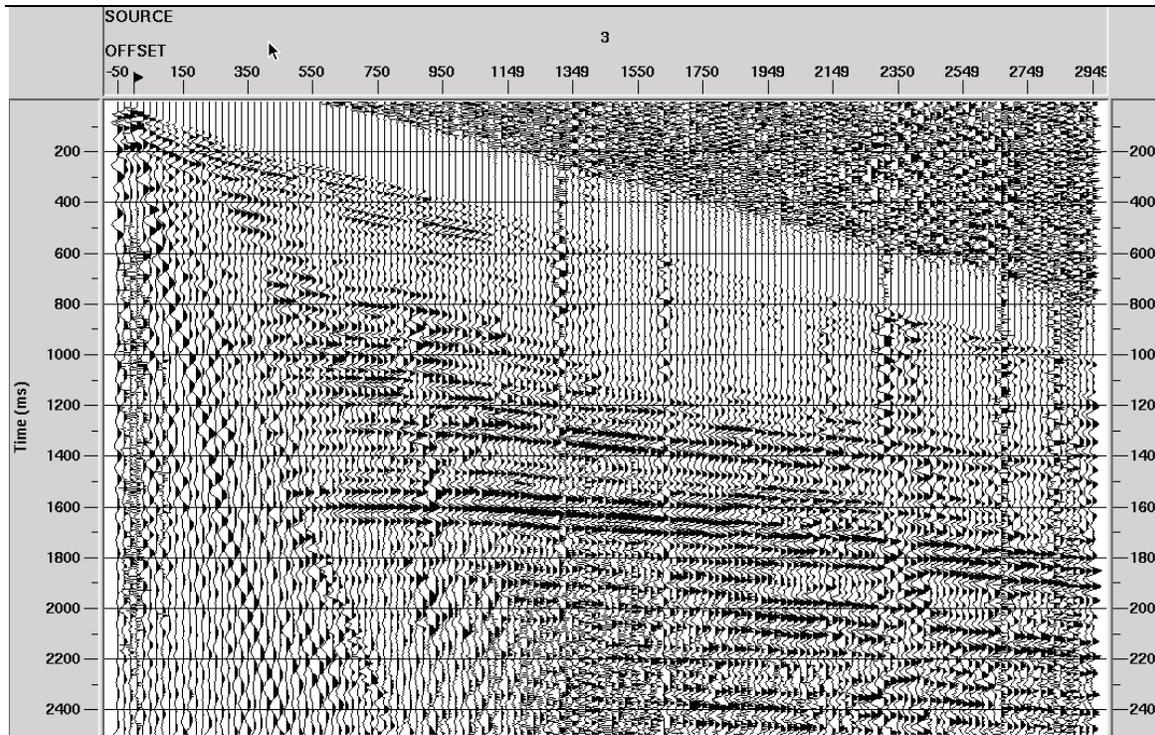
Blackfoot shot 3, no filter

FIG. 1. Unfiltered Blackfoot shot 3

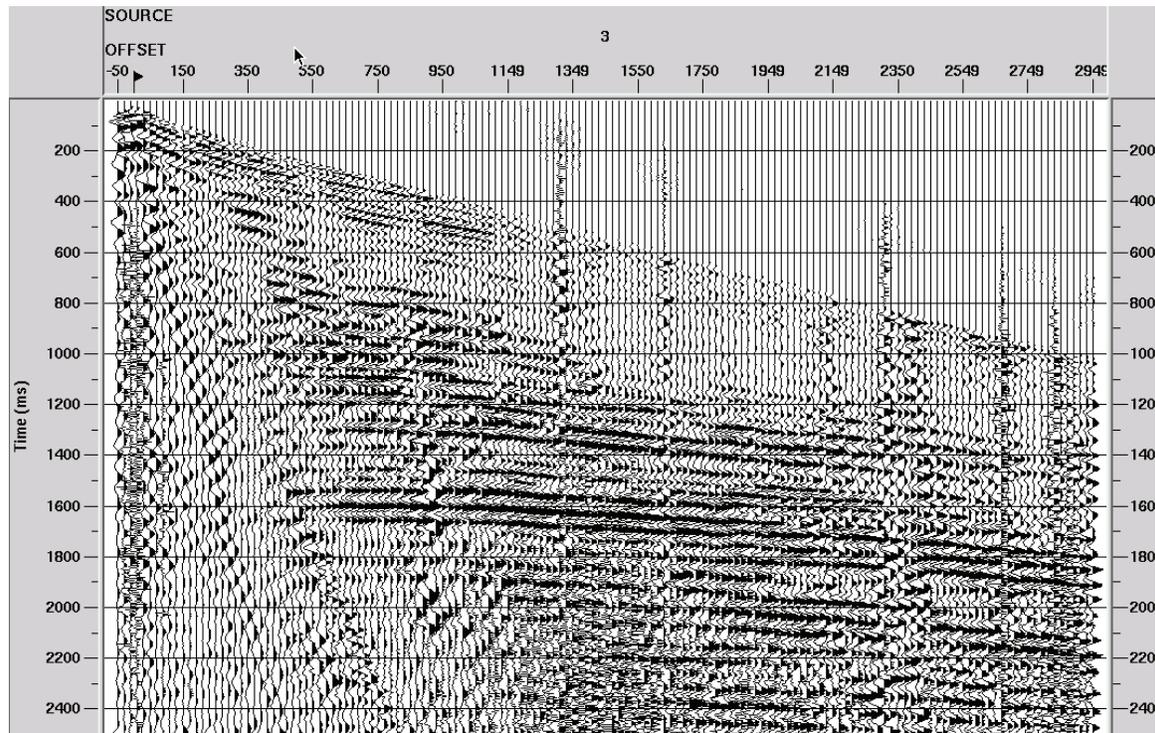
When we apply our standard radial trace fan filter, which applies an 8 – 12 Hz low pass filter in the radial trace domain to estimate noise and converts the noise back to the X-T domain for subtraction from the original shot gather, we get the result shown in Figure 2.

Although we can specify a scalar by which to multiply the noise before subtraction from the raw record, we have defaulted the multiplier to unity. The noise in this gather has been considerably attenuated, particularly the direct arrivals and guided waves. We can see a remnant of the ground roll, however, particularly where it has aliased near the centre of the record. Note that the pre-shot noise on the traces has not been addressed, since the noise subtraction in this mode is a sample-by-sample subtraction, which happens only within the portion of the X-T panel subjected to the radial trace transform (usually, a fan defined by apparent velocities roughly corresponding to the direct arrival velocity).

In the least-squares subtraction mode, the coherent noise estimate is obtained from the input shot gather in the same way as for the straight subtraction mode—a low pass 8 – 12 Hz filter is applied to the radial trace transform of the shot gather. In the least-squares mode, however, the noise estimate is inverse transformed in its entirety back to the X-T domain to form a complete shot gather, including pre-shot noise. Corresponding traces of the input gather and the noise estimate gather are subtracted with the least-squares algorithm to provide the output filtered shot gather. When this process is performed with a 500 ms window, the result is shown in Figure 3. Compared to Figure 2, we can see that significantly more noise is removed, especially in the ground roll cone, and reflection strength and continuity are improved throughout the record. A side benefit is that the pre-shot noise, being the same on both the original and estimated noise gathers, is cancelled. Since the boundaries of the radial trace transform do not include this noise, it is unaltered by the filtering within the transform boundaries.

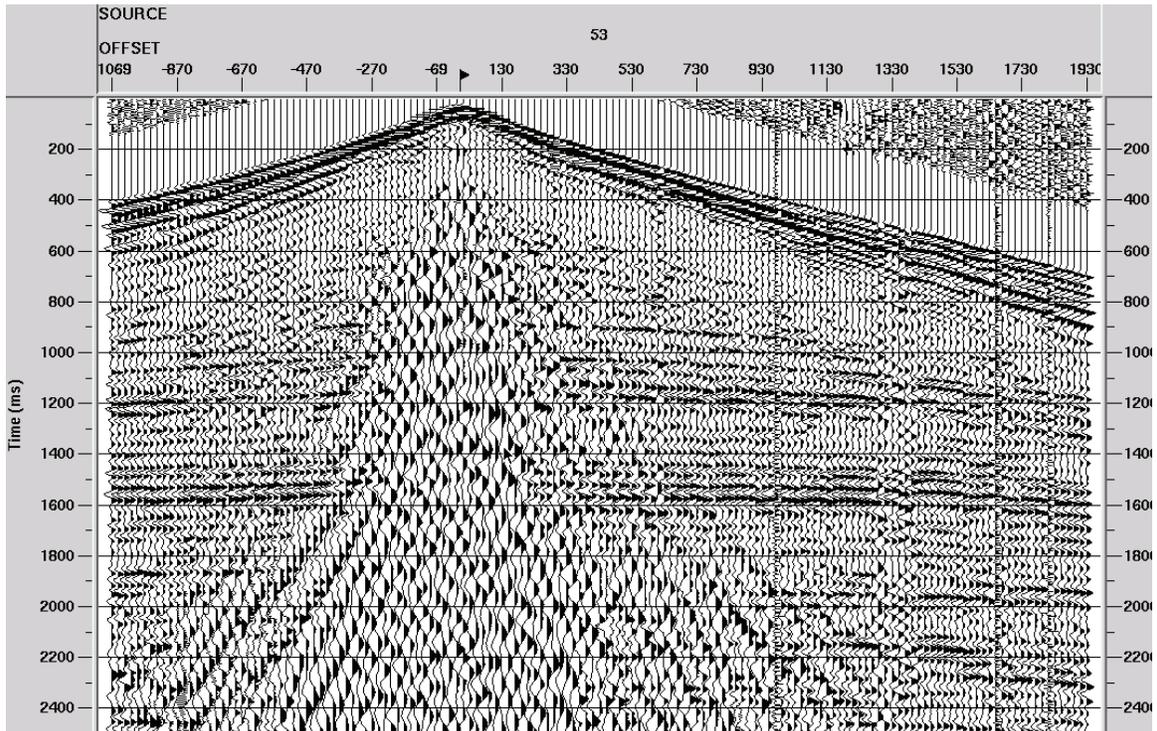


Blackfoot shot 3, subtract RT low pass (8 - 12 Hz) in X-T  
FIG. 2. Blackfoot shot 3 after radial trace filter pass using straight subtraction to remove the noise estimated in the RT domain.



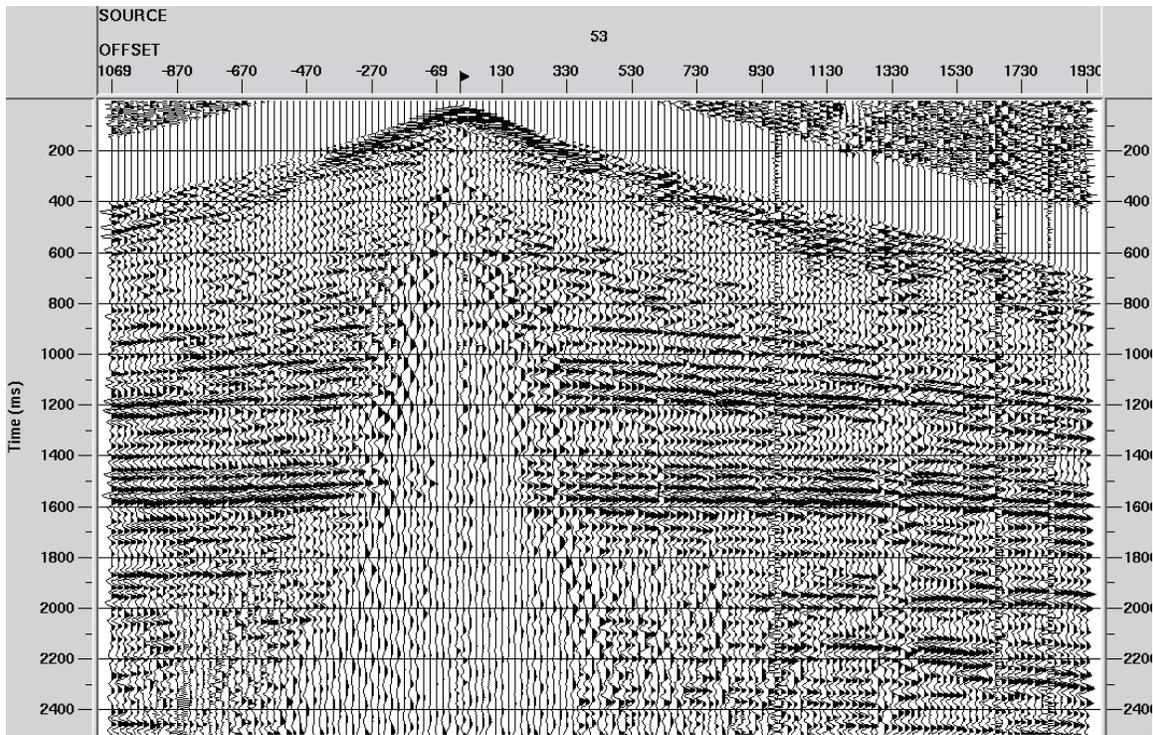
Blackfoot shot 3, least squares subtract low pass RT (8 - 12 Hz) in X-T  
FIG. 3. Blackfoot shot 3 after least-squares subtraction of coherent noise estimated in RT domain.

Figures 4, 5, and 6 show Blackfoot shot number 53 before filtering, after the straight noise subtraction procedure, and after least-squares noise subtraction, respectively; and Figures 7, 8, and 9 show the same representations for shot 100. In both these cases, the least-squares algorithm removes significantly more coherent noise from the input gathers and visibly improves reflection strength and continuity, particularly for those weak reflections deeper in the record which are greatly obscured by the original noise (for example, shot 53, reflections between 1600 – 2000 ms on the edge of the noise cone).



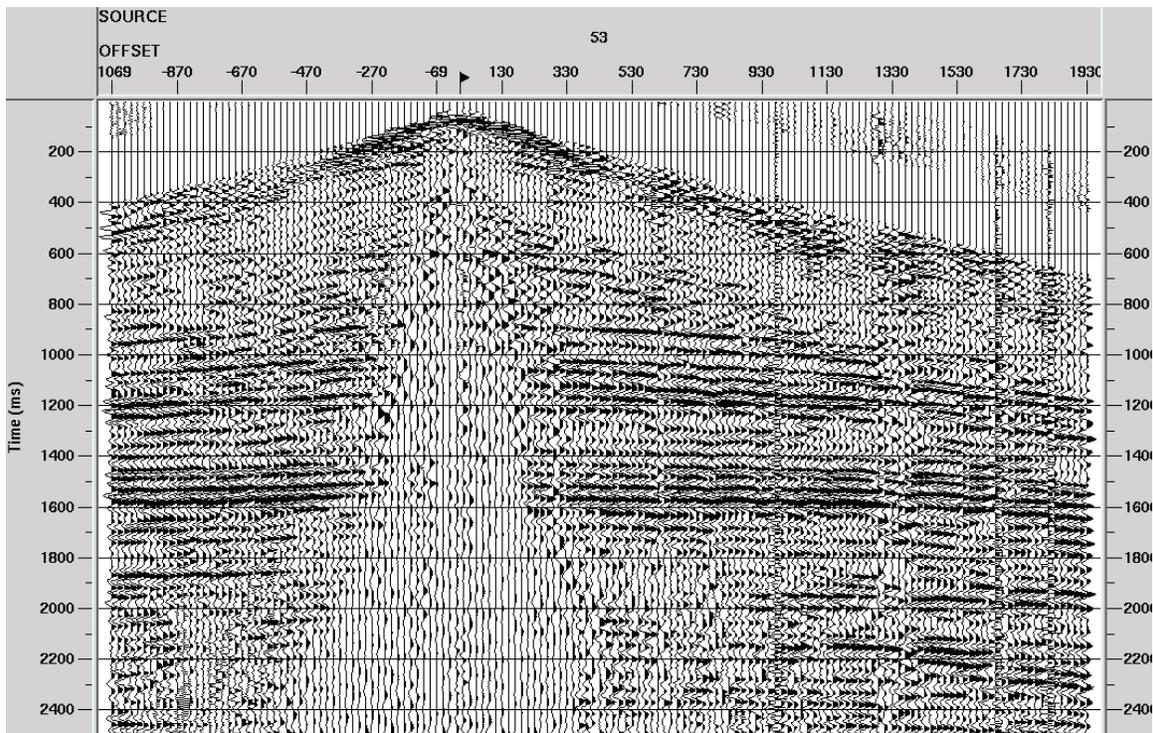
Blackfoot shot 53, no filter

FIG. 4. Blackfoot shot 53 before filtering



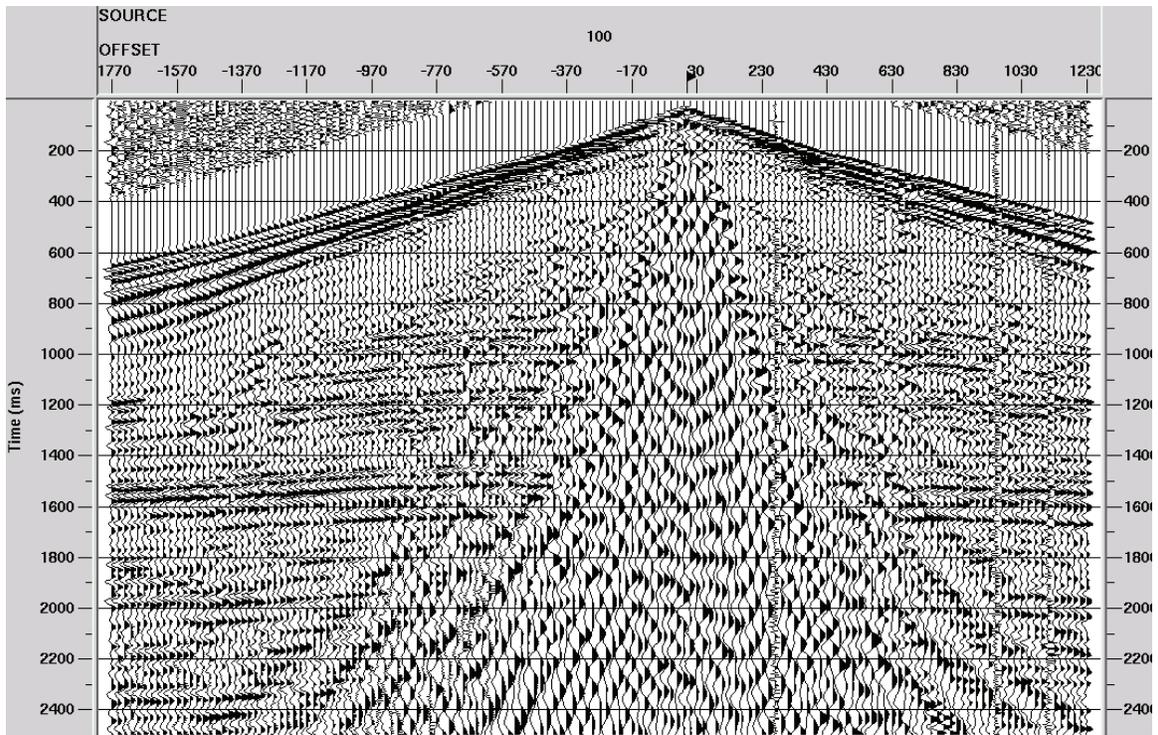
Blackfoot shot 53, subtract RT low pass (8 - 12 Hz) in X-T

FIG. 5. Blackfoot shot 53 after straight subtraction of coherent noise estimated in RT domain.



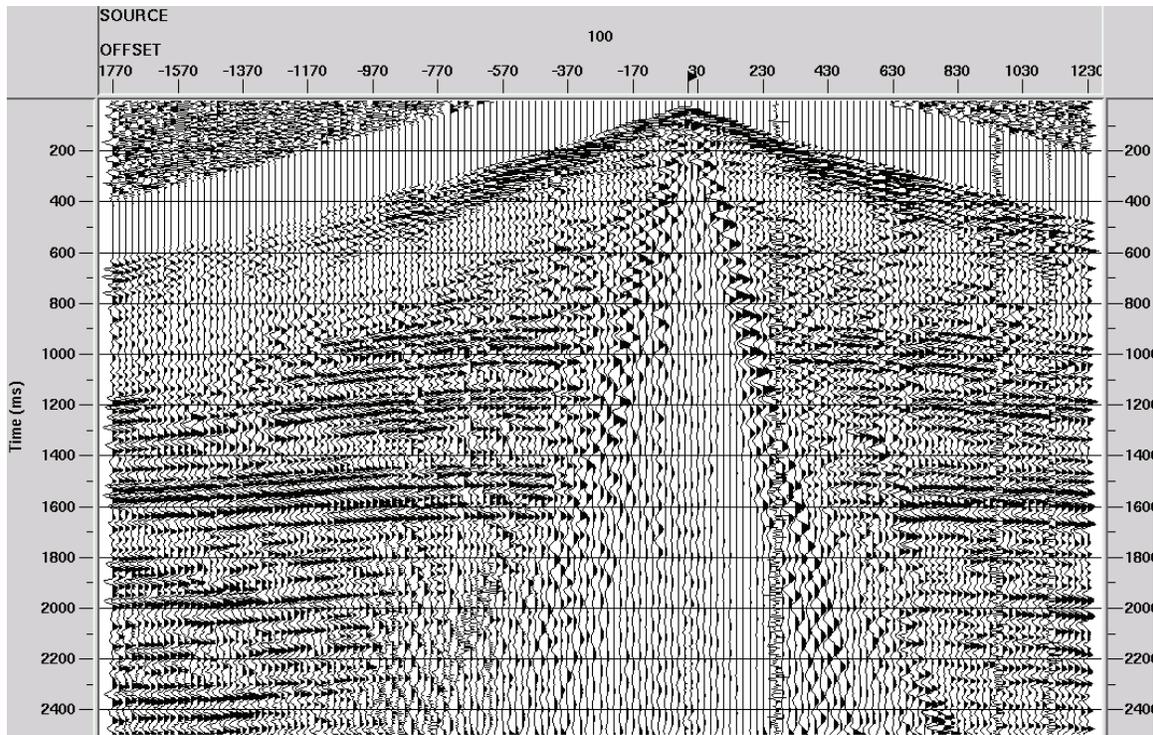
Blackfoot shot 53, least squares subtract low pass RT (8 – 12 Hz) in X-T

FIG. 6. Blackfoot shot 53 after least-squares subtraction of coherent noise estimated in the RT domain.



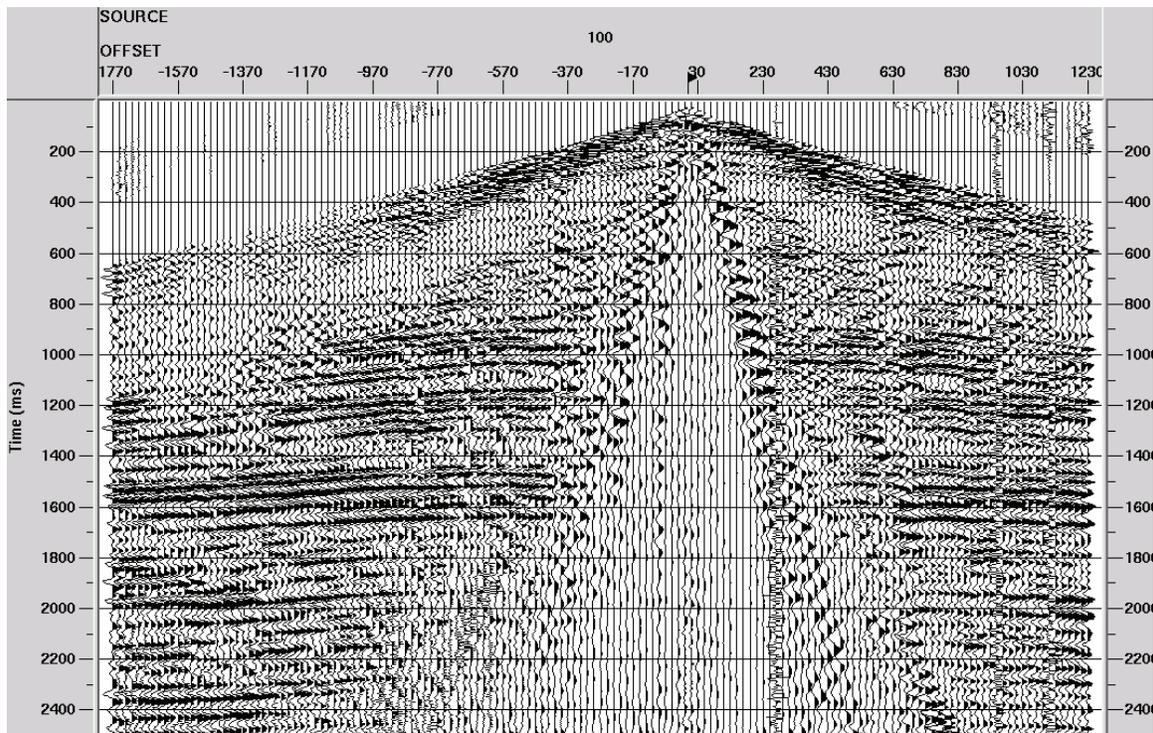
Blackfoot shot 100, no filter

FIG. 7. Blackfoot shot 100 with no filtering



Blackfoot shot 100, subtract RT low pass (8 - 12 Hz) in X-T

FIG. 8. Blackfoot shot 100 after straight subtraction of coherent noise estimated in RT domain.

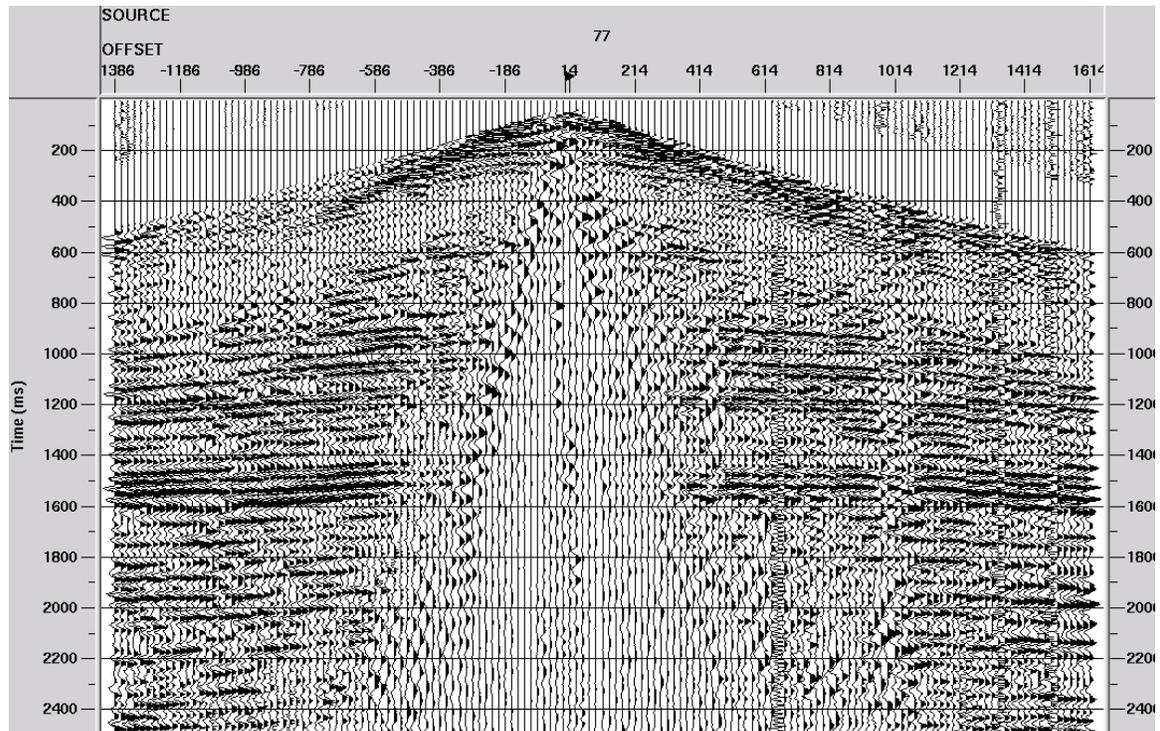


Blackfoot shot 100, least squares subtract low pass RT (8 – 12 Hz) in X-T

FIG. 9. Blackfoot shot 100 after least-squares subtraction of coherent noise estimated in RT domain.

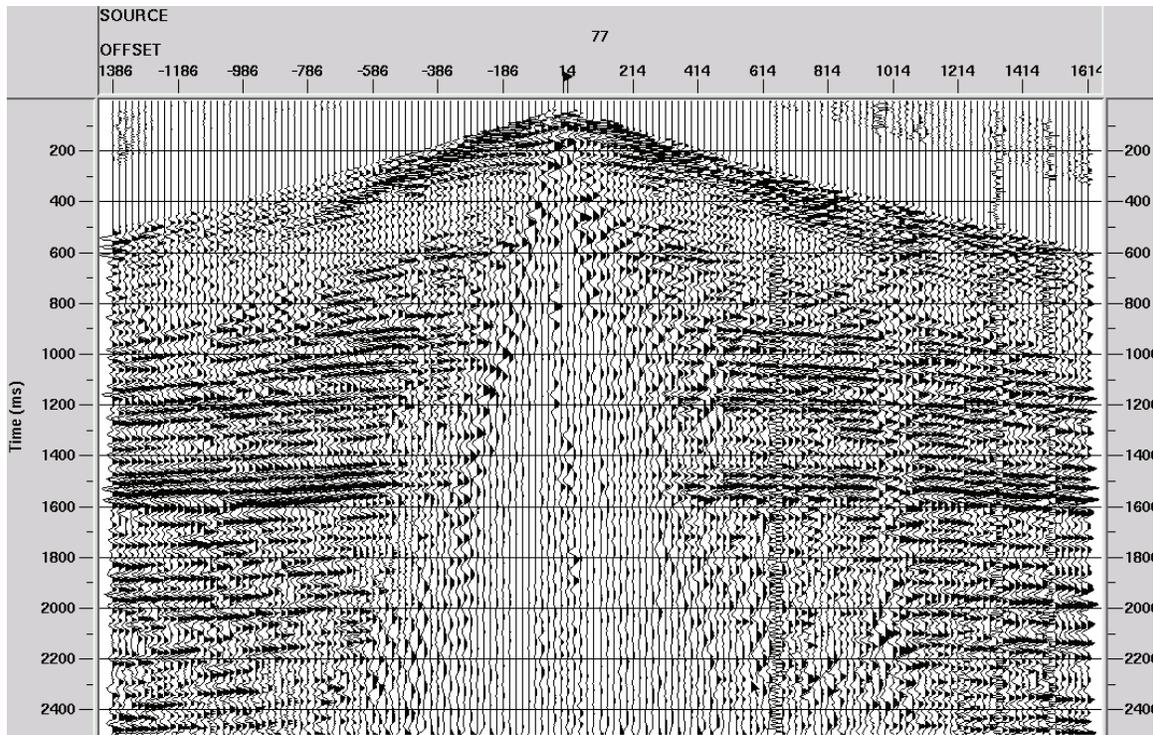
### Gate sensitivity

While the least-squares mode of radial filtering is not greatly sensitive to the length of the gain window, there is at least some visible effect from altering the length. Figures 10 through 16 show the differences between gain windows of 2000 ms, 1000 ms, 500 ms, 250 ms, 125 ms, 67 ms, and 34 ms, respectively. From these figures, it is evident that while the longer gain windows are clearly less effective for subtracting the noise, the shorter ones can cause degradation of reflection continuity and introduce random noise. For this particular shot gather, it is evident that a least-squares gain window with a length of 250 to 500 ms appears to provide good noise attenuation while preserving event continuity.



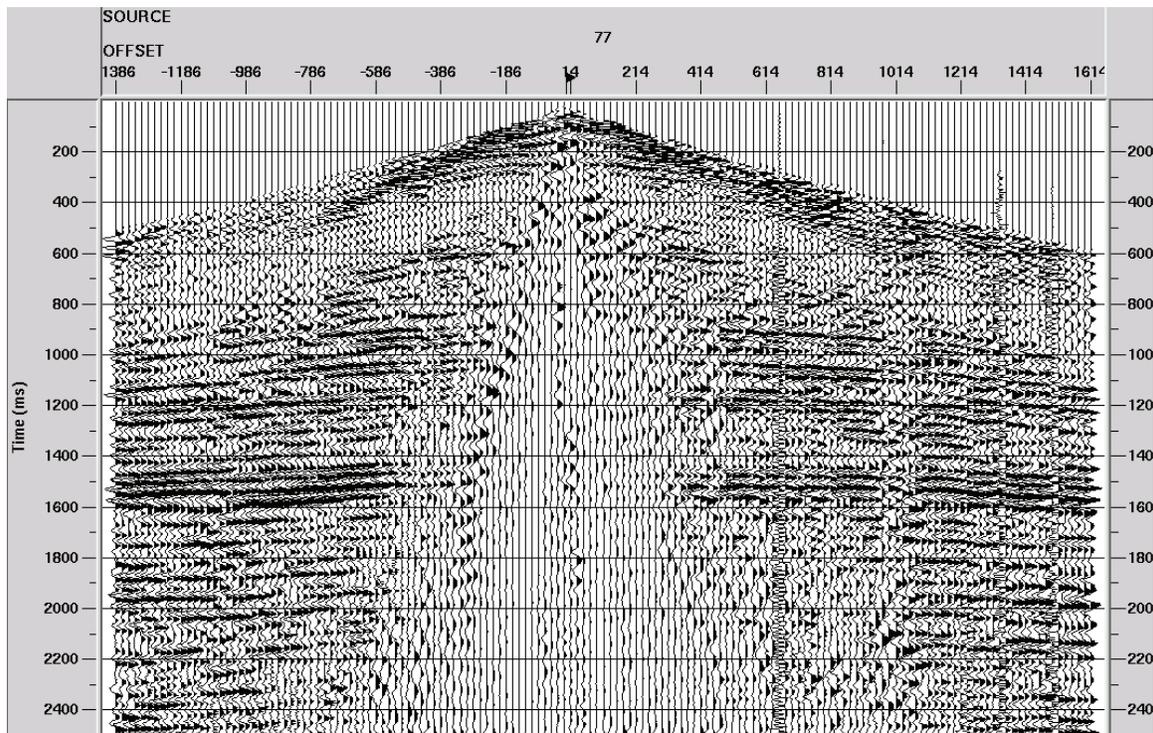
Blackfoot shot 77, least squares subtract low pass RT (8 – 12 Hz) in X-T, 2000 ms gain smooth

FIG. 10. Least-squares radial filter with 2000 ms gain window



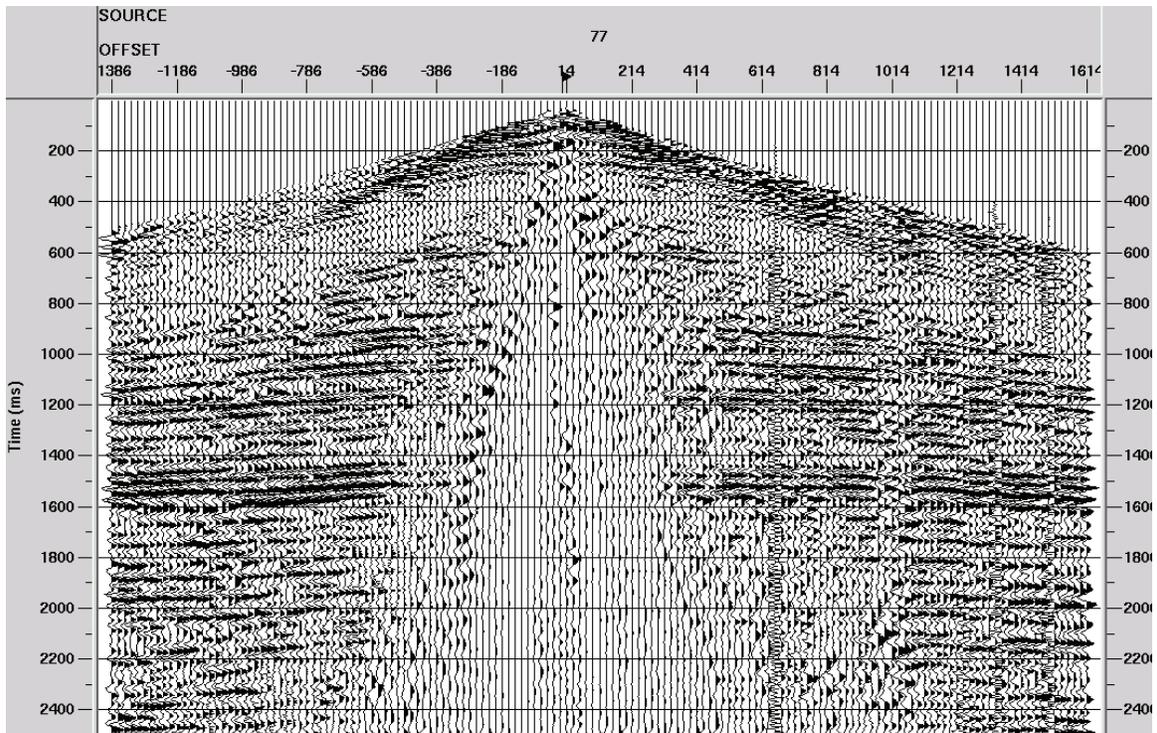
Blackfoot shot 77, least squares subtract low pass RT (8 – 12 Hz) in X-T, 1000 ms gain smooth

FIG. 11. Least-squares radial filter with 1000 ms gain window.



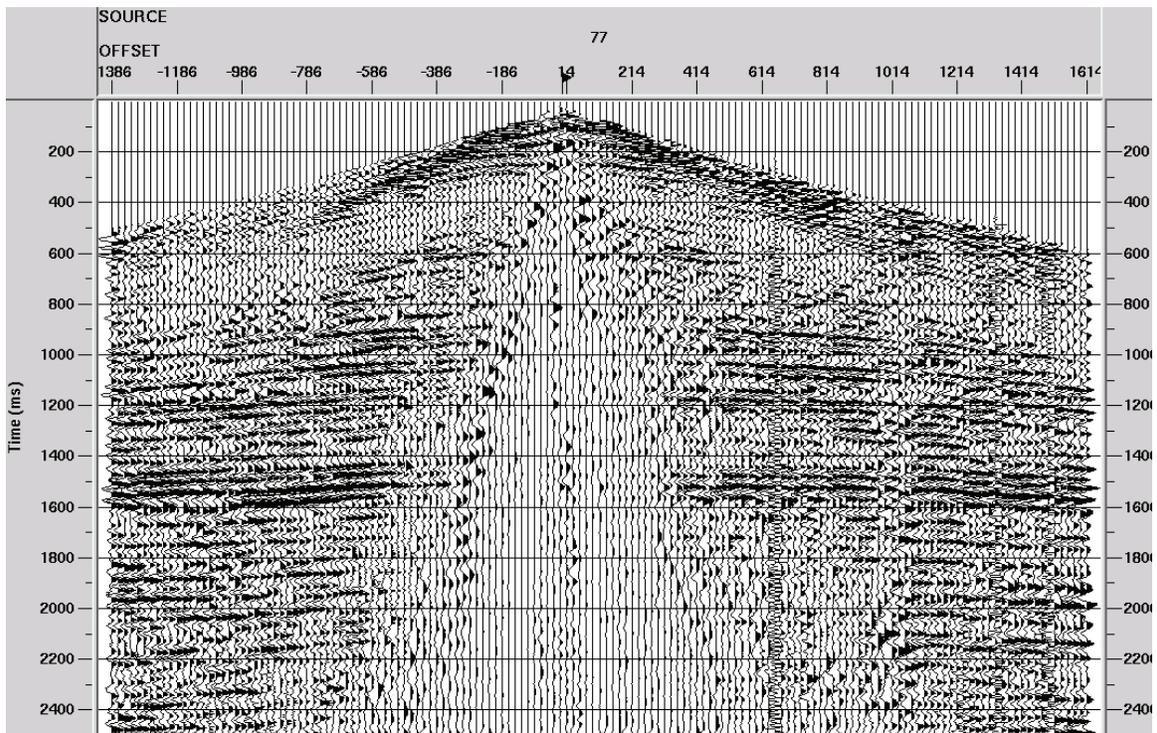
Blackfoot shot 77, least squares subtract low pass RT (8 – 12 Hz) in X-T, 500 ms gain smooth

FIG. 12. Least-squares radial filter with 500 ms gain window.



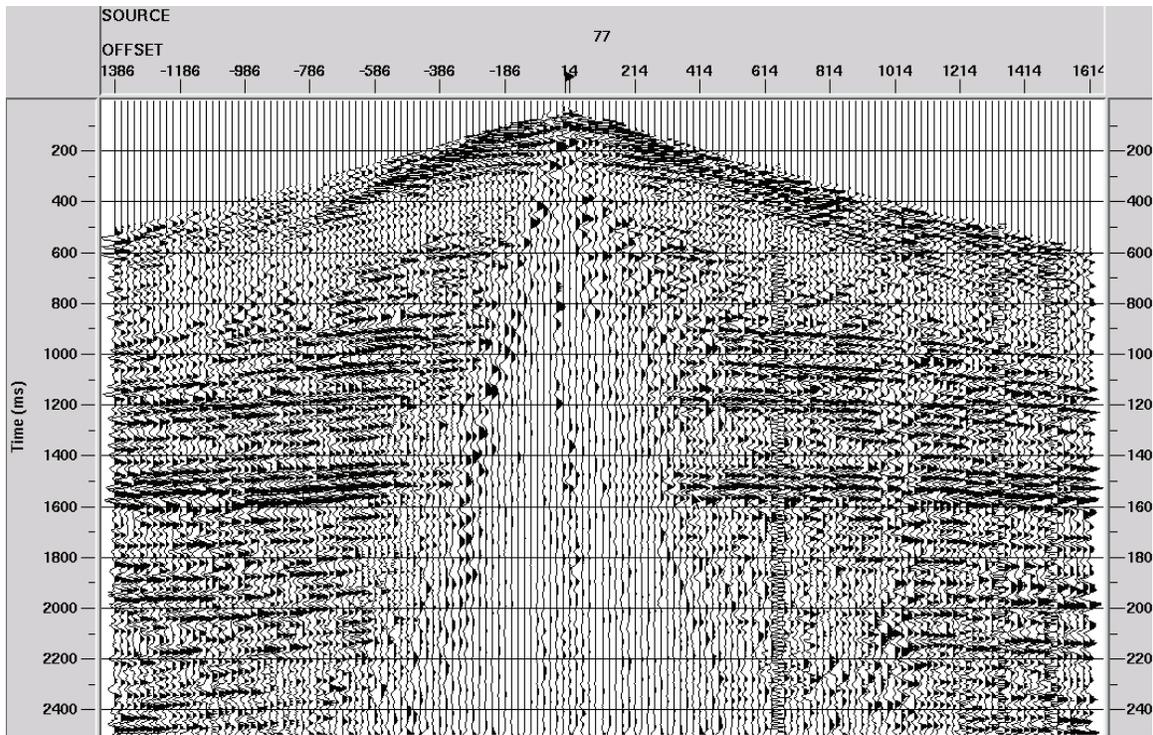
Blackfoot shot 77, least squares subtract low pass RT (8 – 12 Hz) in X-T, 250 ms gain smooth

FIG. 13. Least-squares radial filter with 250 ms gain window.



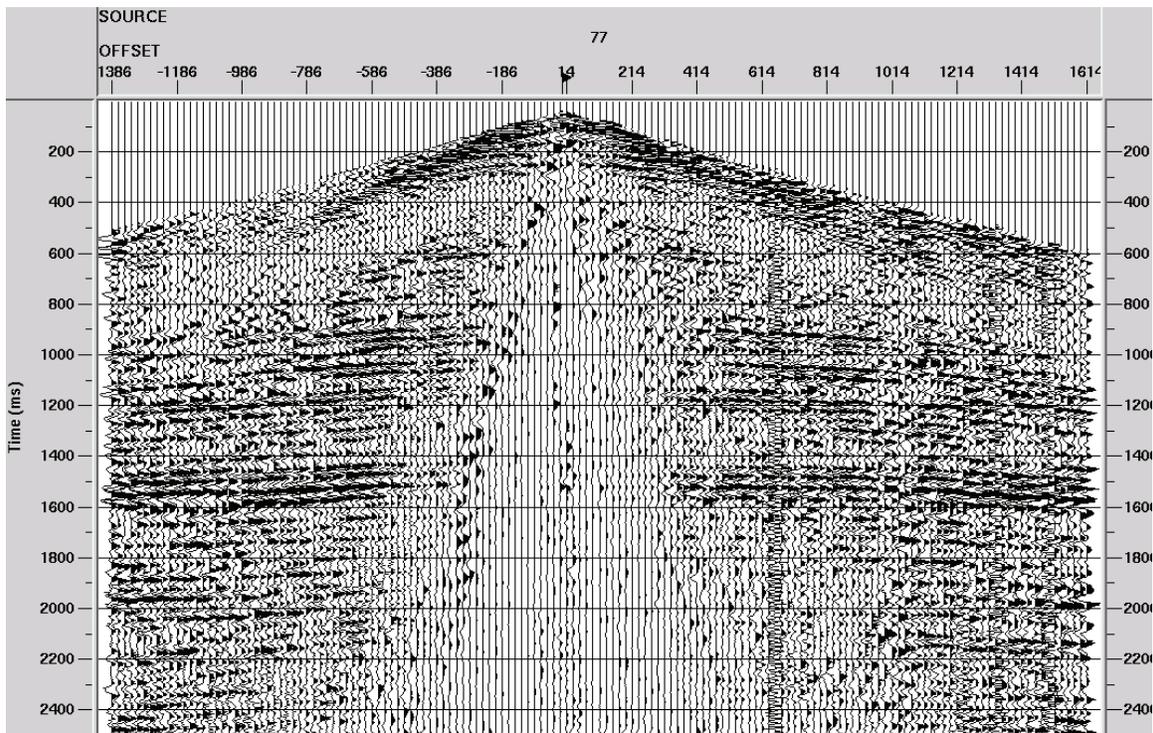
Blackfoot shot 77, least squares subtract low pass RT (8 – 12 Hz) in X-T, 125 ms gain smooth

FIG. 14. Least-squares radial filter with 125 ms gain window.



Blackfoot shot 77, least squares subtract low pass RT (8 – 12 Hz) in X-T, 67 ms gain smooth

FIG. 15. Least-squares radial filter with 67 ms gain window.

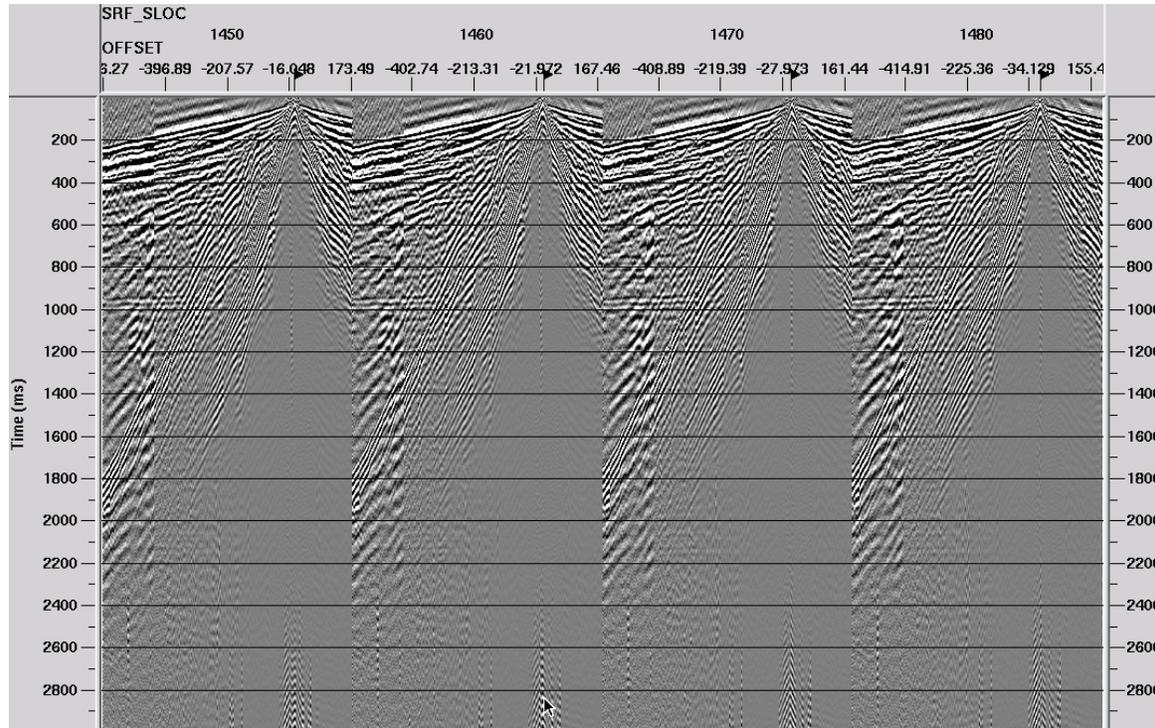


Blackfoot shot 77, least squares subtract low pass RT (8 – 12 Hz) in X-T, 34 ms gain smooth

FIG. 16. Least-squares radial filter with 34 ms gain window.

## Priddis high resolution survey

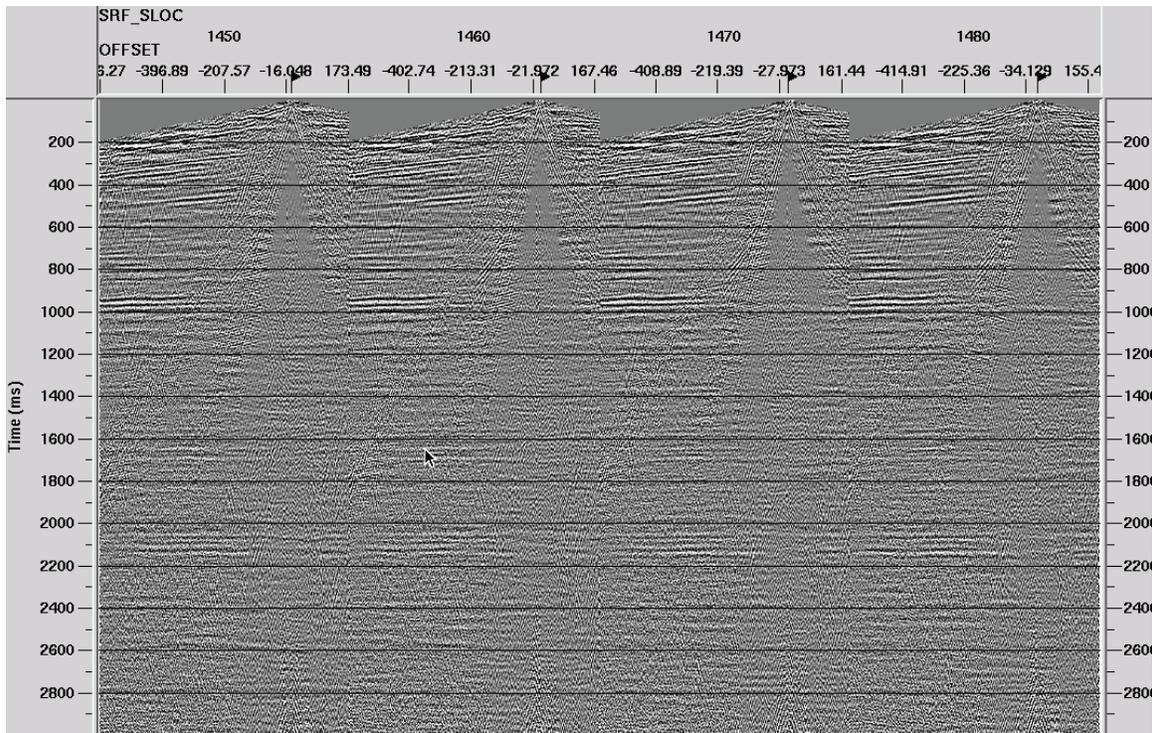
Though our experience with the new algorithm is still limited, we applied it to a recently acquired experimental high resolution data set from our Priddis field test site (Henley, et al 2009). For this experiment, we attenuated coherent noise on receiver gathers, since they were better sampled spatially (2 m) than the shot gathers (4 m). Figure 17 shows some representative receiver gathers before filtering. In our usual technique, we applied a series of radial trace fan and dip filters to all visible linear noise on these records.



Vertical component receiver gathers

FIG. 17. Typical vertical component receiver gathers from the 2009 Priddis 2D 3C line.

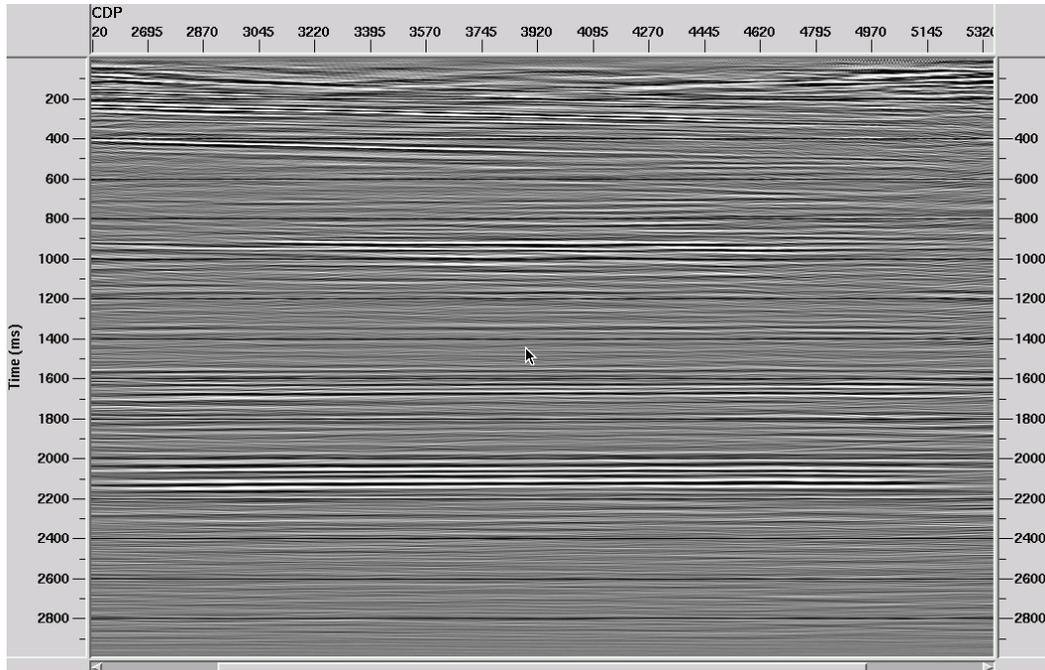
Figure 18 shows the gathers from Figure 17 after application of our filter sequence and Gabor deconvolution.



Vertical component receiver gathers after coherent noise attenuation, deconvolution

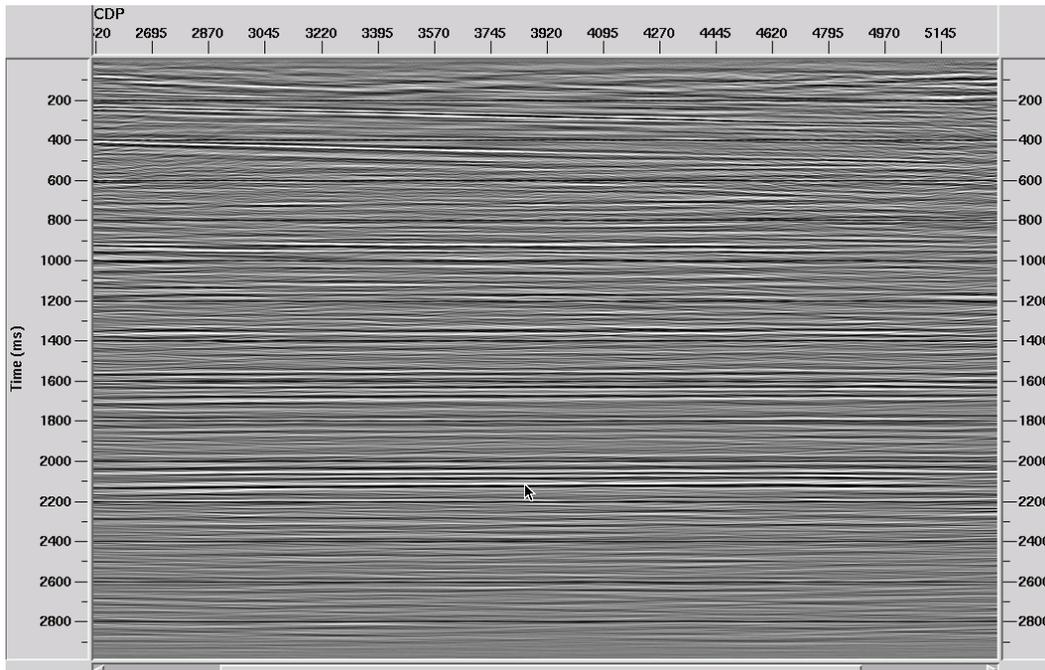
FIG. 18. Priddis survey receiver gathers after several passes of radial trace filtering and Gabor deconvolution.

The migrated stacked section in Figure 19 shows the result of this processing, using the original straight subtraction in the radial trace filters; while the migrated section in Figure 20 shows the corresponding results for the same filters using the new least-squares subtraction option.



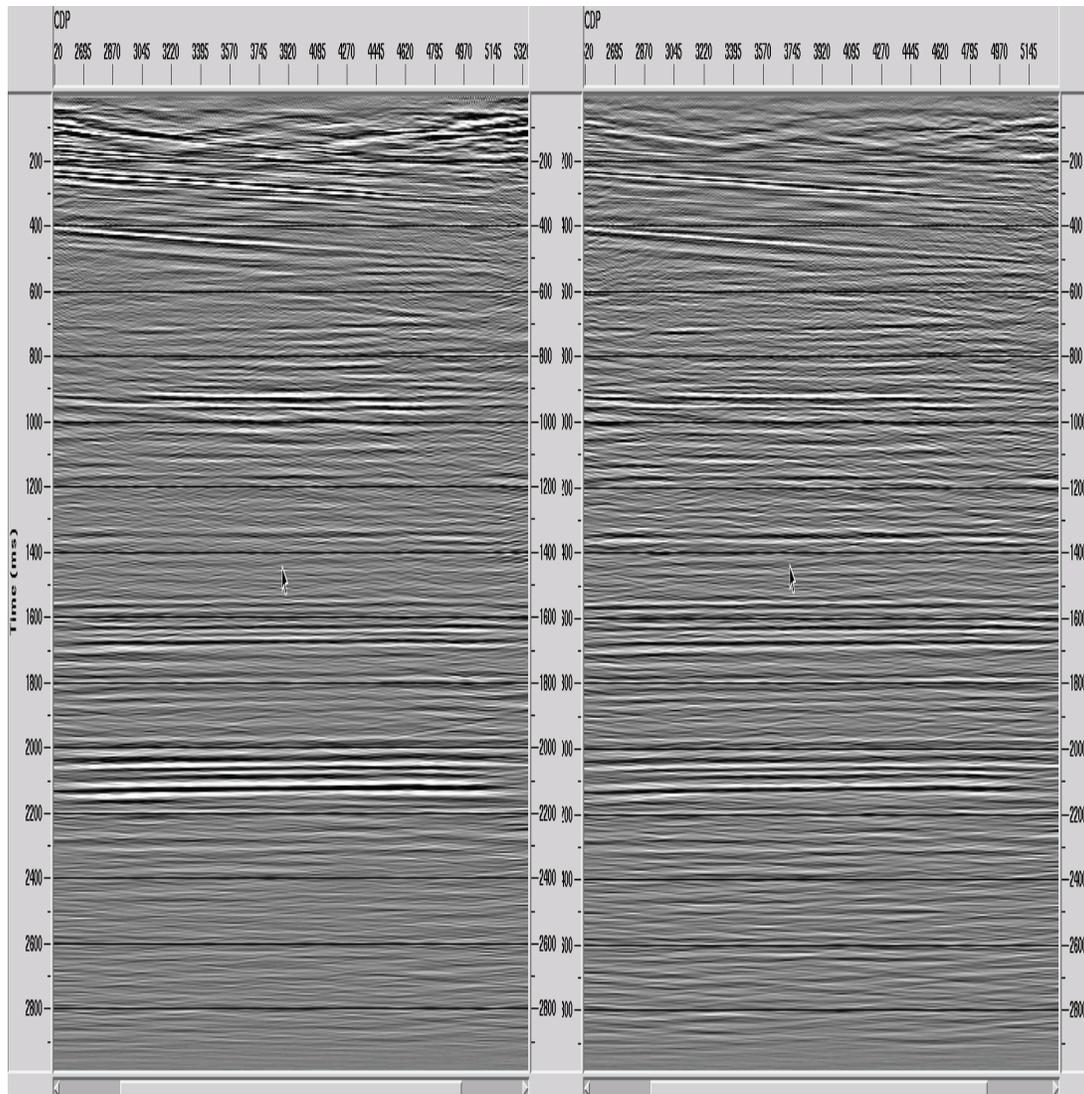
Vertical component migrated stack, after coherent noise attenuation and deconvolution

FIG. 19. Migrated stack of vertical component of 2009 Priddis 2D 3C survey, using conventional radial trace filtering.



Vertical component migrated stack, after coherent noise attenuation and deconvolution, version 2 with new lsq radial filtering

FIG. 20. Migrated stack of vertical component of 2009 Priddis 2D 3C survey, but with least-squares option used in all radial trace filters.



Old radial filters

New lsq radial filters

FIG. 21. Side-by-side comparison of Priddis vertical component migrated stacks, with aspect ratio more nearly in proportion to actual structure.

Figure 21 shows a side-by-side comparison of these two sections, compressed laterally to restore the vertical/horizontal aspect ratio to more nearly unity. The improvement in reflection clarity and continuity is evident.

## DISCUSSION

As can be seen in the previous examples, our new least-squares radial filter algorithm provides visible improvement in noise reduction compared with the earlier straight subtraction method. The effectiveness of the method is not strongly dependent upon the length of the gain window, as long as the window is shorter than the trace but long enough to average over several reflection events. An additional advantage is that in this mode, each X-T trace/noise pair is subtracted in its entirety, so pre-arrival random noise

is automatically removed (this random noise is replicated on the noise estimate traces, even though it is white, because only the data within the radial filter velocity limits are low-pass filtered). In the earlier straight subtraction method, the subtraction itself occurred only within the region defined by the radial filter velocity limits, and so did not address pre-arrival noise.

### ACKNOWLEDGEMENTS

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### REFERENCES

- Henley, David C. 2003, Coherent noise attenuation in the radial trace domain, *Geophysics*, **68**, No. 4, pp1408-1416, 2003.
- Henley, D.C., 2003, More coherent noise attenuation in the radial trace domain, CREWES 2003 research report **15**.
- Henley, D.C., Hall, K.W. Bertram, M.B., Gallant, E.V., Lu, H.X., and Maier, R., 2009, Shaken, not stirred: Priddis 2009 3C-2D hi-res acquisition: CREWES 2009 research report, **21**.

### APPENDIX

The standard derivation of the formula for a coefficient for least-squares subtraction is as follows:

Given a time series

$$y_j, j = 1, 2, \dots, N,$$

and a matching series

$$Ax_j, j = 1, 2, \dots, N,$$

we wish to determine the coefficient,  $A$ , such that the total squared error,

$$E = \sum_{j=1}^N (Ax_j - y_j)^2, \quad 1)$$

Is minimized.

The standard minimization procedure is to set the derivative of  $E$  with respect to  $A$  equal to zero. Hence

$$\frac{dE}{dA} = \frac{d \sum_{j=1}^N (Ax_j - y_j)^2}{dA} = \frac{d \sum_{j=1}^N (A^2 x_j^2 - 2Ax_j y_j + y_j^2)}{dA} = 0, \quad 2)$$

or

$$\sum_{j=1}^N 2Ax_j^2 - \sum_{j=1}^N 2x_jy_j = 0,$$

and

$$A = \frac{\sum_{j=1}^N x_jy_j}{\sum_{j=1}^N x_j^2}. \tag{3)}$$

To generalize Eq. 3), we wish to adapt it to apply a least-squared error gain function to one time series before subtracting it from another. Hence, we assume the two time series  $x_j$  and  $y_j$ , as well as the gain function  $A_j$ , exist over the interval  $j = 1, 2, \dots, M$ , and that we wish to apply the least squares error criterion over an interval of length  $N, N \leq M$ . The samples of the gain function  $A_j$  may be computed from the modified formula

$$A_j = \frac{\sum_{i=\max(1, j-N/2)}^{\min(j+N/2, M)} x_iy_i}{\sum_{i=\max(1, j-N/2)}^{\min(j+N/2, M)} x_i^2}, j = 1, 2, \dots, M. \tag{4)}$$

Examination of Eq. 4) shows that the index  $j$  specifies the centre of the gain window, and that the number of terms in each sum reduces linearly from  $N$  to  $N/2$  as  $j$  approaches either 1 or  $M$ .