Reflection and transmission coefficients at the interface between two transversely isotropic media (Matlab software)

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ABSTRACT

Derivations and related closed form expressions for reflection and transmission coefficients at the interface between two transversely isotropic (TI) media have been available in the literature for several decades. However, the progression from a set of equations to software that produces accurate results is often less obvious than might be expected. This is true for this problem as there are a number of related or ancillary inconveniences related to the original problem that must be addressed. These require either extensive algebraic manipulations or the development of numerical algorithms that deal with the additional points. These are supplemented by typographic errors which may be present in the original works as well as having to acquire an appreciation of at least the basic concepts of wave propagation in an anisotropic medium. This report discusses the use of software for the numerical computation and graphical display of the reflection and transmission coefficients for the media type indicated above using the Matlab programming language. The procedure is initially presented at a fairly basic level for the computation and graphical presentation of the generally complex valued coefficients (amplitude and phase versus incident phase angle). Other options are available to display the coefficients versus the reflected or transmitted phase angle, or the horizontal component of the slowness vector or the incident and reflected or transmitted group angles.

INTRODUCTION

The papers often cited when the topic of reflection and transmission $(R \ \& \ T)$ coefficients at the interface between two transversely isotropic (TI) media is discussed are Daley and Hron (1977) and (1979) and Graebner (1992). The first of these papers presents analytic formulae for the $R \ \& \ T$ coefficients in a form that were designed for a specific application. Once this is understood, the input parameters may be chosen such that proper results may be obtained. Graebner (1992) approached the same problem using the horizontal component of the slowness (p) as the dependent variable. However, rather than solve for individual coefficients, the problem was left in matrix notation. The individual $R \ \& \ T$ coefficients could be obtained from this using numerical methods. Analytical expressions for the $R \ \& \ T$ coefficients were derived by this author using Graebner's formulation. Although it would have possibly been more efficient to accomplish this using Mathematica® or Maple®, Cramer's rule was used and the 16 $R \ \& \ T$ coefficients at an interface between two TI media where obtained employing an archaic method. Again this was done so that the expressions for the $R \ \& \ T$ coefficients were in a specific form. More efficient formulae could have been produced.

It should be mentioned that as in many cases in the literature, all three papers cited above have typographic errors. The liquid/solid case in Graebner's paper has a few problems. In this case, the solid/solid formulae presented here may be used to obtain $R \ \&$

T coefficients for the liquid/solid case. This is done by setting the shear wave velocity in the liquid to "numerical zero". A safe estimate of this quantity is $(\varepsilon_M)^{1/3}$, where ε_M is "machine epsilon" or the minimum floating point value a number may be assigned on a computing device.

Included in this package are:

1. A very simplistic Matlab main calling script ***script_coef***. This routine requires that the user interact to a great extent by editing the file to change the anisotropic parameters and the particular reflection (*R*) or transmission (*T*) coefficient being considered. More user friendly front ends exist and can be supplied. However, this method was chosen to require the user to develop some understanding of the package.

2. A set of functions which consist of the 16 possible *R & T* coefficients at solid – solid interface between 2 transversely isotropic media. A number of functions required for the computation of specific quantities or intermediate results are part of this package.

3. Some related utility functions about which more will be said later are included.

PROGRAM INPUT

At present, for the simple main script, all input is hardwired in the program and it is up to the user to modify ***script_coef*** for a specific coefficient and anisotropic parameters. Required input is:

1. The anisotropic coefficients in the upper and lower layers are required to be specified. These are A_{11}, A_{33}, A_{55}, A_{13}, ρ_1. The A_{ij} are not compliances C_{ij} but rather density normalized compliances $A_{ij} = C_{ij}/\rho$, so that the A_{ij} have the dimensions of velocity squared. ρ is density. The transformation from the A_{ij} to the C_{ij} required in the computations is done within the program.

```
a11(1) = 14.8246;     % Highly anisotropic upper medium:
a33(1) = 12.2080;
a55(1) = 1.3059;
a13(1) = 6.7032;
 rr(1) = 2.0;
 ad(1) = (a13+a44)^2 - (a11-a44)*(a33-a44);
%
a11(2) = 25.0;    % Lower medium isotropic in this case:
a33(2) = 25.0;
a55(2) = 7.4;
 ad(2) = 0.0;
a13(2) = sqrt((b11-b44)*(b33-b44)) - b44;
 rr(2) = 2.2;
%
```

Another parameter that is required at some times is defined by the formula $A_D = \left(A_{13} + A_{55} \right)^2 - \left(A_{11} - A_{55} \right)\left(A_{33} - A_{55} \right)$. In an isotropic medium $A_D \equiv 0$ which may be required to be used to determine A_{13} in the medium.

2. Type of *R* or *T* coefficient, from the list of 16 possible: *p1p1, p1s1, p1p2, p1s2, s1p1, s1s1, s1p2, s1s2, p2p1, p2s1, p2p2, p2s2, s1p1, s1s1, s1p2, s1s2*. The coefficient designation is of the form: $\left[\ xiyj, (x, y = p \text{ or } s)(i, j = 1 \text{ or } 2) \right]$ Here, *x* is the incident wave type, *i* the medium of incidence $\left[\ i, j = 1 \rightarrow (\text{upper}) \ \right]$, $\left[\ i, j = 2 \rightarrow (\text{lower}) \ \right]$, *y* reflected or transmitted wave type, *j* the medium of reflection or transmission.

```
ponv = pi/180.0;
%
for ii = 1 : 91          % Loop Over Incident Angle:
bng_mod = ii-1;          % Phase Incident Angle:
ang_mod(ii) = bng_mod/ponv;
pr = sin(bng_mod/ponv);
xn = complex(pr,0.0);
vv = velnp(xn,a11,a33,a44,ad);    % Incident Normal
                                  % Velocity:
pr1 = pr/real(vv);       % Real component of
                         % incident horizontal
                         % slowness vector:
[amp,phase] = p1s1( a11(1),a33(1),a55(1),a13(1),...
        rr(1),a11(2),a33(2),a55(2),a13(2),rr(2),pr1 );
ampr(ii) = amp;
fazr(ii) = phase;
ang_mod(ii) = bng_mod;
end
```

PROGRAM OUTPUT

Only one coefficient may be computed per run in the present form of the ***script_coef***. The user specifies coefficient type and anisotropic parameters in the two media. Files are opened to which the incident phase angle is written (**aang**), the amplitude of the coefficient (**aamp**) and the phase of the generally complex coefficient (**afaz**).

Additionally, the utility functions **anglep(*)** or **angles(*)** compute the ray (group) angle of the incident or reflected/transmitted wave type – file (**rang**). This angle is given by either:

```
ponv = pi/180.0;
%
phase_angle = sang;          % In degrees:
rang_s = angles(a11,a33,a44,a13,sang*ponv)/ponv;
```

% or

```
phase_angle = pang;          % In degrees:
rang_p = anglep(a11,a33,a44,a13,pang*ponv)*ponv;
```

Output arrays are defined to be used as input to MATLAB graphical displays.

SAMPLE INPUT AND RESULTS

A model consisting of a highly anisotropic layer overlying an isotropic medium will be used as a test case. The coefficients *p1p1, p1s1* and *s1s1* will be considered. The anisotropic parameters are given in Table 1. The A_{ij} have the dimensions of km^2/s^2 and the densities (ρ), gm/cm^3 .

The *p1p1* and *p1s1* reflection coefficients are shown in Figures (1) and (2) for the model described in Table1. In both cases the amplitudes and phases of the generally complex reflection coefficients are plotted against both phase and group angles of incidence. The group angles are obtained from the phase angles using the utility functions **anglep(*)** or **angles(*)**.

In Figure (3) the *p1s1* reflection coefficient (amplitude and phase) is plotted versus the reflected (*s1*) reflected group angle.

Layer	a_{11}	a_{33}	a_{55}	a_{13}	ρ
1	*14.8246*	*12.2080*	*1.3059*	*6.7032*	*2.0*
2	*25.0*	*25.0*	*7.4*	*10.2*	*2.3*

Table 1. Anisotropic parameters of test model. The a_{ij} have the dimensions of km^2/s^2 and ρ - gm/cm^3 See text for details.

The *s1s1* reflection coefficient is considered for the same model used in the previous cases. The reflected amplitude and phase is plotted against phase angle in Figure (4) and group angle of incidence in Figure (5).

CONCLUSIONS

An overview of the translation of Fortran code to a Matlab software package is discussed. The problem considered is reflection and transmission coefficients at the interface between two transversely isotropic media. Although the Fortran version of this software was written circa 2000 and used on a regular basis since that time, the Matlab code should be considered as a preliminary release. Any potential users having problems with this release are asked to contact the author at pdaley@ucalgary.ca.

ACKNOWLEDGEMENTS

REFERENCES

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Graebner, M., 1992, Plane-wave reflection and transmission coefficients for a transversely isotropic solid, Geophysics, 57, 1512-1519.
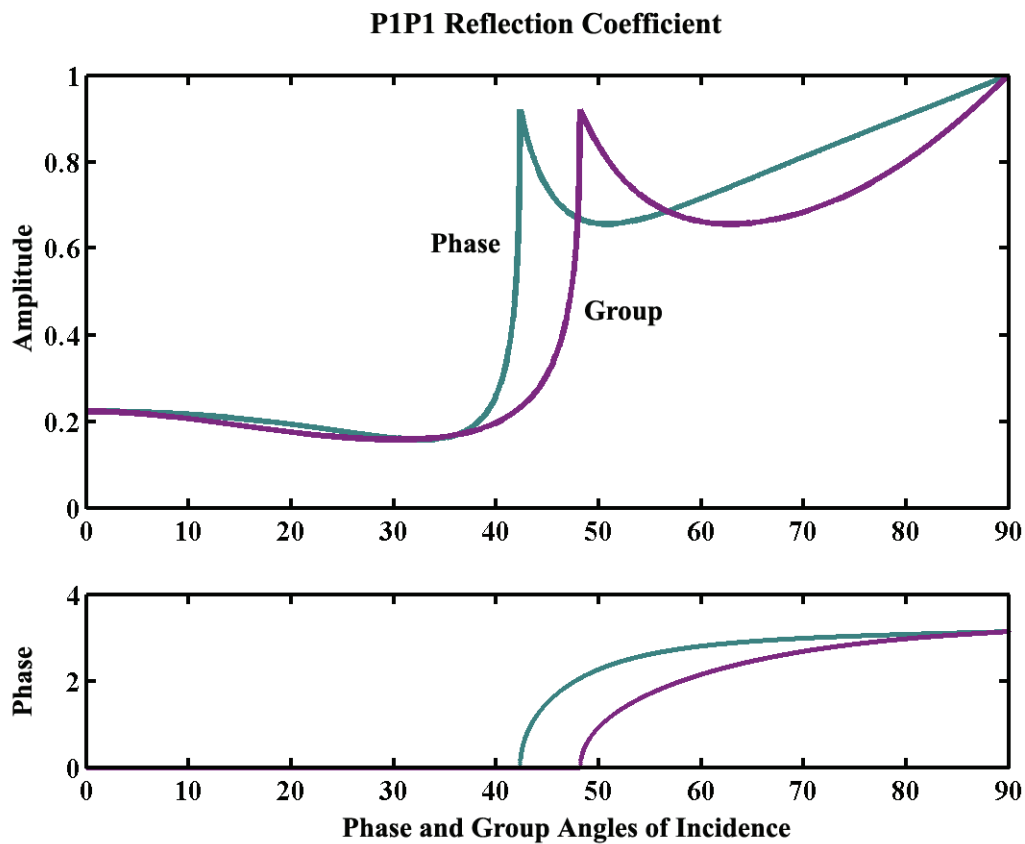
Fig. 1. *p1p1* reflection coefficient – amplitude and phase versus phase and group angles of *p* incidence.
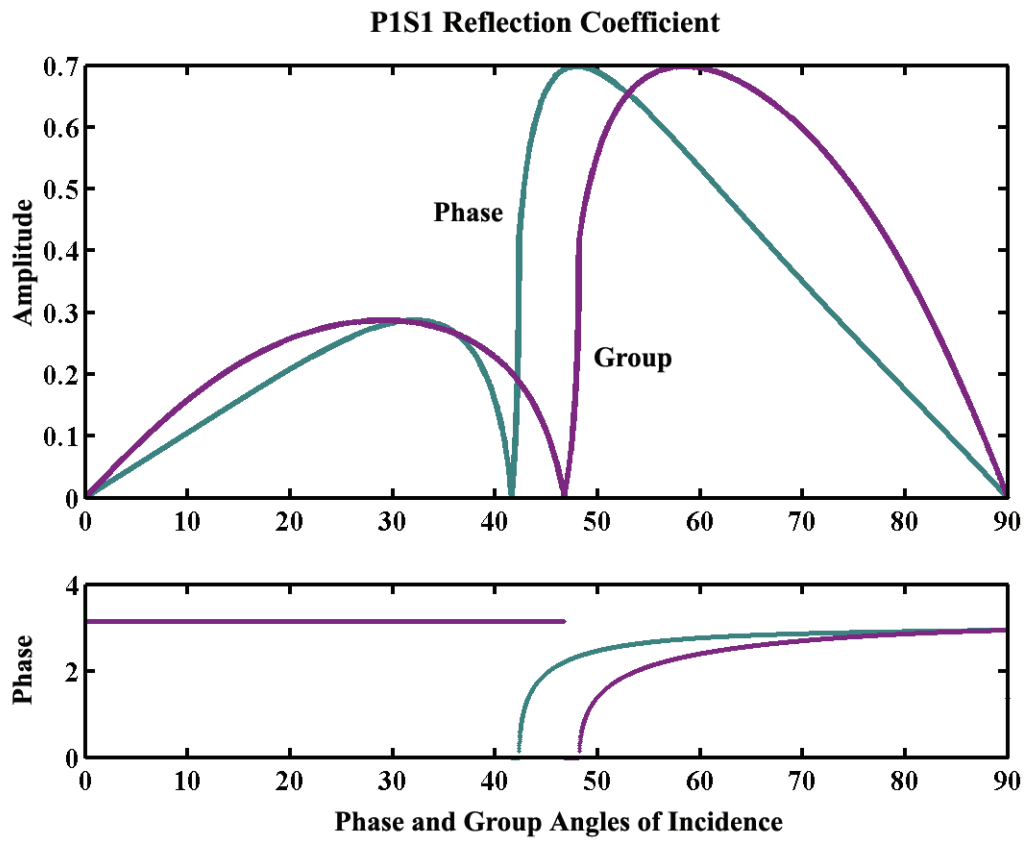
P1S1 Reflection Coefficient

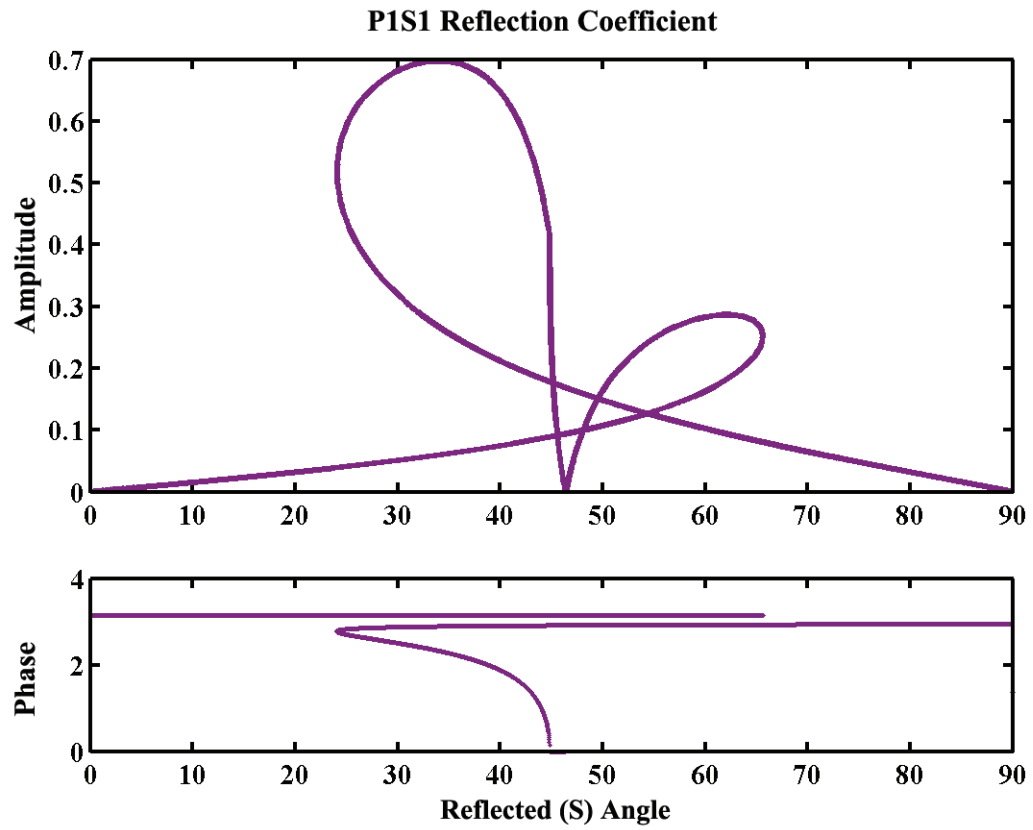Fig. 2. *p1s1* reflection coefficient – amplitude and phase versus phase and group angles of *p* incidence.

Fig. 3. *p1s1* reflection coefficient – amplitude and phase versus the reflected *(s)* group angle.
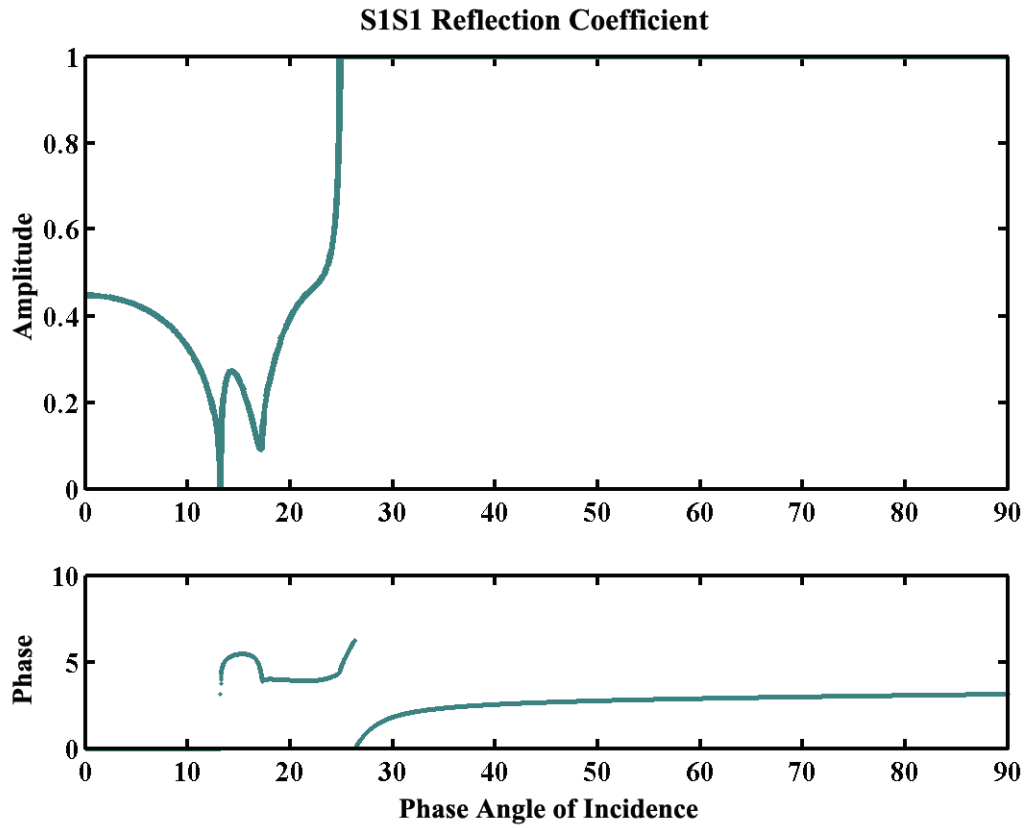
S1S1 Reflection Coefficient

Fig. 4. *s1s1* reflection coefficient – amplitude and phase versus phase angle of *s* incidence.
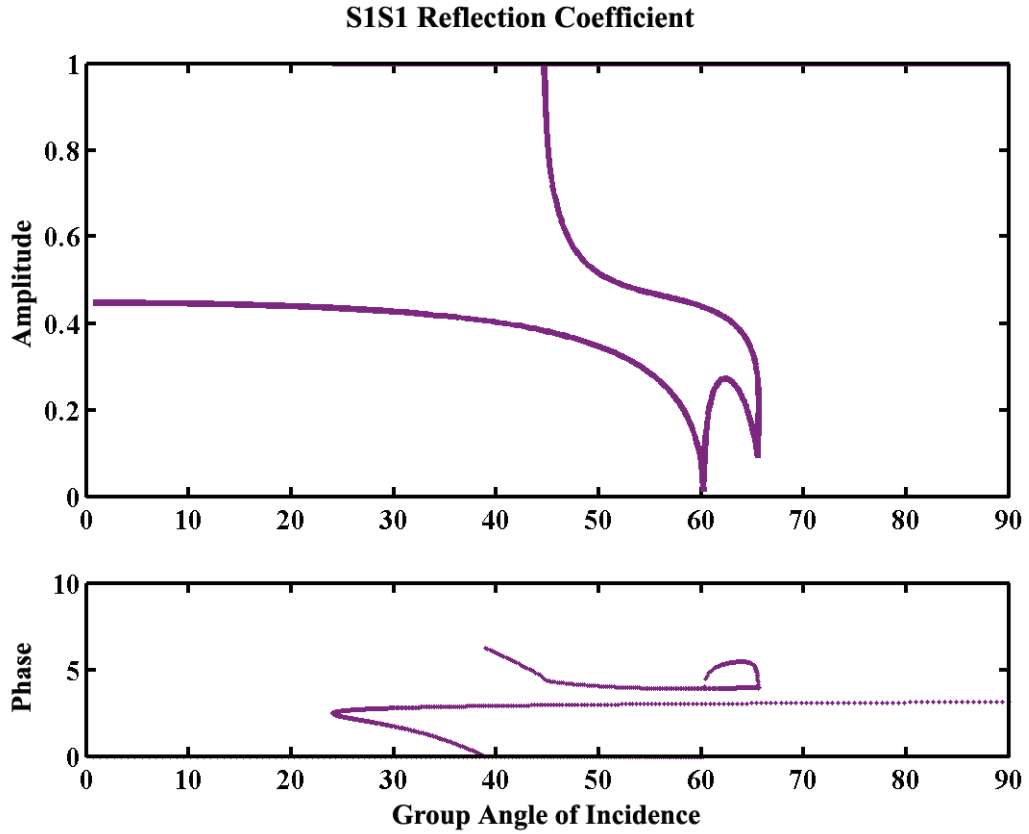
S1S1 Reflection Coefficient

Fig. 5. *s1s1* reflection coefficient – amplitude and phase versus group angle of *s* incidence.