Choosing reference velocities for PSPI migration

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ABSTRACT

A method of selecting reference velocities for PSPI migration is designed. Reference velocities will be chosen based on the complexity of velocity model. Migration results show that this method is basically satisfied.

INTRODUCTION

Phase shift migration (Gazdag, 1978) is unconditional stable which applies a phase shift in the Fourier domain to extrapolate wavefield at one depth to another. The limitation of this method is that it requires either constant medium velocity or velocity is just function of depth. To adapt for complex geological structure area where lateral velocities have considerable variations, phase shift migration was amended into phase shift plus interpolation (PSPI) migration (Gadag and Squazzero, 1984). PSPI migration uses a number of reference velocities to extrapolate wavefield to next depth. Since then methods of extrapolating wavefield and choosing reference velocities were present in literature (Margrave and Ferguson, 1999; Ma, 2008).

When performing PSPI migration, the accuracy of migration result is mainly credited to how and how many reference velocities being chosen for a given downward extrapolation step. A method of selecting reference velocities is designed in this paper which can select reference velocities based on the complexity of velocity model.

THEORY OF PSPI MIGRATION

Start with the 2D scalar wave equation which describes propagation of compression wavefield $\psi(x, z, t)$ in a medium with constant material density and P-wave velocity v,

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial z^2}=\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}, \tag{1}$$

where x and z are the horizontal axis and vertical axis (positive downward) respectively and t is time.

Wavefield $\psi(x, z, t)$ can be expressed in 2D frequency domain,

$$\psi(x,z,t)=\frac{1}{4\pi^2}\iint_{-\infty}^{+\infty}\varphi(k_x,z,\omega)e^{-i(\omega t-k_x x)}dk_x d\omega, \tag{2}$$

where ω is radial frequency and k_x is radial wavenumber.

Rewrite the scalar wave equation by substituting equation (2) into equation (1),

$$\frac{1}{4\pi^2}\iint_{-\infty}^{+\infty}\{\frac{\partial^2\varphi}{\partial z^2}+(\frac{\omega^2}{v^2}-k_x^2)\varphi\}e^{-i(\omega t-k_x x)}dk_x d\omega=0. \tag{3}$$

Equation (3) holds true for all ω and k_x, which means

$$\frac{\partial^2\varphi}{\partial z^2} = -(\frac{\omega^2}{v^2} - k_x^2)\varphi. \quad (4)$$

If v is constant, the solution to above equation is

$$\varphi(k_x, z = \Delta z, \omega) = \varphi(k_x, z = 0, \omega)e^{ik_z\Delta z}, \quad (5)$$

where

$$k_z = \pm\sqrt{\frac{\omega^2}{v^2} - k_x^2}. \quad (6)$$

This solution also holds true for $v(z)$ as long as Δz is small enough.

Equation (5) shows that wavefield at depth Δz can be calculated by phase shifting surface data in frequency domain. Substitute equation (5) back into equation (2) and let $t=0$, we will get

$$\psi(x, \Delta z, t = 0) = \frac{1}{4\pi^2}\int_{-\infty}^{+\infty}\{\int_{-\infty}^{+\infty}[\varphi(k_x, 0, \omega)e^{ik_z\Delta z}]e^{ik_x x}dk_x\}d\omega, \quad (7)$$

where $exp(ik_z\Delta z)$ is the phase difference between wavefield at depth Δz and surface. The part in the bracket is inverse Fourier transform about x, then integrating over frequency ω we will get a sample of migrated depth section $\psi(x, z, t=0)$ at depth $z=\Delta z$, i.e., a single line in (x, z) section. Depth migration can be obtained by repeating this process until desired depth.

In the presence of lateral velocity variation, i.e., v is a function of both x and z, $v=v(x,z)$, the phase difference of wavefield between depth $z+\Delta z$ and z cannot be simply computed through dispersion equation (6). To overcome this limitation, PSPI migration was introduced by Gadag and Squazzero in 1984.

PSPI migration is based on the assumption that the wavefield in point x_k is equivalent to a reference wavefield as long as the actual velocity in this point $v(x_k)$ equals to the reference velocity v_i, i.e., $v(x_k)=v_i$.

$$\varphi(x_k, z, \omega) = \varphi_{v_i}(x_k, z, \omega). \quad (8)$$

Implementing of PSPI migration consists of two steps. In the first step, phase shift is applied to obtain a set of reference wavefield $\{\varphi_{vi}(x, \omega, z)\}$ using a set of reference velocities $\{v_i\}$.

After we get these reference wavefields, an approximation to $\varphi(x, z, \omega)$ is estimated by linear interpolating over reference wavefields $\{\varphi_{vi}(x, \omega, z)\}$ if $\{v_i\}$ is close to $v(x_k)$.

$$\varphi(x_k, z, \omega) = Interpolate\{\varphi_{v_i}(x, z, \omega)\}. \quad (9)$$

To maintain high accuracy for small dip, a laterally varying time-shift is applied in the space-frequency domain as a pre-processor for the input data (Gazdag and Sguazzero, 1984).

$$\varphi^*(x,z,\omega) = \varphi(x,z,\omega)e^{i\frac{\omega}{v(x)}\Delta z}. \quad (10)$$

This extra time-shift will be compensated later in k_x-ω domain, which means that $exp(ik_z\Delta z\text{-}i\omega/v(x)\ \Delta z)$ is used as the phase difference instead of $exp(ik_z\Delta z)$ when extrapolating wavefield to next depth.

The flow chart of PSPI migration is shown in Figure 1.

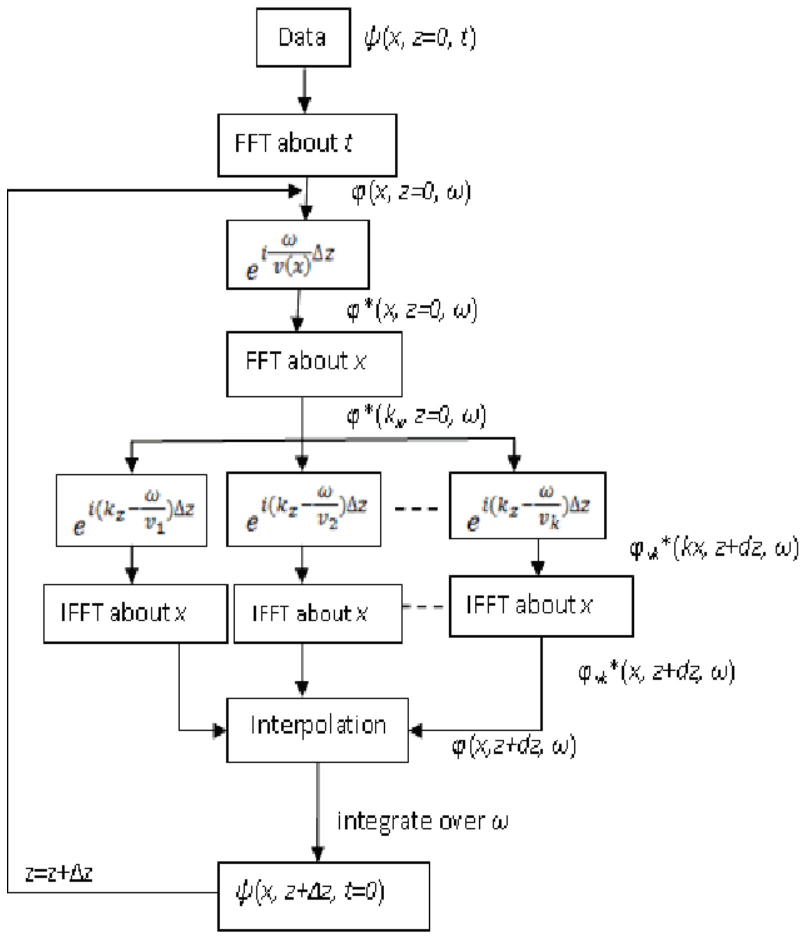

Figure 1. Flow chart of PSPI migration.

SYNTHETIC EXAMPLE

PSPI migration software and a method of choosing reference velocities were designed. Synthetic data was used to test the software and the method.

Figure 2 shows the synthetic data with receiver spacing dx=20m and sampling rate dt=2ms.

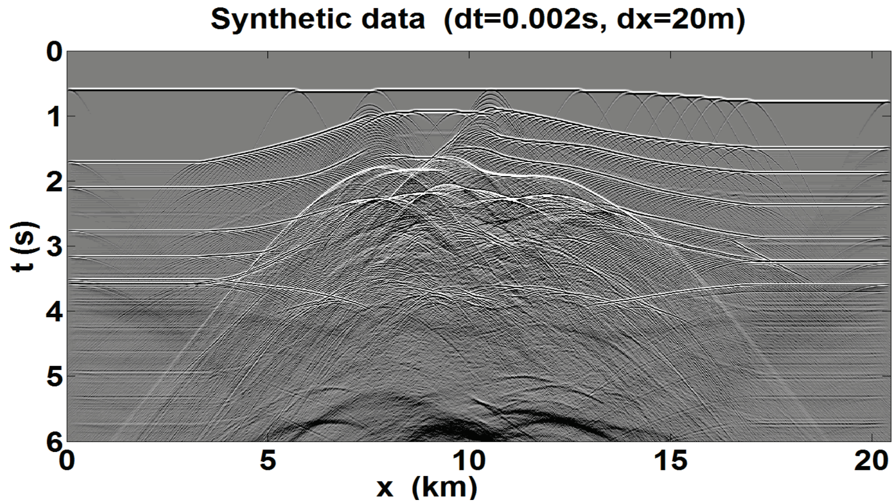

Figure 2. Synthetic data with sampling rate 2ms and receiver spacing 20m.

Figure 3 shows velocity model to be used in PSPI migration. It presents strong velocity variation in horizontal direction and a high-velocity-salt locates in the centre. The dash red line is marked at depth=2.4km in order to compare true velocity model with reference velocity models.

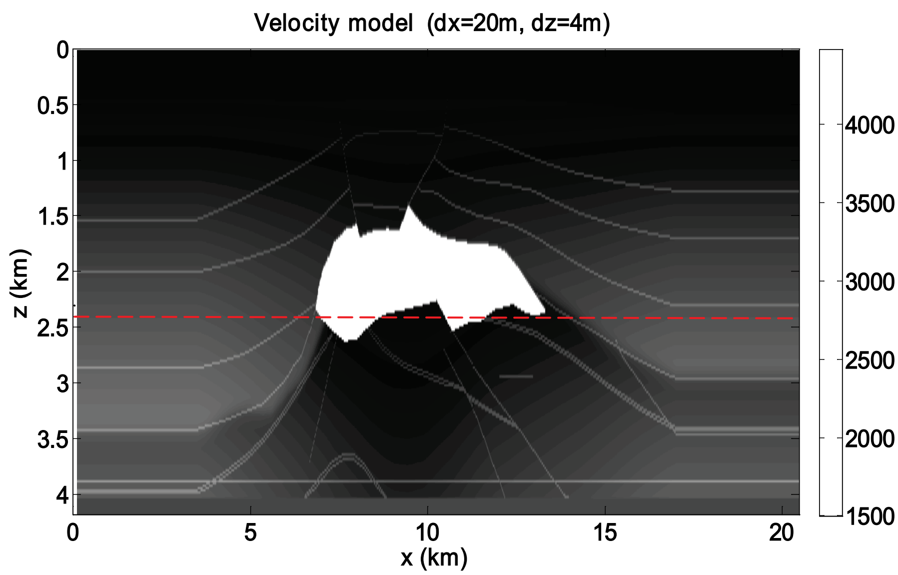

Figure 3. Velocity model with *dz*=4m and *dx*=20m.

Two reference models were used to test their effects on migration result. First, a simple model is used. Figure 4 shows the difference between true model and reference model at depth z=2.4km. Only four reference velocities are used at depth 2.4km. It is obvious that this model is a very rough estimation of true model. For example, from x=3km to x=7km, velocity changes from v_{true}=2563m/s to v_{true}=2350m/s, but there is only one constant reference velocity v_{ref}=2517m/s corresponds to this region.

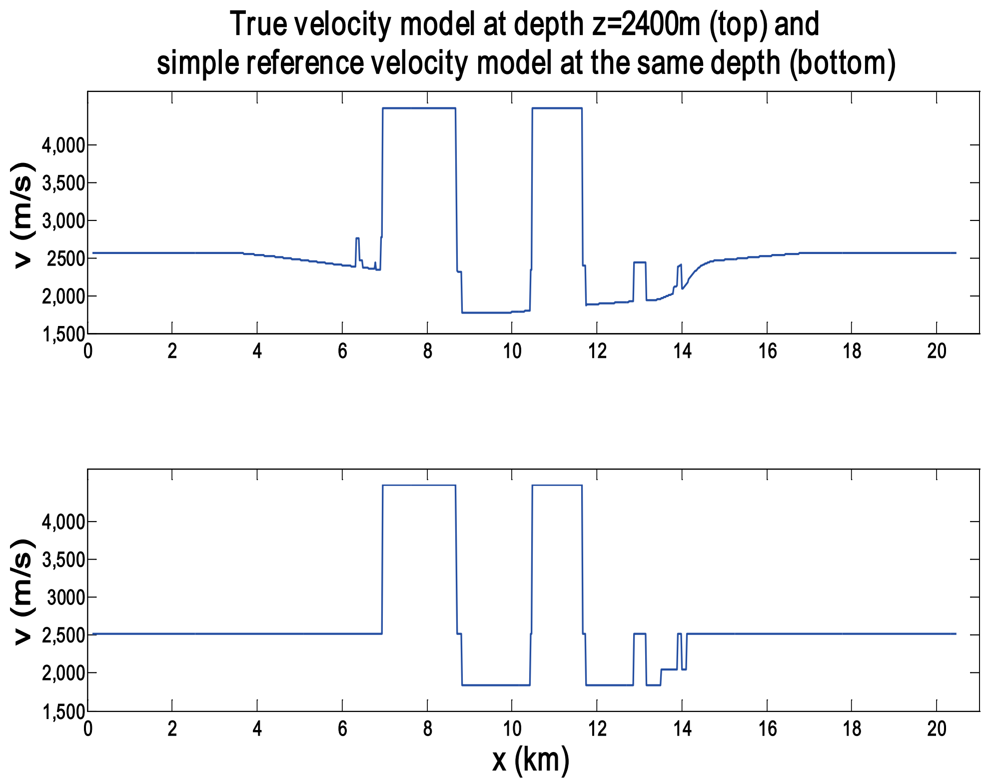

Figure 4. Comparison of true model (top) and simple model (bottom) at depth 2.4km.

Figure 5 shows the migration result using this simple reference velocity model.

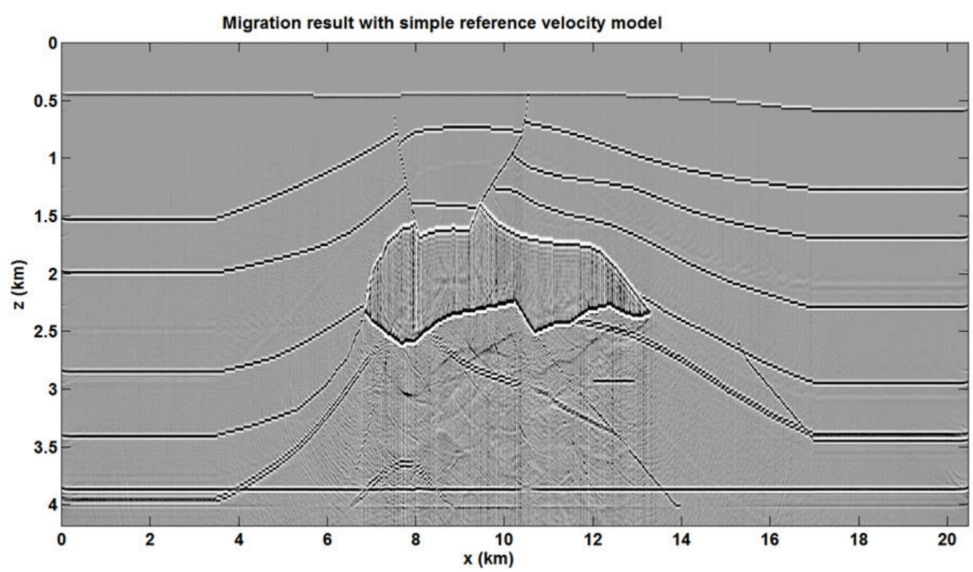

Figure 5. Migration result using simple reference velocity model.

In order to have better migration result, more reference velocities should be employed in PSPI migration. Figure 6 shows the comparison of true model (top) and a reference model (bottom). This new model is built by choosing velocity interval *dv*=40m/s. We can see that this model can be considered as a close approximation of the true model.

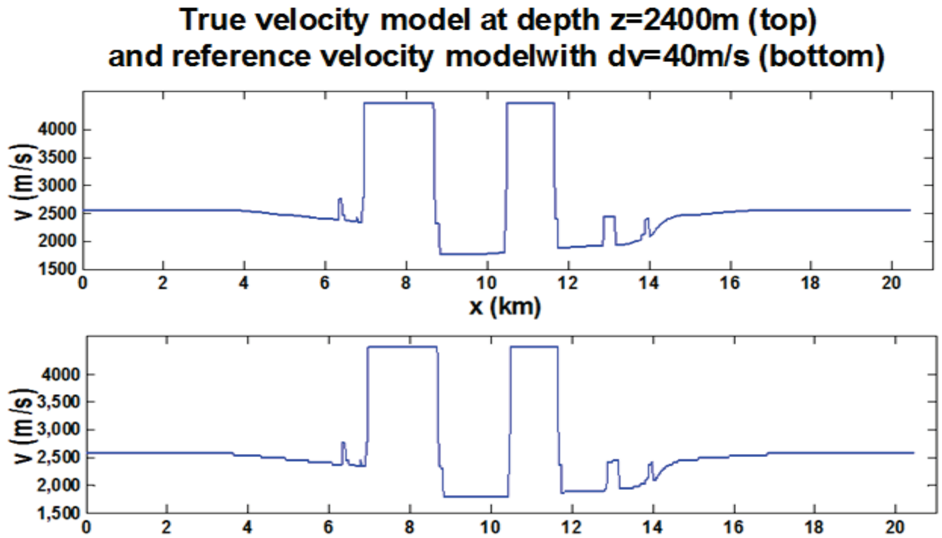

Figure 6. Comparison of true model (top) and reference velocity model with *dv*=40m/s.

Linear interpolation method was used to estimate wavefield at one point from a set of reference wavefields. Figure 7 shows the reference model from receiver R_{311} to receiver R_{331} at depth z=2.4km.

There are two questions associated with wavefield interpolation. First, do we need to calculate reference wavefields from v=2500m/s to v=2700m/s? The answer is negative because no point from R_{311} to R_{331} is associated with these velocities. Second, which two reference velocities should be chosen at R_{317} (Point B)? Obviously the reference velocity at R_{318} cannot be chosen.

This paper presents a method which can judge how many reference velocities should be used in PSPI migration; it can also determine which reference velocities should be used at given position.

For example, for R_{314}, neighbour reference velocities are used; for R_{317}, only reference velocities at its left side are used; for R_{318}, the reference velocity is true velocity, so interpolation is not necessary at this point; for R_{322}, only reference velocities at its right side are used.

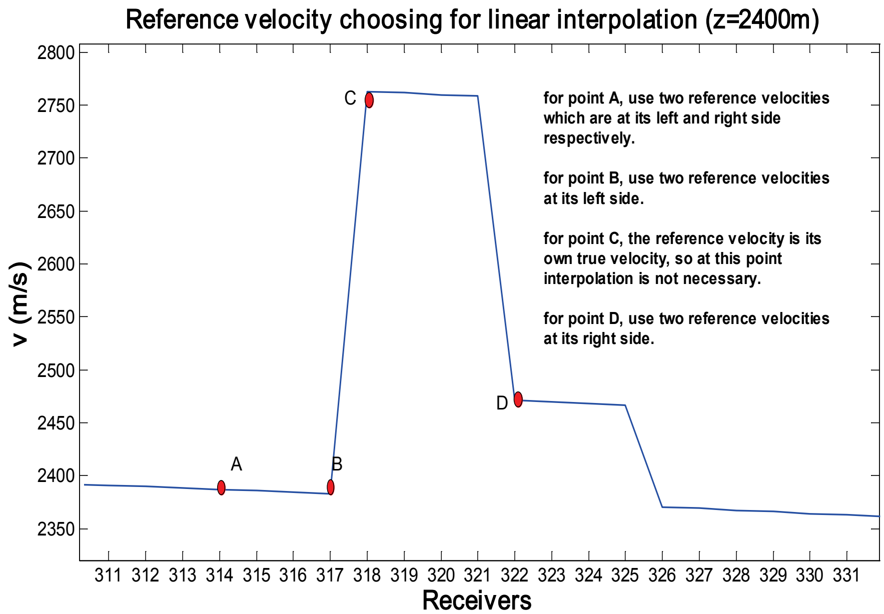

Figure 7. Rule of choosing reference velocities. Four typical points are marked by red dot.

Migration result using new reference model and linear interpolation is shown in Figrue 8. We can see that the quality of migration is improved especially inside and under the salt.

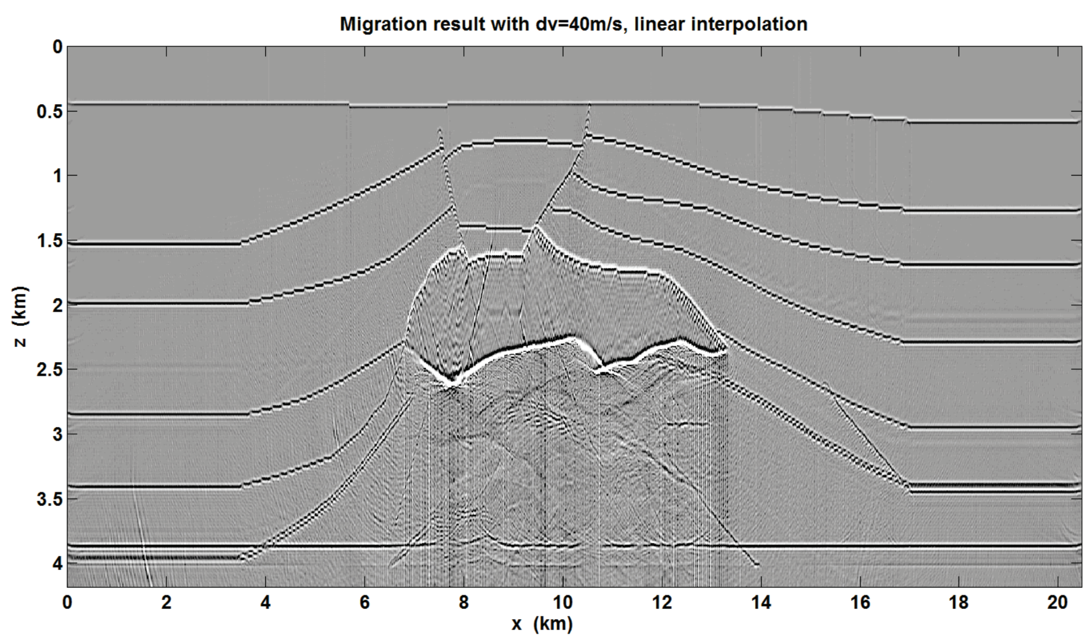

Figure 8. Migration result using new reference model (*dv*=40m/s) and linear interpolation.

CONCLUSION

PSPI migration software and a method of choosing reference velocities were designed. Reference velocities will be chosen based on the complexity of velocity model. Migration results show that this method is basically satisfied.

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