Mud-rock line estimation via robust locally weighted scattering smoothing method

A. Nassir Saeed, Laurence R. Lines, and Gary F. Margrave

ABSTRACT

The robust locally weighted scattering smoothing (LWESS) is another regression method that not only smoothes the scatter plot but also guard against outliers that distort the smoothed points. The mud-rock line produced by the robust locally weighted smoothing scattering method, LOWESS, has shown superiority over the Castagna linear regression method in mapping the Glauconitic sand reservoir of Blackfoot area. The produced graph has demonstrated to be a good visualizing tool for an effective direct hydrocarbon indicator (DHI) as well as in discriminating sand and carbonate lines.

INTRODUCTION

The linear regression method is substantially influenced by the presence of noise or outliers that might give inaccurate estimation of the slope and intercept points; thus erroneous predicted data. The main objective of this study is to research for another regression method that is less affected by the presence of noise in order to estimate the mud-rock line more precisely.

MUDROCK LINE ESTIMATION

Castagna et al., (1985) used the Vp-Vs linear relation to predict V_p from V_s via linear regression method. The Castagna's mud-rock line equation is written as

$$V_p = 1.16\,V_s + 1.36 \tag{1}$$

where sonic velocities are in *Km*/*sec*.

A classical procedure for noisy data usually require some editing before performing linear regression so as to reduce influences of outliers on estimating slope and intercept points used in best fit-line equation. Another approach is by fitting polynomial of higher degrees, such as quadratic equation, where the curvature factor is included in the best fit-line equation. The latter procedure will violate the simple Vp-Vs linear relation sought in predicting missing log data.

Hence, in order to map mud-rock line more precisely, we should look for another regression method that take into consideration the effect of neighbor points in the scattering graph as well as to prove it's robustness against the outliers.

ROBUST LOCALLY WEIGHTED SMOOTHING SCATTERING (LOWESS)

Cleveland, (1979) has introduced the robust locally weighted scattering smoothing (LOWESS) method. This is another regression method that not only smooths the scatter plot but also guard against outliers that normally distort the smoothed points in the scattering plot.

Given (x_i, y_i) points, let d are the distance between x_i and it's b^{th} nearest neighbors along the x-axis. b is the nearest integer to $(f \cdot n/2)$, where n is the number of data point and f is the smoothing factor ($0 < f < 1$). A greater value of f leads to smoothing the curve line, while small values of f produce rougher curve. A practical value of f is usually between 0.2 and 0.8.

The weighted slope ($b_{estimate}$) and the intercept ($a_{estimate}$) of line derived from robust locally weighted regression method can be written as,

$$b_{estimate} = \frac{\sum W_k^2 (x_i - \bar{x}) \cdot (y_i - \bar{y})}{W_k^2 (x_i - \bar{x})^2} \quad (2)$$

$$a_{estimate} = \bar{y} - b_{estimate} \,.\bar{x} \quad (3)$$

Where,

$\bar{y}$ *and* $\bar{x}$ are weighted mean values, and

$$W_k = \varpi\left(\frac{x_i - x_k}{d_i}\right), \quad \varpi(x) = \begin{cases} (1-|x|^p)^p & for |x| < 1 \\ 0 & for |x| \geq 1 \end{cases}$$

The best fit line equation is then written as

$$y_w = a_{estimate} + b_{estimate} \bar{x} \quad (4)$$

The weight function can be either bi-weight function (p=2) or, tri-cube weight function (p=3). In this study, tri-cube weight function is applied. Note that the size of weight depends on the magnitude of residual $y_i - \hat{y}_i$. Large residuals results in small weights and small residuals result in large weights (Cleveland, 1979).

The robust locally weighted smooth scattering, LOWESS, is used in this study in order to map the mud-rock line. Figure (1) shows the mud-rock line produced by LOWESS method. As expected, this method is less influenced by noise that substantially affects the estimation of slope and intercept of fitting line.

Figure (2) shows the mud-rock lines produced by LOWESS (in red color), locally linear-regression (in magenta color) and by using Castagna's mud-rock equation (in green color). The data points are colored by gamma ray values, where sand usually has low gamma ray values. The slope and intercept values estimated via LOWESS method are also used in equation (1), and produced mud-rock line (in purple color) in figure (2). The resulting weighted mud-rock line shows better estimate for the intercept value compared to the locally derived linear regression method, and demonstrated to be less influenced by the outliers.

The mud-rock line estimated by LOWESS method has successfully delineated channel sand, whereas linear regression and ARCO mud-rock equation did not. Furthermore, the carbonate line can be easily distinguished by the LOWESS method, where it starts from the intersection point of Vs (around 2.8 Km) and Vp of 4.8Km/sec and above.

In figure (3), velocity values of the sonic shear (Vs) log of the productive reservoir sand channel (black dots) are superimposed in the section. Indeed, the mud-rock line estimated by LOWESS regression method has proven to be effective DHI tool. In figure (4), the sonic P-wave of the log and P-wave velocity produced via LOWESS method show good resemblances. The curve in green color represents the P-wave velocity produced in smoothing scattering domain.

CONCLUSIONS

The robust locally weighted smoothing scattering method successfully maps the productive sand reservoir as well as the carbonate line. It proves to be a good interpretive tool to be used in hydrocarbon and lithology discrimination. The predicted log of P-wave velocity shows excellent resemblance to the original sonic P-wave log

REFERENCES

Castagna, J. P. Batzle, M.L. and Eastwood, R.L., 1985, Relationships between compressional-wave and shear-wave velocities in elastic silicate rocks: Geophysics, **50**, 571-581.

Cleveland, W.S., 1979, Robust locally weighted regression and smoothing scatterplots: Journal of the American Statistical Assoc., **74**, 829-836.

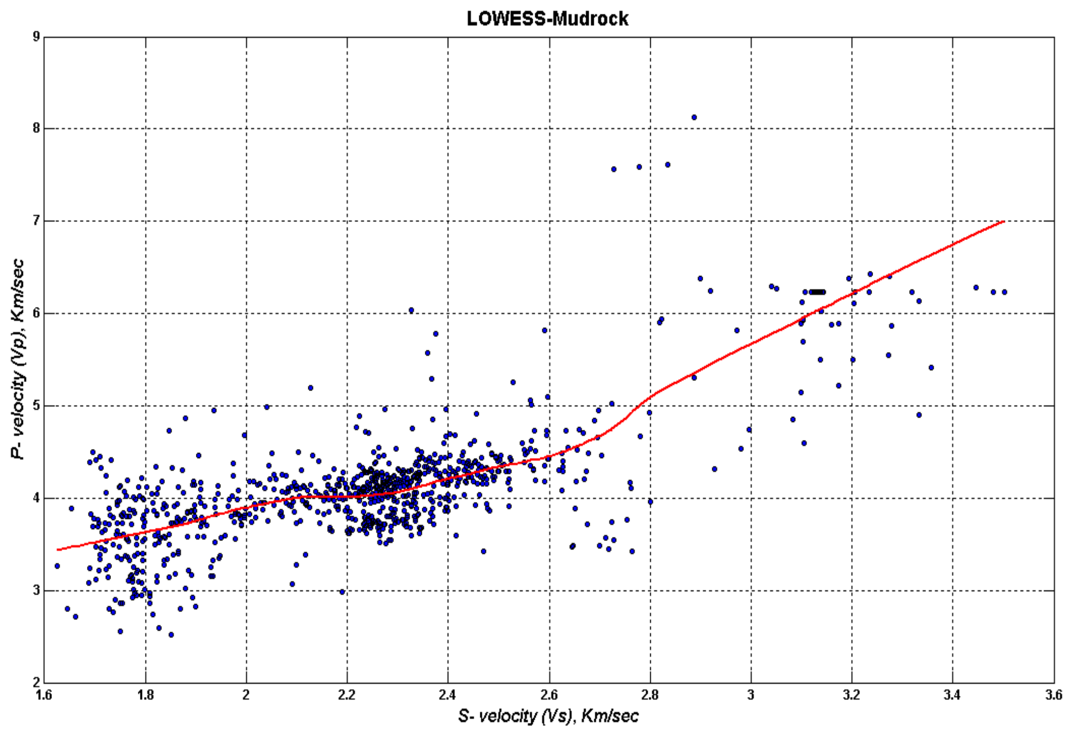

FIG.1.Robust locally weighted regression method, LOWESS, for estimating mud-rock line.

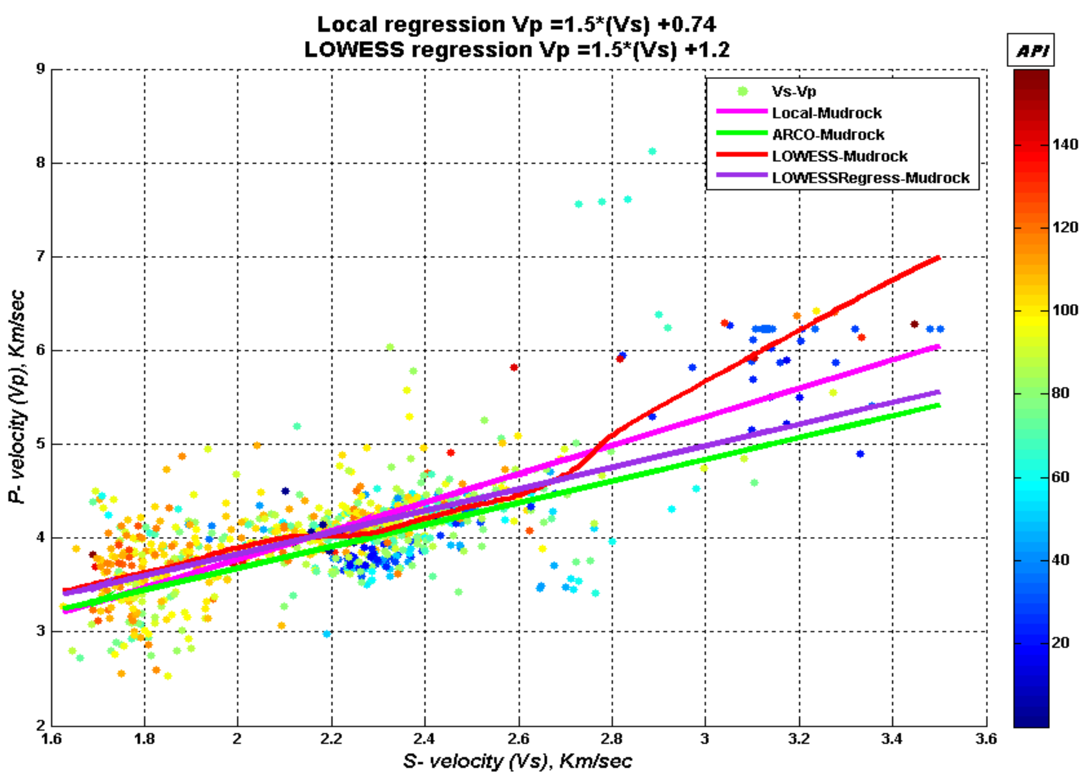

FIG.2. Robust locally weighted regression method for estimating mud-rock line. Data points are colored by Gamma ray

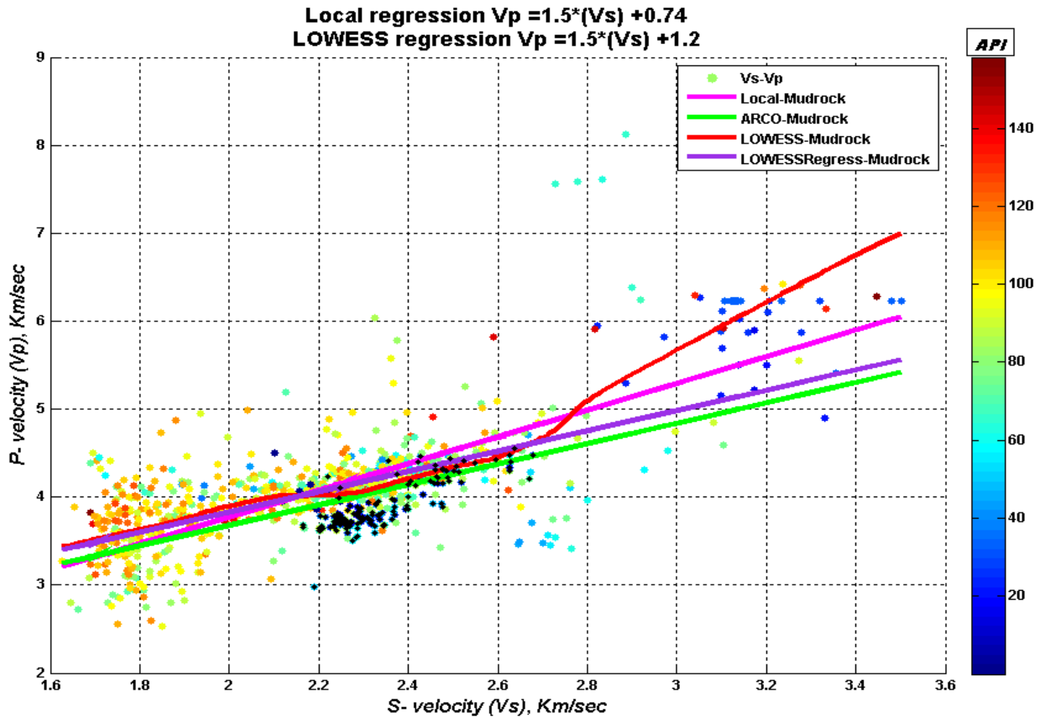

FIG.3. Robust locally weighted regression method for estimating mud-rock line. V_s values from the productive sand channel reservoir (black dots) are superimposed.

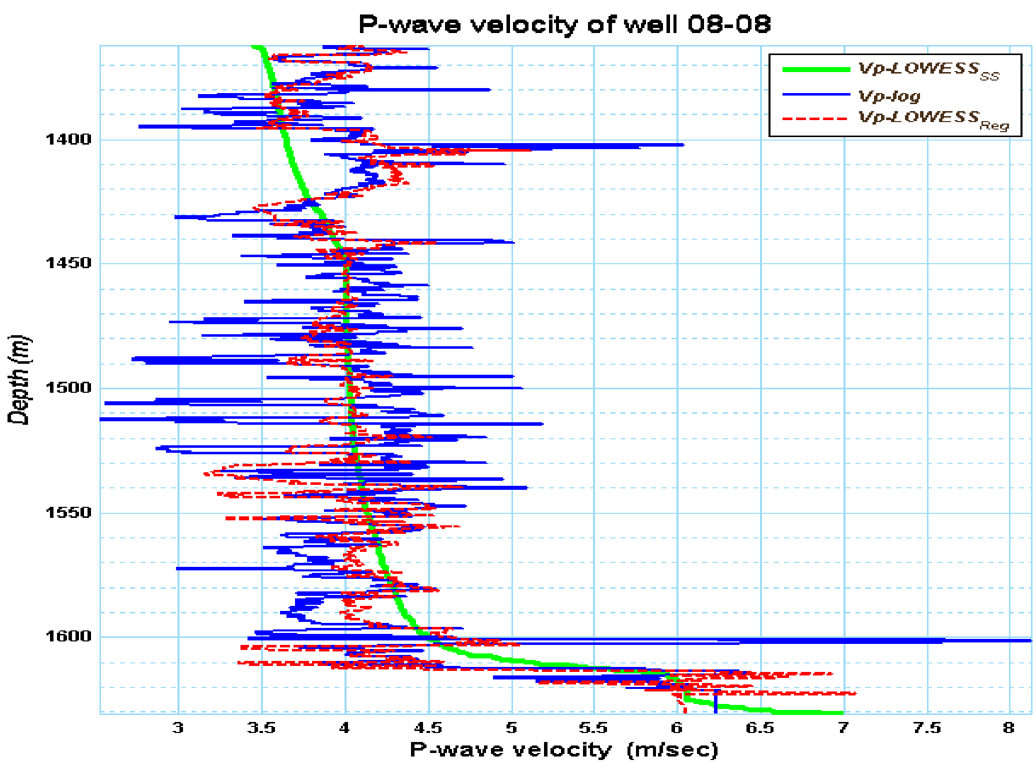

FIG.4. P-wave sonic log, and derived P-wave velocity via the LOWESS method of well 08-08.