Elimination of seismic multiples by anisotropic, pre stack depth-migration and filtering

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ABSTRACT

Group and phase velocities are derived such that, when used in constant velocity migration, a set of seismic multiples from the sea surface are focused such that they separate from reflection primaries. The velocities are found to be homogeneous but anisotropic. The focused multiples are then deleted from the data and surrounding data are used to fill in the deleted areas. Un-migration restores the data to it's input stat sans the multiples. A before and after comparison using synthetic data verifies that this is a very clean process in that multiples are removed without loss of primaries.

INTRODUCTION

The following is a derivation of a group velocity and a phase velocity. The purpose is, given only water velocity, to collapse all identifiable multiples in the section for removal by filtering in what is, effectively, a prestack depth migration transform domain. This approach is restricted to first order multiples. Data input to this scheme are common offset gathers or common source gathers, and migration proceeds using the group velocity with, for example, a ray-based algorithm like Kirchhoff migration (Schneider, 1978) for constant velocity. Based on the phase velocity, migration proceeds with, for example, Stolt migration (Stolt, 1978). A caveat here is the requirement is that whatever migration method is used, it must be reversible, and so I employ the method of Burnett and Ferguson (2011) to achieve this.

Development begins with a moveout equation scattering from a point source in a homogeneous medium. It is assumed that wavefields associated with the seafloor reflection and the corresponding surface multiple are well modelled as a superposition of Huygens point scatterers, and that a recorded diffraction is the fundamental element upon which to construct a wavefield. A recording array of hydrophones at a datum close to the sea surface is assumed throughout.

A second diffraction equation is derived for a second point scatterer. This one models scattering from the original point source through a multiple generated at the sea surface. Such a moveout curve is be identical to the primary curve with a bulk shift in time to account for the two-way travel time from the surface. Analytically, this second moveout equation is given here as a function of the difference in depth between the actual point source and a virtual point source deeper down.

This second moveout curve is equated to a third who's free variable is an effective velocity that causes the necessary bulk shift. The result is then solved for the effective velocity that is shown to be anisotropic.

Synthetic seismic data are then depth migrated with the effective velocity whereby the target first order multiples focus relative to the rest of the wavefield. The focused multiples are then simply erased, and the data are un-migrated. Subtraction of the original data from the filtered data isolates the removed multiples. The absence of reflection energy on this result suggests that this method is very effective.

THEORY

Beginning with the model of reflection and multiple reflection as a superposition of point sources, the moveout equation for constant velocity is

$$\Delta t_0\left(x\right) = \frac{z_0}{v_0}\sqrt{1+\left(\frac{x}{z_0}\right)^2}, \tag{1}$$

where Δt_0, z_0, and v_0 are hyperbolic moveout time, depth, and velocity respectively, x is the distance between a receiver and the normal to the recording surface that bisects the diffractor. Velocity v_0 is associated with a point $(x = 0, z_0)$ in the subsurface. A diffraction at a deeper coordinate is then used to model a surface multiple due to the primary source. The result is a shifted version of equation 1 where the shift represents the difference in depth $z - z_0$ between the two coordinates according to

$$\Delta t_z\left(x\right) = \Delta t_0\left(x\right) + \frac{z-z_0}{v_0}. \tag{2}$$

In terms of a new velocity $v_z \neq v_0$, equation 2 is written

$$\Delta t_z\left(x\right) = \frac{z}{v_z}\sqrt{1+\left(\frac{x}{z}\right)^2}, \tag{3}$$

where v_z causes the time shift due to depth difference. Equate equations 2 and 3 and solve for v_z to get

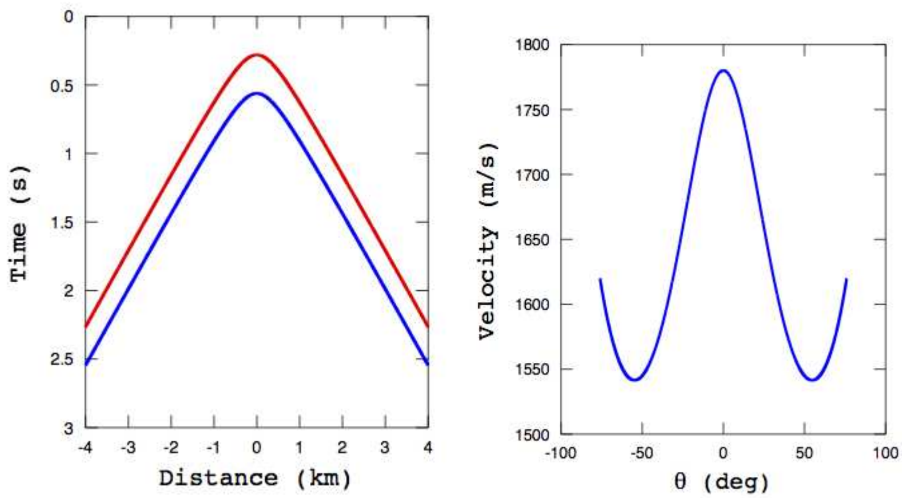
$$v_z\left(x, z, z_0\right) = z\, v_0\,\frac{\sqrt{1+\left(\frac{x}{z}\right)^2}}{z - z_0 + z_0\sqrt{1+\left(\frac{x}{z_0}\right)^2}}, \tag{4}$$

In terms of group angle, equation 4 is given by

$$v_z\left(\theta, z, z_0\right) = \frac{z\, v_0}{\cos\theta}\left[z - z_0 + z_0\sqrt{1+\left(\frac{z}{z_0}\tan\theta\right)^2}\right]^{-1} \tag{5}$$

where θ is the angle between a receiver at distance x from normal to the recording surface that bisects the diffractor, and $\tan\theta = x/z$.

Group velocity $v_z\left(\theta, z, z_0\right)$ is a function of constant velocity directly and through angle θ, as well as through z and z_0. Depth z is simply the depth coordinate of the migration output space and so it is known everywhere. Seafloor depth z_0 varies with distance x of course, but if the variation is not too strong, then v_0 is modulated by the user until a satisfactory migration result is obtained.



(a) Diffractions. (b) Anisotropic velocity.

FIG. 1. 1(a) The primary diffraction (red) is computed with equation 2, and the deeper diffraction (blue) is computed with equation 3 and velocity v_z from equation 5 (v_0 = 1780 m/s). 1(b) v_z for v_0 = 1780 m/s.

Figure 1(a) demonstrates the simple relationship between the primary diffraction and it's multiple. The primary is a simple hyperbola, and it's multiple is just a shifted version of the primary. Note that traveltimes for the multiple in Figure 1(a) are generated with equation 3 and the anisotropic velocity profile given in Figure 1(b).

For use with reversible migration operators like Stolt migration, group velocity is converted to phase velocity v_p according to

$$|\mathbf{v}_z|^2 = \left(\frac{\partial \omega}{\partial k_x}\right)^2 + \left(\frac{\partial \omega}{\partial k_z}\right)^2 \tag{6}$$

(Thomsen, 1986), where

$$\mathbf{v}_z = |\mathbf{v}_z| \ \hat{\mathbf{n}}, \tag{7}$$

is the vector form of the group velocity of equation 5, and $\hat{\mathbf{n}}$ is normal to the advancing wavefront as in Figure 2. Phase velocity v_p is related to wavenumbers k_x and k_z, and frequency ω through the dispersion relation

$$k_z^2 = \left(\frac{\omega}{v_p}\right)^2 - k_x^2 \tag{8}$$

(Thomsen, 1986). Thus, a fairly involved calculation is required to make the conversion, and the solution will be provided in a subsequent offering.

EXAMPLES

The anisotropic focusing velocity for first-order multiples is tested on a synthetic data set from the SMAART JV. An example of a common offset gather from this data volume is given in Figure 3. Here, a set of point sources are embedded just below the sea floor, and these register in the data as a set of diffractions just below the reflection from the sea floor.

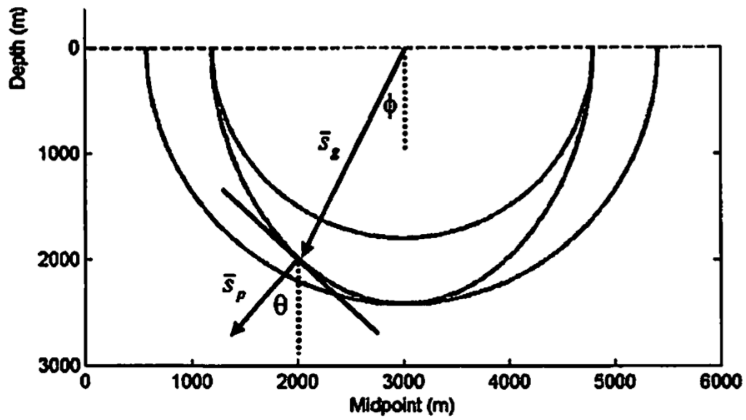


FIG. 2. Group slowness $\bar{s}_g$ and phase slowness $\bar{s}_p$ defined in depth and midpoint coordinates for a number of isochrons. The tangent indicated corresponds to an advancing wavefront in an anisotropic medium. Normal to the tangent $\hat{n}$ (equation]ref) indicates phase direction; the ray labelled $\bar{s}_g$ traces the direction to the tangent point from the origin (3000 m, 0 m) is the group direction. Angles ϕ and θ are group and phase angles respectively.

On the left-hand side of the image, the sea floor reflection arrives at 2.2 s, and the diffraction primaries arrive at 2.5 s. The water bottom multiple arrives at about 3 s on the left-hand side, and the multiples of the diffractors arrive 200 ms later. Based on a velocity slighter than water velocity, migration of the input data focuses the diffracted multiples (Figure 4). Though a depth migration code is employed, data are output in time for direct comparison to the input. The multiples are now distinct, compact, and they are separate from local primaries. The focused multiples are then erased as in Figure 5. Sinc interpolation is used to smooth the edges associated with setting to zero the focused multiples, and the data are un-migrated (Figure 6) (Burnett and Ferguson, 2011). Un-migration of the edited data returns a data volume that is free of the multiples of the point diffractors. The goodness of this process is evaluated by simple subtraction of the input data and the edited, un-migrated data as in Figure 7. No primary reflection energy is present in this difference plot - only the multiples and some Fourier wraparound error are revealed.

CONCLUSIONS

The group and phase velocities derived here can be used to focus a set of first-order seismic multiples so that they might more readily separate from nearby primary reflections and there be removed. Un-migration of the edited data returns a data volume that is free of these multiples. Conventional migration methods are used in this approach, however, they must be reversible to return a usable data set.

An example based on synthetic data with known multiples is provided as a demonstration. A constant velocity was chosen by trial and error until the multiples of interest focused. These were then set to zero, and surrounding data was used to infill the zeroed regions through sinc interpolation. The resulting data volume is multiples free, and this is verified through examination of an input / output difference plot. No primary energy has leaked through, and only Fourier wraparound energy is present alongside the multiples.

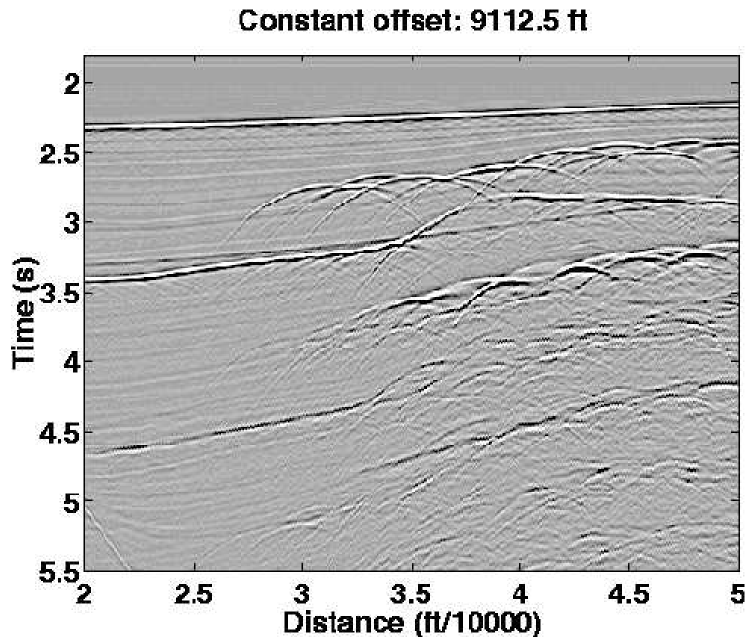


FIG. 3. A constant-offset (9112.5 ft) gather of marine seismic data. The first arriving reflection event (2.2 s on the left hand side) corresponds to the water bottom, and below the water bottom, a number of diffractors are present (2.5 s on the LHS). Multiples of these diffractions follow the water bottom multiple.

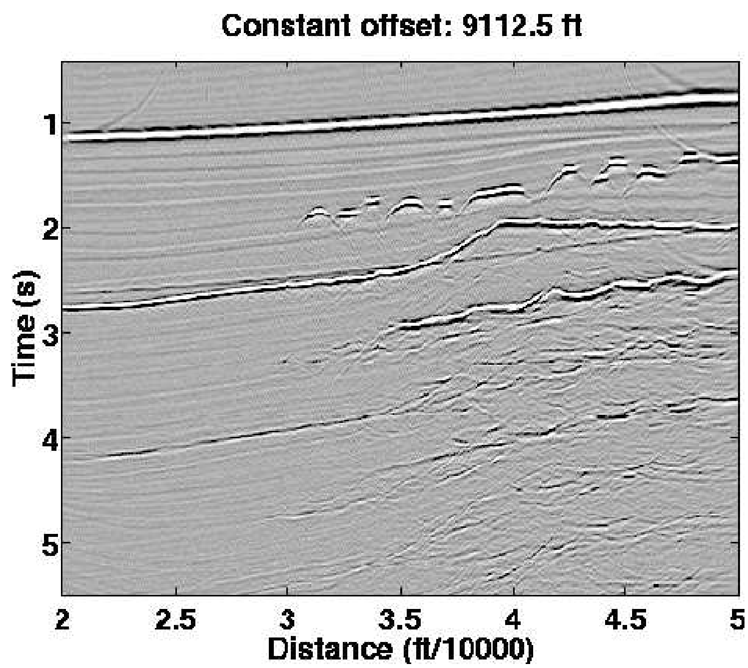


FIG. 4. Constant-offset migration of Figure 3 using the anisotropic velocity of Figure 1(b) such that the multiples are focused and distinct from surrounding reflections. Data are rendered in time for direct comparison to the input.

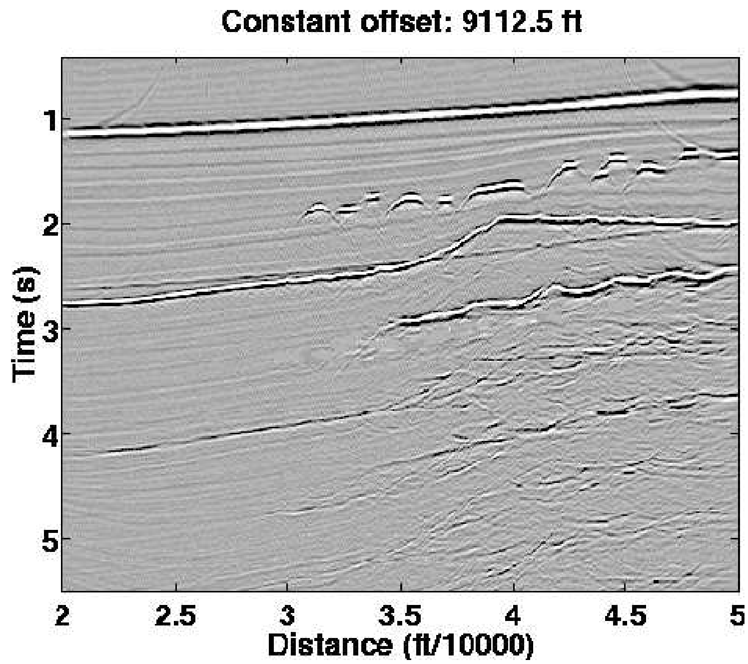


FIG. 5. The focused multiples are erased.

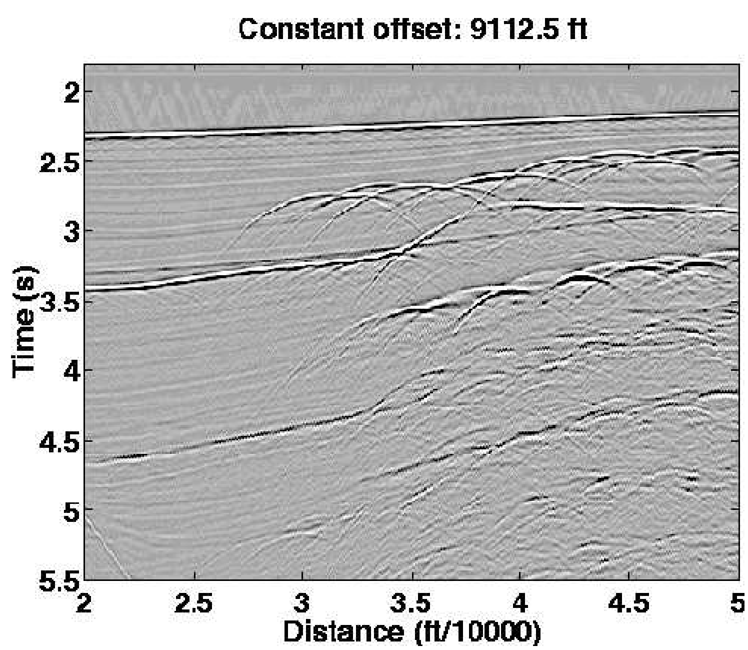


FIG. 6. The data of Figure 5 are un-migrated. Compare with Figure 3 to see that the multiples are now removed.

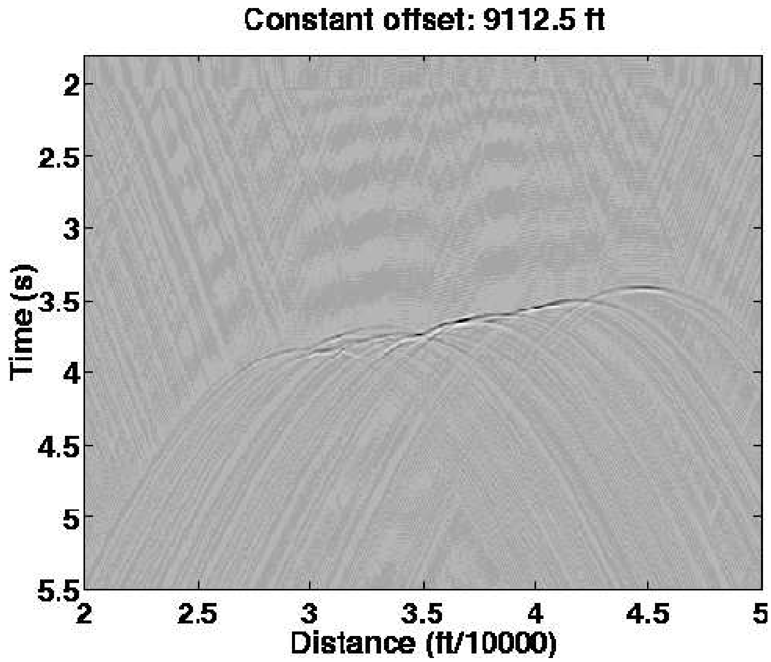


FIG. 7. The data of Figure 5 are un-migrated. Compare with Figure 3 to see that the multiples are now removed.

ACKNOWLEDGEMENTS

I wish to thank Dr. Ray Ergas for the inspiration for this work as well as the staff and sponsors of CREWES for their support. I also than NSERC for their support of this work through CRD grant CRDPJ 379744-08.

REFERENCES

Burnett, W., and R. J. Ferguson, 2011, A reversible transform for seismic data processing: Journal of Geophysics and Engineering, 477–486.

Schneider, W. A., 1978, Integral formulation for migration in two-dimensions and three-dimensions: Geophysics, **43**, 49–76.

Stolt, R. H., 1978, Migration by Fourier transform: Geophysics, **43**, 23–48.

Thomsen, L., 1986, Weak elastic anisotropy: Geophysics, **51**, 1954–1966. (Discussion in GEO-53-04-0558-0560 with reply by author).