On the extraction of angle dependent wavelets from synthetic shear wave sonic logs

David Cho, Craig Coulombe and Gary F. Margrave

ABSTRACT

The extraction of angle dependent wavelets requires the use of a shear wave sonic log. However, shear wave measurements are often not acquired in a conventional logging suite and must be estimated to produce a synthetic result. The errors associated with the synthetic shear propagate through to the angle dependent reflectivity with a sine squared dependence of the incidence angle. Therefore, the reflectivity becomes unreliable at larger angles and a least squares extraction using the convolutional model could yield erroneous results. To reduce the errors associated with wavelet extraction at larger angles, a near angle wavelet was estimated with an acceptable amount of error using a least squares approach. Subsequently, an estimate of the angle dependent wavelet amplitude spectrum and a constant Q attenuation model was used to evolve the amplitude and phase respectively to estimate the wavelets at larger angles.

INTRODUCTION

In an amplitude variation with offset (AVO) inversion, the seismic wavelet is used in the forward modeling process where it is convolved with a model reflectivity to generate the synthetic data. Due to the attenuation effects of the Earth, the wavelet is non-stationary and alterations in the amplitude and phase occur throughout its propagation history. Therefore, in an AVO inversion where multiple angle dependent measurements of the reflection response are used, separate wavelets for each angle can be used to compensate for the attenuation effects as a result of increased propagation distance with larger angles.

The extraction of the wavelet is typically performed using the concept of the convolutional model, which states that the seismogram is the result of a convolution between a reflectivity series and a wavelet. Using this method, a reflectivity series must be generated and is typically performed using well log measurements and an appropriate formulation of the angle dependent reflectivity. The required well logs then consist of the compressional and shear wave velocity and the density. The compressional wave velocity and density logs are typically included in a standard logging suite. However, the shear wave velocity is often not acquired. In such cases, statistical methods can be used for wavelet estimation but will lack the associated phase information. An alternative option is to generate a synthetic shear wave sonic log using an appropriate method. However, finite errors will exist in the synthetic shear and will subsequently propagate through the wavelet extraction process, degrading the final result. This study investigates the errors associated with wavelet extraction using a synthetic shear wave sonic log and proposes the use of a constant Q model approach as suggested by Cho et al. (2010) for reducing the errors associated with the final extracted wavelets.

SYNTHETIC SHEAR GENERATION

Numerous methodologies exist for estimating the shear wave velocities including the use of rock physics models or making simple assumptions such as a constant compressional to shear wave velocity ratio. Here we implement a simple method using a linear transform between the compressional and shear wave velocities derived from nearby wells with measured shear wave information. The log data used to derive the transform relationships are first separated into distinct lithologies using various well log measurements as illustrated in Figure 1.

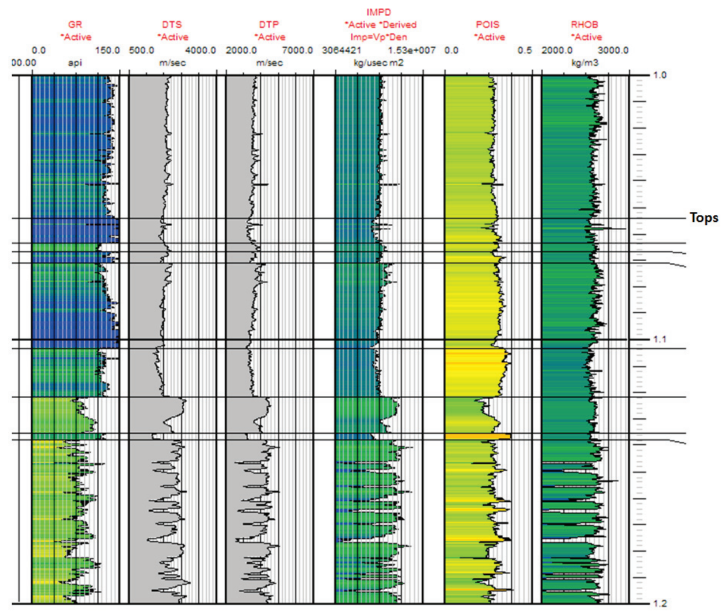

FIG. 1. Defined lithologies using a suite of well log measurements.

Subsequently, a linear regression is performed for each defined lithology using a scatter plot of the compressional and shear wave velocity values as illustrated in Figure 2. These relationships are then applied to the compressional wave velocities for the estimation of the synthetic shear.

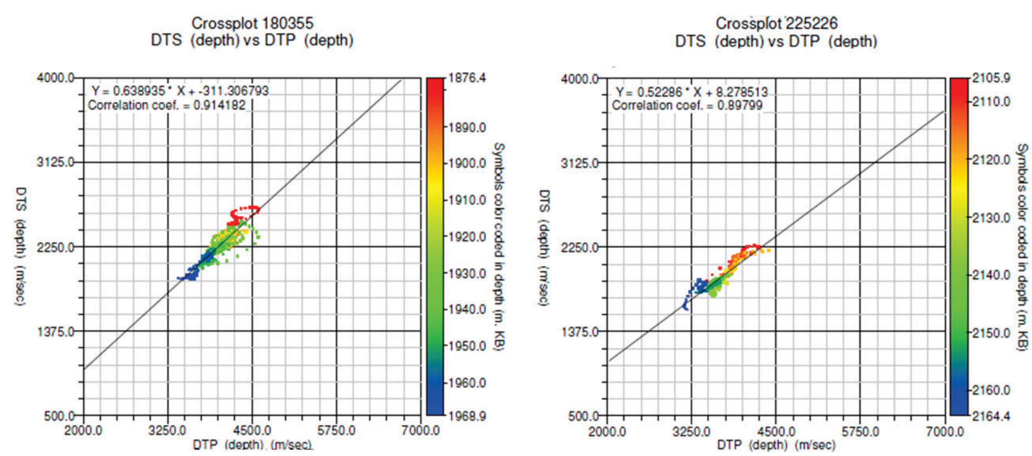

FIG. 2. Regression analysis for compression and shear wave velocities.

The regression line derived forms a sort of mudrock line for each defined lithology. A measure of the associated errors can be obtained by the scatter of points around the mudrock line and will have a value that is on the order of the standard deviation for the associated distribution. Typically, shale formations exhibit a tighter distribution and therefore, are more ideal for the analysis.

WAVELET ESTIMATION ERRORS

For the extraction of the angle dependent wavelets using well logs, we use the convolutional model, which is given by

$$\underline{s(\theta)} = \underline{\underline{R(\theta)}}\,\underline{w(\theta)} \tag{1}$$

where s is the seismogram, w is the wavelet, R is the reflectivity convolutional operator and the single and double underline represents a vector and matrix respectively. In addition, each quantity is dependent on the incidence angle, θ. The resulting least squares solution for the wavelet is given by

$$\underline{w(\theta)} = \left[\underline{\underline{R(\theta)}}^T \underline{\underline{R(\theta)}}\right]^{-1} \underline{\underline{R(\theta)}}^T \underline{s(\theta)}\,. \tag{2}$$

To solve equation 2, we must calculate the reflectivity from the well logs and construct the corresponding operators. Consider a simple two term AVO equation given by

$$r(\theta) = A + B\sin^2\theta\,, \tag{3}$$

where r is the reflectivity and A and B represent the compressional and shear wave velocity terms respectively. Differentiation of equation 3 with respect to the shear wave velocity term yields

$$dr(\theta) = \sin^2\theta dB\,. \tag{4}$$

Therefore, given an error in the shear wave velocity, the corresponding error in the reflectivity as a function of θ has a sine squared dependence and is illustrated in Figure 3.

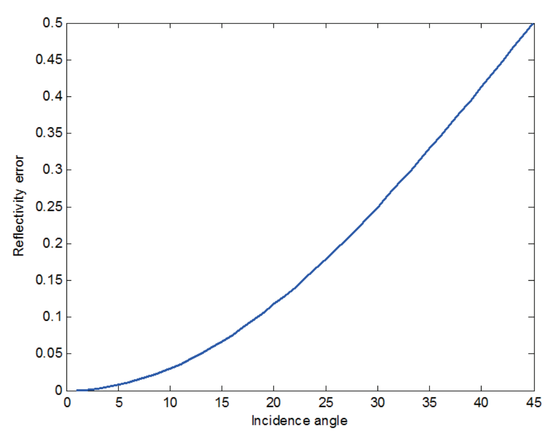

FIG. 3. Reflectivity error as a function of incidence angle.

The error in reflectivity is low at near angles and increases for far angles. Therefore, the corresponding angle dependent wavelets extracted using equation 2 will be increasingly unreliable with increasing angle. Using the above extraction method, the only wavelets that can be relied upon are the near angle wavelets that contain a relatively small amount of error.

WAVELET ESTIMATION USING CONSTANT *Q*

Consider the smoothed spectrum of four angle stacks used in an AVO inversion as shown in Figure 4. By assuming that the reflectivity series is random in nature with a white spectral signature, we can attribute all spectral character to the seismic wavelet. Figure 4 then represents an approximation for the amplitude spectrum of the wavelets.

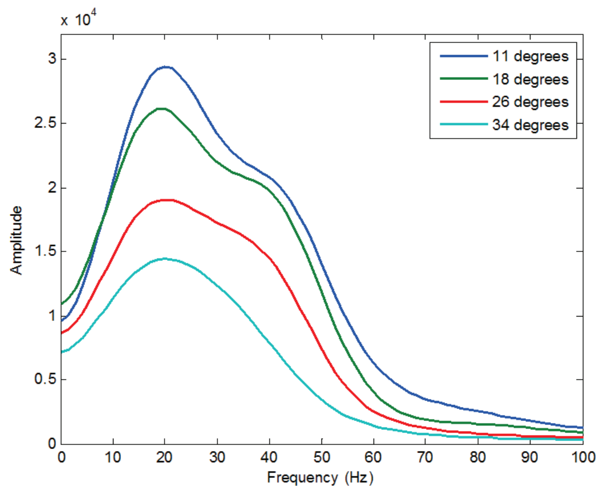

FIG. 4. Smoothed amplitude spectrum of four angle stacks used in an AVO inversion.

Note that as the incidence angle and hence the propagation distance increases, the higher frequencies are lost. This is characteristic of constant Q attenuation and therefore we can implement the associated properties to assist in the wavelet extraction process. Since the amplitude spectrum for each wavelet is known, we simply need the associated phase spectrum, which can be derived using the fact that constant Q attenuation is necessarily coupled with minimum phase dispersion (Futterman, 1962).

Given that we can estimate the near angle wavelet of 11 degrees (we call this the pilot wavelet) using equation 2 with an acceptable amount of error, it can be represented by

$$w_p(t) = F^{-1}\left[A_p(\omega)\exp\left(i\phi_p(\omega)\right)\right], \tag{5}$$

where ω is the frequency, A_p and ϕ_p are the amplitude and phase spectrum respectively and F^{-1} is the inverse Fourier transform. The phase can subsequently be written as the combination of an unknown source signature phase term, $\phi_p^{(ss)}$ and a minimum phase filter, $\phi_p^{(mp)}$ (Cho et al., 2010), which is represented by

$$\phi_p(\omega) = \phi_p^{(ss)}(\omega)\phi_p^{(mp)}(\omega) = \phi_p^{(ss)}(\omega)H\left[\ln\left(A_p(\omega)\right)\right], \tag{6}$$

where H represents the Hilbert transform. Assuming that the only attenuation mechanism is associated with constant Q, each separate wavelet can be written in a similar fashion. Subsequently, a filter can be derived to evolve the phase and is given by

$$\gamma_i = \frac{\exp\left(i\phi_i^{(mp)}(\omega)\right)}{\exp\left(i\phi_p^{(mp)}(\omega)\right)} \tag{7}$$

where γ_i represents the filter between the pilot wavelet and the i^{th} subsequent wavelet. The resulting angle dependent wavelet is given by

$$w_i(t) = F^{-1}\left[A_i(\omega)\exp\left(i\phi_p(\omega) + i\phi_i^{(mp)}(\omega) - i\phi_p^{(mp)}(\omega)\right)\right]. \tag{8}$$

Figure 5 shows the resulting angle dependent wavelets derived from equation 8 where the amplitudes have been normalized.

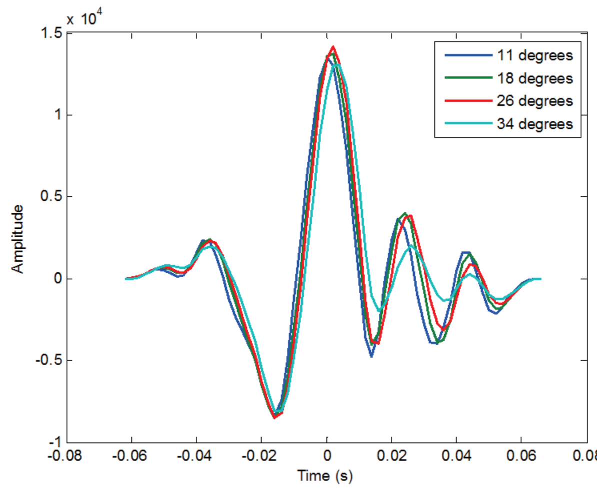

FIG. 5. Normalized angle dependent wavelets derived from equation 8.

CONCLUSIONS

The errors associated with a least squares extraction of the seismic wavelet using synthetic shear wave velocities grow with an increasing incidence angle. Given that a near angle wavelet can be extracted with an acceptable amount of error, a methodology for estimating larger angle wavelets was proposed. Using an estimate of the angle dependent wavelet amplitude spectrum and a constant Q attenuation model, we can evolve the amplitude and phase spectrum respectively to avoid the errors associated with the reflectivity calculations using synthetic shear wave velocities.

REFERENCES

Cho, D., Close, D., Horn, F., 2010, Marine Source Signature Estimation for AVO Inversion with no Well Control Using a Constant Q Attenuation Model: 80th Annual International Meeting, SEG, Expanded Abstracts, 473-477.

Futterman, W. I., 1962, Dispersive body waves: Journal of Geophysical Research, 67, no. 13, 5279–5291.