Short note: Shaping / Matching filters

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ABSTRACT

In this paper, we briefly present a review of one of the most common filters in geophysics: that is matching filter. Matching filter is one of the simplest filters to apply, but one needs to consider two important criteria: $i)$ the matching filter length and $ii)$ an optimum crosscorrelation lag. We provide two examples that analyze the effects of the filter length and the correlation between two dissimilar input traces to be matched. These examples show that an optimum filter length needs to be tested by plotting filter lengths vs residuals norm (L-curve method). In some cases a filter length that is less than the input trace length is ideal and reduces the computational time significantly. In the second example, two dissimilar traces are matched and the result is a larger residual error. The value of this example stresses the importance that both traces to be matched need to be correlatable.

INTRODUCTION

Sheriff (1994) defined matching filter as one "which maximizes the output in response to a signal of particular shape." In literature, one might hear about the Wiener least-square filtering, matching or shaping filters (Robinson and Treitel, 1980; Claerbout, 1976), cross-correlation filters (Anstey, 1964), and correlator (Karl, 1989), to name just a few, and think they are different where in fact they share the same definition.

One of the common practices in geophysics is the need to alter the shape of an input signal in order to obtain a desired output signal. This alteration is performed through what's known as shaping or matching filters. In this paper, we will be looking at how matching filters operate, what is required to obtain an optimum matching filter and if these filters are limited.

Following Robinson and Treitel (1980) notations, we consider the problem of finding a filter $f_t = (f_0, f_1, ..., f_m)$ of length $m+1$ that shapes an input waveform $b_t = (b_0, b_1, ..., b_n)$ of length $n+1$ into a desired output trace $d_t = (d_0, d_1, ..., d_{m+n})$ of length $m+n+1$ so that the error between the desired output d_t and actual output c_t is minimum (Figure 1). The actual output is written as follow:

$$c_t = \sum_{s=0}^{m} f_s b_{t-s}, \tag{1}$$

where this is known as the convolution of the shaping filter with the input signal.

The expression for this residual vector is

$$\begin{aligned} \mathbf{e} &= \mathbf{Bf} - \mathbf{d} \\ &= \mathbf{C} - \mathbf{d}, \end{aligned} \tag{2}$$

where $\mathbf{B}$ is the convolution matrix formed from $\mathbf{b}$. $\mathbf{f}$ is the computed shaping filter. Our

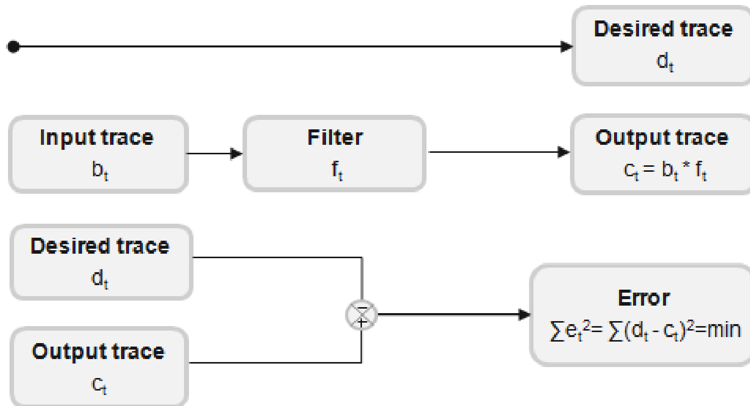

FIG. 1: General least-squares filtering model (after Robinson and Treitel (1980)).

notation convention in this paper for matrices are upper case boldface and vectors are lower case boldface.

Minimizing **e** in equation 2 in the least-squares sense results in solving the system of normal equations:

$$\begin{aligned} \mathbf{B}^T\mathbf{B}\mathbf{f} &= \mathbf{B}^T\mathbf{d} \\ \mathbf{f} &= (\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T\mathbf{d} \end{aligned} \tag{3}$$

In equation 3, the filter coefficients of **f** given by the cross-correlation of **b** with **d**, filtered by the inverse of the autocorrelation of **b**. Equation 3 can be written in matrix form

$$\begin{bmatrix} \phi_0 & \phi_1 & \phi_2 & \phi_3 & . & \phi_m \\ \phi_1 & \phi_0 & \phi_1 & \phi_2 & . & . \\ \phi_2 & \phi_1 & \phi_0 & \phi_1 & \phi_2 & . \\ \phi_3 & \phi_2 & \phi_1 & \phi_0 & \phi_1 & . \\ . & . & . & . & \ddots & . \\ \phi_m & \phi_{m-1} & . & . & . & \phi_0 \end{bmatrix} \begin{bmatrix} f_0 \\ f_1 \\ . \\ . \\ \vdots \\ f_m \end{bmatrix} = \begin{bmatrix} S_0 \\ S_1 \\ . \\ . \\ \vdots \\ S_m \end{bmatrix}, \tag{4}$$

where ϕ_m is the m^{th} lag of the autocorrelation of **b** (the input signal) and **s** is the cross-correlation of **b** with **d**. This filter is known as the correlator (Karl, 1989) since convolution with the signal's time reversed terms is the same as the correlation without reversing the signal's time terms.

To design a matching filter one needs to consider two criteria: i) filter length and ii) an optimum lag. Increasing the filter length improves the performance of the matching filters. If we consider matching two signals **s1** and **s2** using **m** then an optimum length of **m** is the length of the input signal, **s1**. If **m** is a perfect matching filter that converts **s1** into **s2**, which means

$$\mathbf{m} * \mathbf{s1} = \mathbf{s2} \tag{5}$$

where * denotes convolution. In the Fourier domain, equation 5 is

$$M(\omega) = \frac{S_2(\omega)}{S_1(\omega)}. \tag{6}$$

Equation 6 means that the Fourier transform of a perfect matching filter is a spectral ratio. It follows that a least-squares matching filter, which minimizes the L_2 norm, has a Fourier transform which is an approximate matching filter.

The least-squares matching filter is preferred because it is stable in the presence of noise while a spectral ratio computed directly is not.

In the following examples, we will be examining the effects of the matching filter length and the noise on the performance of the filter.

EXAMPLES

Figure 2 shows a reflectivity series and two traces generated from this reflectivity but using two different wavelets. To match trace #1 to trace #2, we examine three least-squares matching filters with different operator lengths. In the first example, we use an operator length of $0.4s$, less than the length of the input trace. Figure 3 shows the result of this matching filter with an RMS error of 0.00014.

In example two, we use an operator length that is the same length as the input trace (operator length = $1.0s$). Figure 4 shows the result of the least-squares matching filter with a slightly smaller RMS error of 0.00012.

The last example in Figure 5, the operator length is longer than the length of the input trace. The RMS error is the smallest compared to the previous two examples. The error is 0.00011.

Examining the previous examples closely, we notice that both traces are highly correlatable. The choice of the matching filter length did not affect the performance of the filter. Although we observe that increasing the filter length corresponds to decreasing RMS error, but the decrease is not very significant. These examples show that an optimum filter length needs to be tested by plotting filter lengths vs residuals norm (L-curve method). Figure 6 shows that $0.4s$, the smallest window, is close to the optimum filter length.

In addition to the filter length tests, we examine two traces that are generated from the same reflectivity but the two wavelets from which they are generated from differ significantly (Figure 7. Figure 8 show the result of a $0.4s$ length matching filter. The residual error is ten times the error shown in the previous examples. This example demonstrates that high correlation between the matched traces is important and results in minimum error, whereas less correlatable traces result in larger residual.

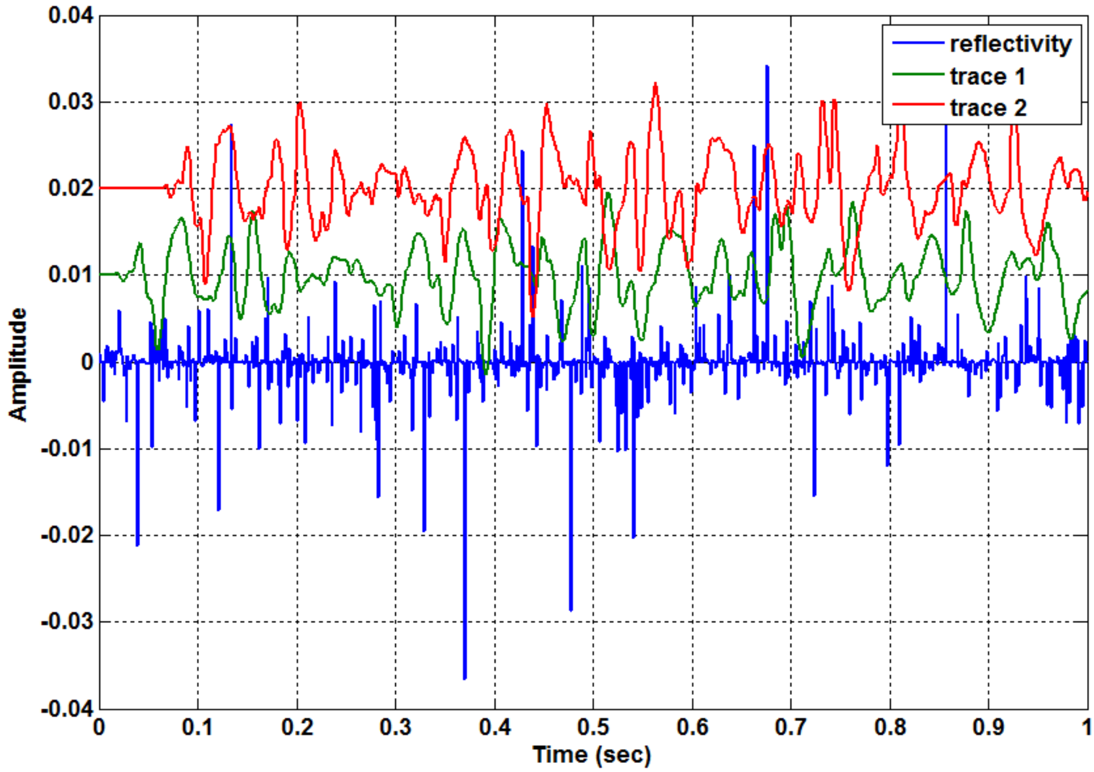

FIG. 2: An example of random reflectivity and two traces generated from the same reflectivity but two different wavelets.

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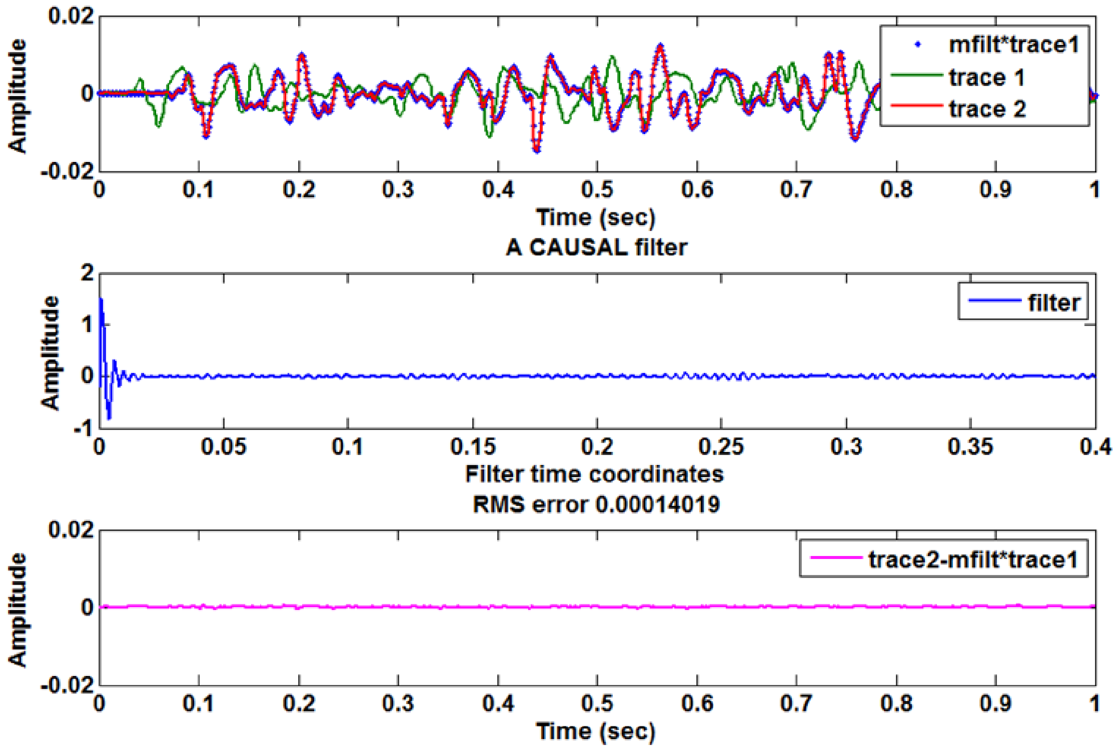

FIG. 3: A matching / shaping filter designed to match trace #1 to trace #2 with an operator length of $0.4s$ that is less than the length of the input trace #1. Top panel shows the first trace (green), second trace (red) and the matched first trace (dotted blue). The middle panel is the matching filter and the bottom panel shows the difference between the second trace and the matched first trace.

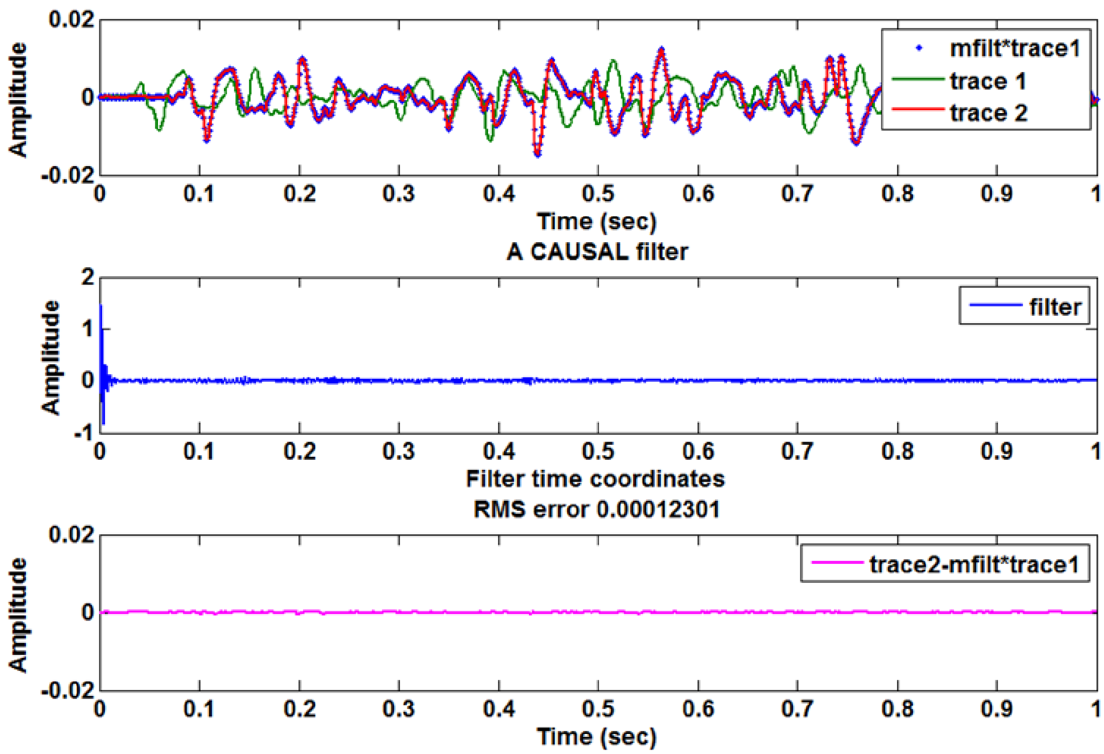

FIG. 4: A matching / shaping filter designed to match trace #1 to trace #2 with an operator length of $1.0s$ that is the same length as the input trace #1. Top panel shows the first trace (green), second trace (red) and the matched first trace (dotted blue). The middle panel is the matching filter and the bottom panel shows the difference between the second trace and the matched first trace.

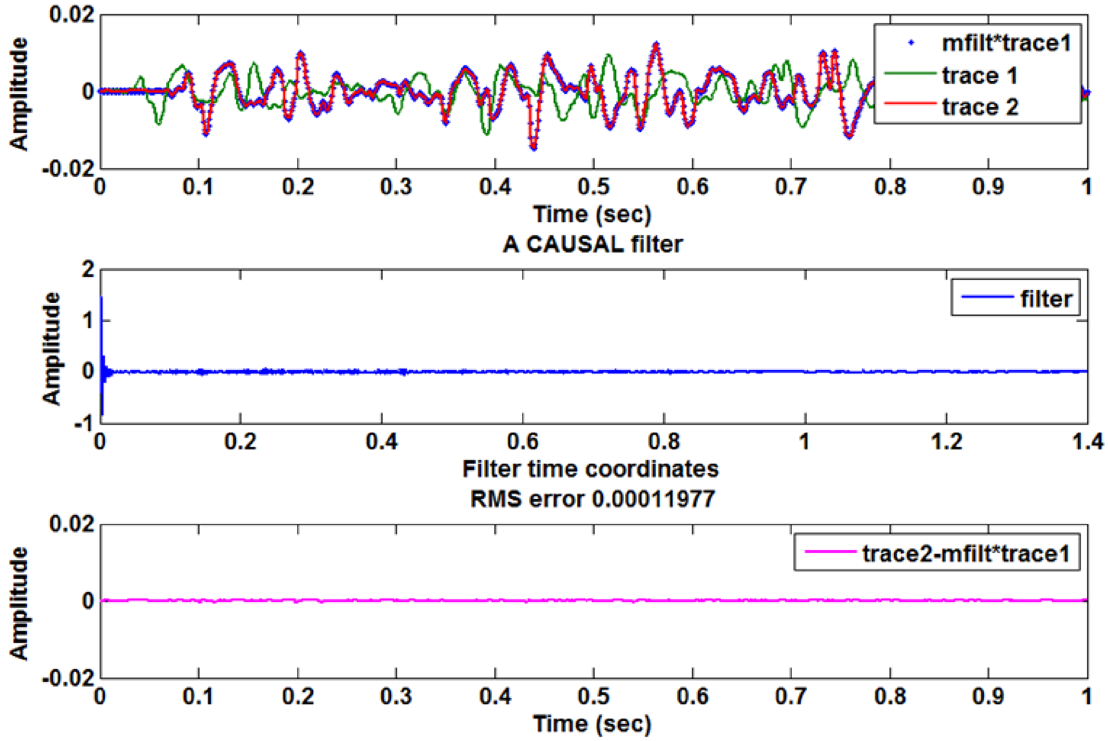

FIG. 5: A matching / shaping filter designed to match trace #1 to trace #2 with an operator length of $1.4s$ that is larger than the length of the input trace #1. Top panel shows the first trace (green), second trace (red) and the matched first trace (dotted blue). The middle panel is the matching filter and the bottom panel shows the difference between the second trace and the matched first trace.

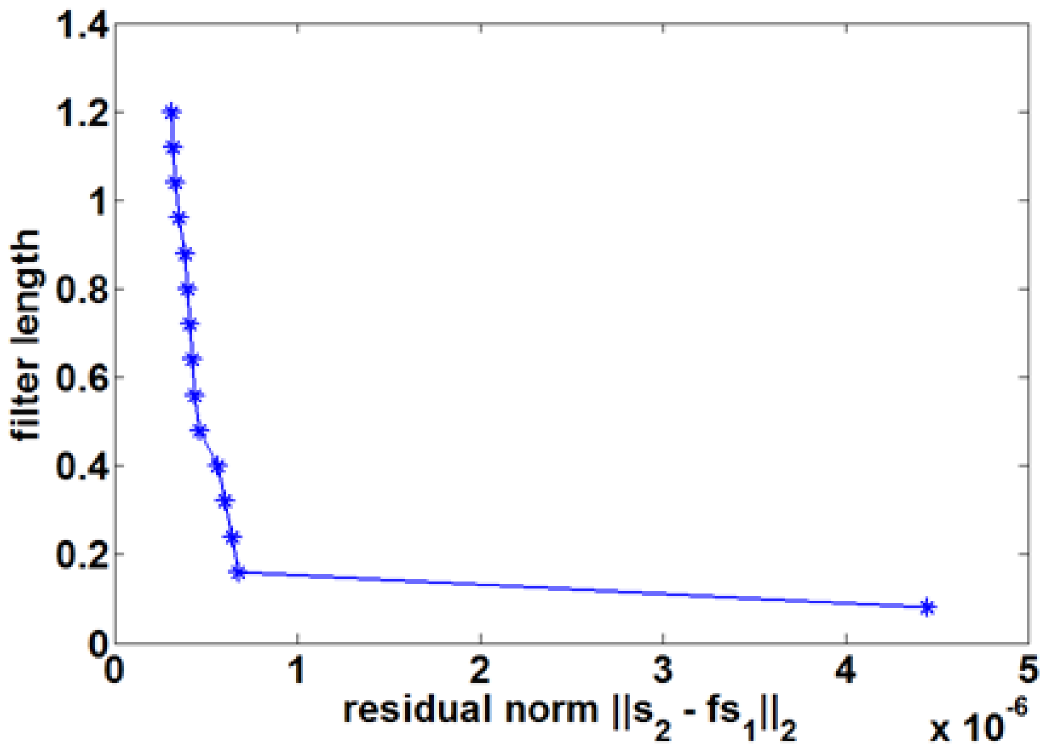

FIG. 6: L curve of the filter length vs L_2 norm.

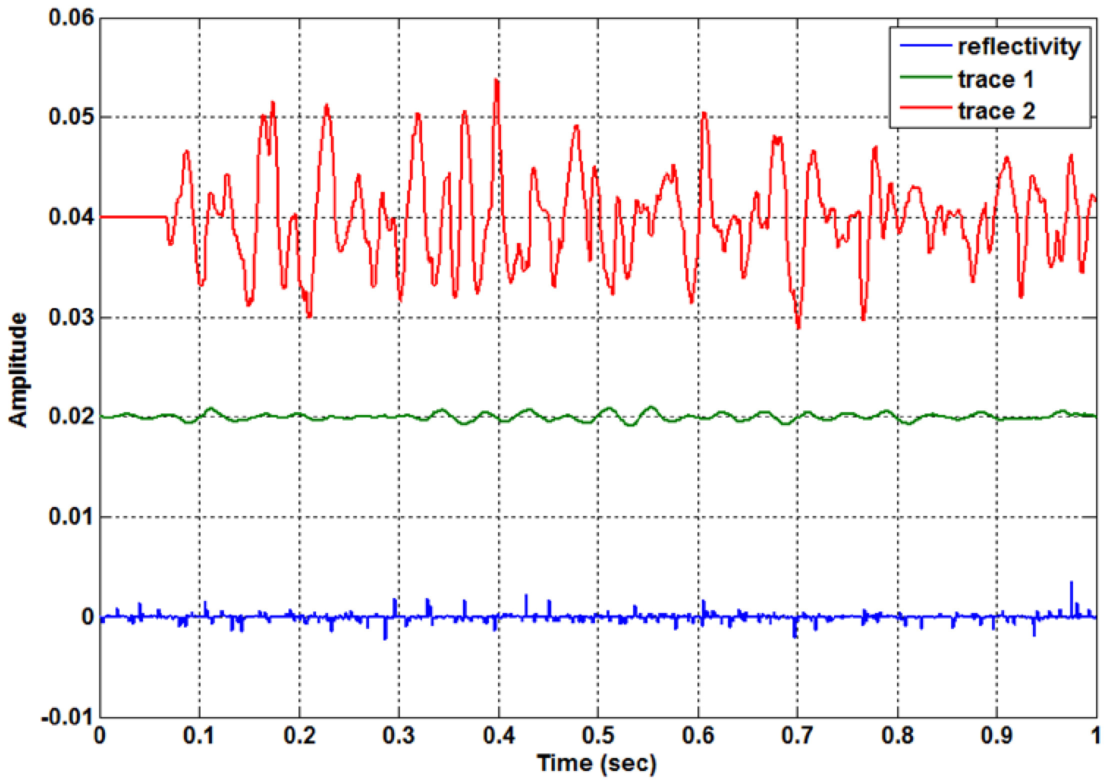

FIG. 7: An example of random reflectivity and two traces generated from the same reflectivity but two different wavelets.

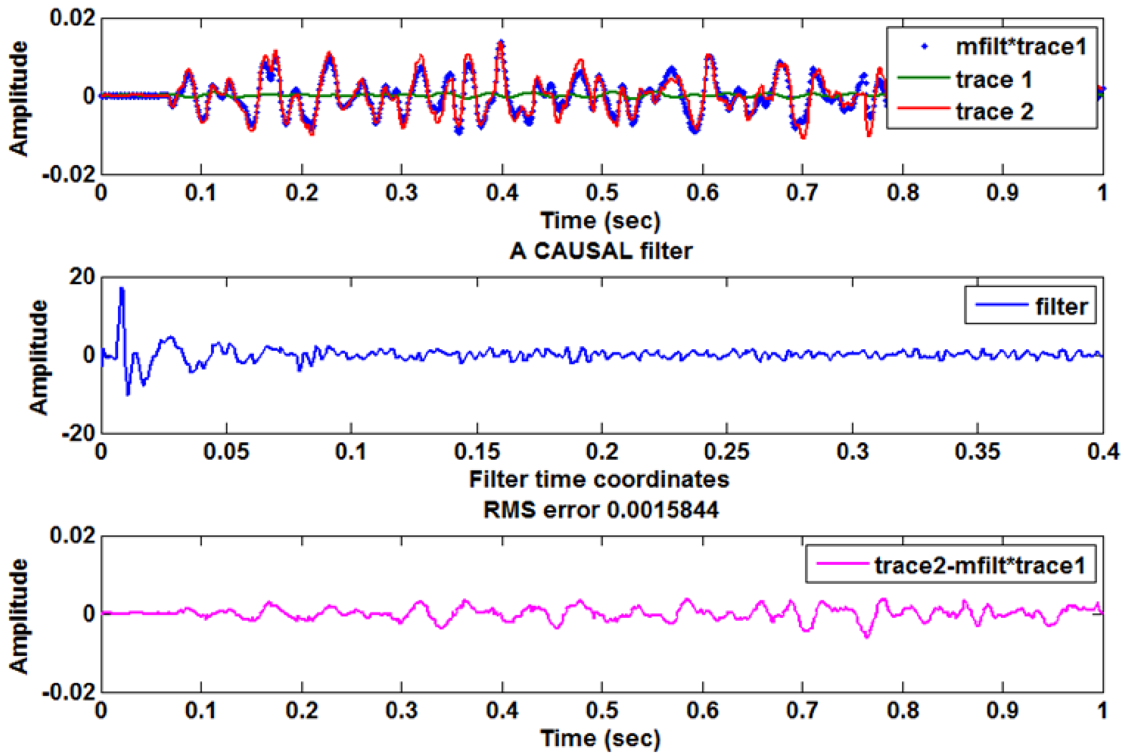

FIG. 8: A matching / shaping filter designed to match trace #1 to trace #2 with an operator length of $0.4s$ that is less than the length of the input trace #1. Top panel shows the first trace (green), second trace (red) and the matched first trace (dotted blue). The middle panel is the matching filter and the bottom panel shows the difference between the second trace and the matched first trace.

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