Decomposition of surface consistent statics

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ABSTRACT

Traveltime differences between traces and a model trace may be decomposed into source and receiver statics. The decomposition of these statics is a difficult problem because the inversion process is rank deficient. The rank can be increased by adding a stabilizing equation to make the process invertable. Three stabilizing equations are tested and evaluated to demonstrate their effectiveness in stabilizing the process.

INTRODUCTION

Traces are formed into a gather for stacking. Before stacking, the traces may have small time shifts that will degrade the resolution of the stacked trace. These estimated times shifts cannot be applied directly to the traces as they can produce distortions in the stacked data. Decomposing these time shifts into source and receiver components, the statics, reduces the problem of the distortion. This is accomplished by:

- cross correlating each trace with a model trace
- obtaining the time shift from the cross correlation, and
- decomposing these time shifts into source and receiver components, and then
- applying the decomposed time to the corresponding traces.

We will focus on the decomposition of the time shifts t into the source and receiver components s and r.

Consider N shots whose reflection energy is acquired by M receivers. There will be a static s_i for each shot i, and a static r_j for each receiver j. These are not known at this time. There will be $N*M$ traces, each with a total static of $t_{ij} = s_i + r_j$. We estimate the value of t_{ij} from techniques such as cross-correlation between each input trace and a model trace. We now want to estimate the $M+N$ values of s_i and r_j from the $N*M$ measurements of t_{ij}. We have many more equations of t_{ij} than the unknowns s_i and r_j, and one might assume that a simple least squares formulation would solve the problem. That is not the case. Even though we have many more equation than unknowns, many of the equations are linearly dependent and could be reduced to one less than the number of unknown statics making the solution of s_i and r_j impossible.

In practical applications, there are many more receivers than shots, and only some of the receivers are associated with a given shot. Even then, the number of independent equations will be one less than required. There may be other complications that arise when the shot interval is four time the receiver interval. The estimate of t_{ij} may also contain random noise and other coherent noise or biases due to incorrect velocities.

We will simplify the problem to one with jitter or random noise added to the estimates of t_{ij}.

STATIC MODEL

We simplify the problem by assuming we have three sources (shots) shooting into the same five receivers giving fifteen traces. Fifteen correlation times will be designated t_k, where k varies from 1 to 15, as defined by the noiseless equations

$$\begin{aligned} t_1 &= s_1 + r_1 \\ t_2 &= s_1 + r_2 \\ t_3 &= s_1 + r_3 \\ &\ldots \\ t_{14} &= s_3 + r_4 \\ t_{15} &= s_3 + r_5 \end{aligned} \quad . \tag{1}$$

These equations may be written in matrix form

$$\begin{bmatrix} 1 & & & 1 & & & & \\ 1 & & & & 1 & & & \\ 1 & & & & & 1 & & \\ 1 & & & & & & 1 & \\ 1 & & & & & & & 1 \\ & 1 & & 1 & & & & \\ & 1 & & & 1 & & & \\ & 1 & & & & 1 & & \\ & 1 & & & & & 1 & \\ & 1 & & & & & & 1 \\ & & 1 & 1 & & & & \\ & & 1 & & 1 & & & \\ & & 1 & & & 1 & & \\ & & 1 & & & & 1 & \\ & & 1 & & & & & 1 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \end{bmatrix} = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \\ t_7 \\ t_8 \\ t_9 \\ t_{10} \\ t_{11} \\ t_{12} \\ t_{13} \\ t_{14} \\ t_{15} \end{bmatrix}, \tag{2}$$

or

$$\mathbf{Gm} = \mathbf{t}\,. \tag{3}$$

Note in equation (2) that we can produce two identical equations by subtracting line 2 from line 1, and from subtracting line 7 from line 6 to get $r_1 - r_2 = t_?$, illustrating the problem of linear dependence. In the absence of noise, the resulting times would be equal. In the presence of noise, the resulting times would be the average.

The rank of **G** is 7, not 8, as may be initially expected, and the unknown parameters cannot be evaluated by matrix inversion. Noise can also be added to the t_k parameters to further complicate the problem.

METHOD

We increase the rank of **G** by adding a new equation to the system of equations in (1). From our knowledge of the acquisition geometry and geology, we can relate the statics of a source and receiver if they are at the same location. When using a surface source such as Vibroseis, then we can assume the two statics are equal, for example at location 1,

$$s_1 - r_1 = 0 . \tag{4}$$

If the source is down-hole dynamite, then we may measure the "up-hole" time t_{up} to give

$$s_1 - r_1 = \pm t_{up} , \tag{5}$$

where the sign is dependent on user definitions.

The question now becomes, is equation (4) linearly independent? When substituted into **G**, we do find the matrix now has a rank of 8. With no noise on t_k, the number of equation can be reduced to 8 and **m** computed exactly. When there is noise, **m** is estimated using the least squares method, i.e.,

$$\mathbf{m} = \left(\mathbf{G}^{\mathrm{T}}\mathbf{G}\right)^{-1} \mathbf{G}^{\mathrm{T}}\mathbf{t} . \tag{6}$$

We will also consider two additional equations: a second one that assumes each source has the same static as a corresponding receiver, and a third one where the average of all source statics is equal to the average of all receiver statics.

If we assume the three source locations coincide with the receiver locations r_1, r_3, and r_5, then the second stabilizing equation is

$$s_1 + s_2 + s_3 - r_1 - r_3 - r_5 = 0 . \tag{7}$$

If the averages are equal, then we could use the third stabilizing equation

$$\frac{1}{3}\sum_1^3 s_i - \frac{1}{5}\sum_1^5 r_k = 0 . \tag{8}$$

Times could be added to these two equations if up-hole times are appropriate.

NUMERICAL TESTS

Statics for each source and receiver were chosen randomly, and then combined to define the combined static t_k. Noise was then added to t_k and the parameters s_i and r_j were estimated by least squares, and then compared with the initially defined values. This procedure was repeated for 100 trials, each using the same source and receiver statics. Standard deviations of the error were estimated and the process was repeated for each of the three stabilizing equations. The results are shown in Figure 1 where the errors of the first four trials are shown along with the standard deviation of the 100 trials. The true static had a standard deviation of 0.010 sec, and the noise a standard deviation of 0.001 sec, for a signal-to-noise (SNR) ratio of 10. These tests were then repeated with the noise reduced to 0.0001 sec, (SNR = 100) and displayed in Figure 2.

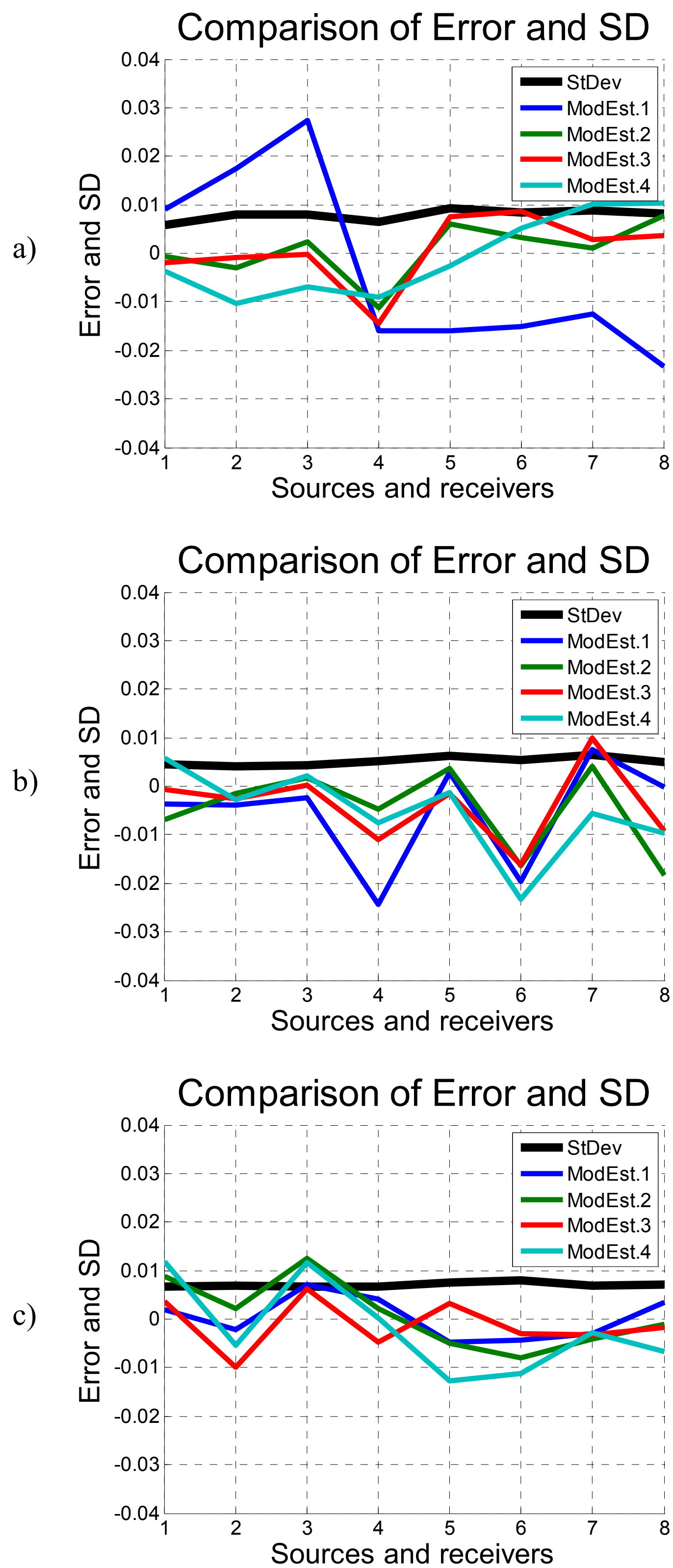

FIG 1. The errors from four trial solutions and the standard deviation of 100 trials are shown in a) using equation (4), b) equation (7), and c) equation (8). The noise was 10%.

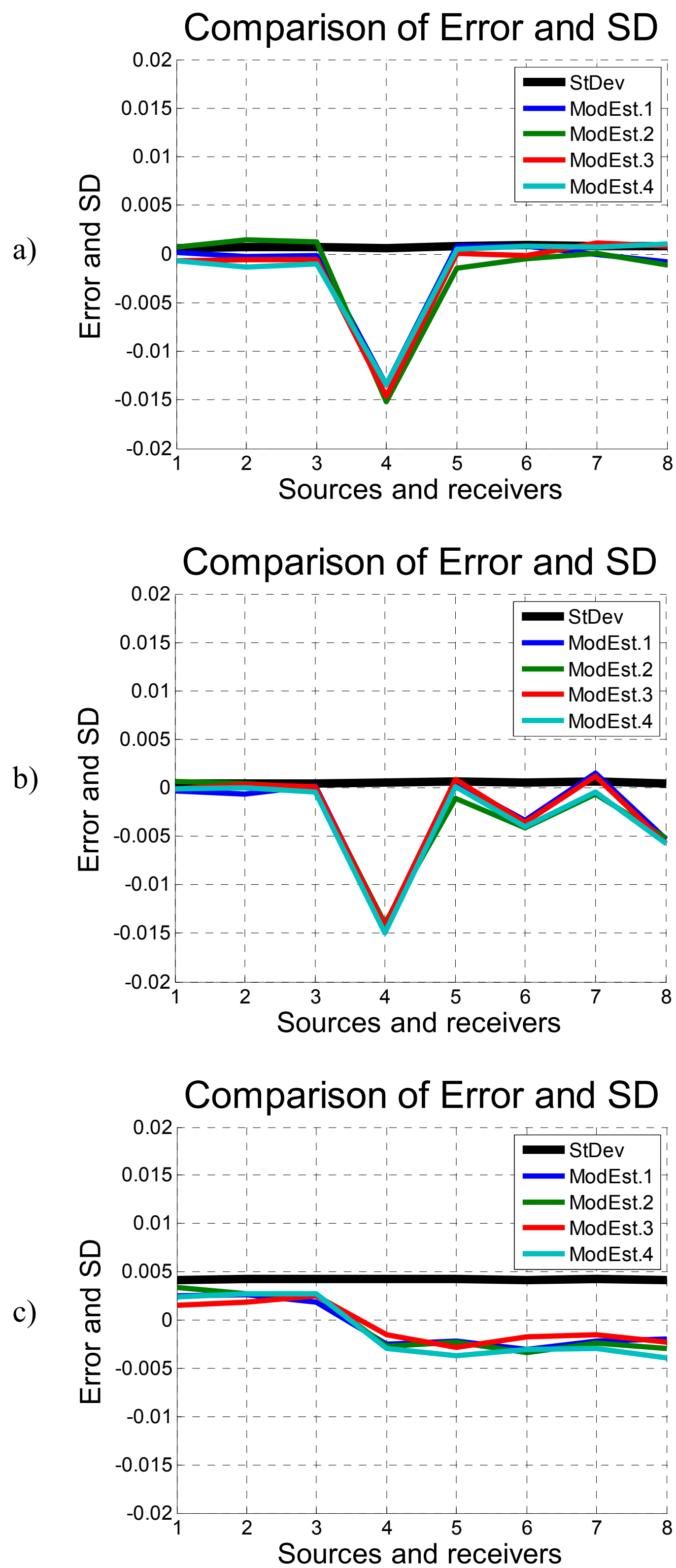

FIG 2. Similar to Figure 1, but now the noise is reduced to 1%.

COMMENTS

The test above used a simple geometry to validate the proposal of using a single stabilizing equation. More testing is required to draw conclusions from these results. We may assume that equations which use more parameters may have a more stabilizing effect on the solution. That will require tests that match real acquisition geometries with large numbers of sources and receivers. In addition, the realistic tests may have sources located at large receiver intervals that cause a number of independent solutions within the receivers. In this case, a number of stabilizing equations may be required for each matrix **G**. In this test, only one of the stabilizing equations was used in **G**.

The stabilizing equations may reduce the number of independent variables by tying the statics together. This may modify the rank of the matrix, and may make stabilizing equations with a large number of parameters appear to be more accurate or have a lower SD.

The similar trends of the error in Figures 1b, and 2b indicate that the error may be dependent on the values used for the actual static. The testing should be modified to require the defined source and receiver statics to be identical in each trial of the equation tests. We should apply 100 trials of jitter to 100 different defined statics for s_i and r_j.

More advance testing of the results can be use such as singular value decomposition (SVD) to evaluate the effect of each stabilizing equation on the estimated parameters.

CONCLUSION

Adding stabilizing equations to the decomposition process make the inversion process possible. Three stabilizing equation were tested with positive results. The results of each equation provided different standard deviations (SD) of the estimated parameters. The significance of these estimates requires further consideration.

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