Imaging of time-lapse structural changes with linearized inverse scattering

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ABSTRACT

Inverse scattering theory has been used widely in many applications in seismology including time-lapse problems. The difference data during the change in a reservoir from the baseline survey to monitor survey is determined using the linear approximation of the Born series. The linear Born approximation is used to derive a forward operator, mapping the model or perturbation to the measured data, and an adjoint operator, mapping the measured data to the perturbation based on the work of Kaplan (2010). The reference medium in time-lapse problem is the baseline survey medium and the perturbed medium is the monitor survey medium. A difference data is formed by applying structural change in the baseline survey and subtracting it from monitor survey. As the reference medium is as complicated as the perturbed medium, some difficulties such as spurious multiples in the difference data are encountered. To eliminate these unwanted events extending the full version of the Born series is strongly suggested. This paper reviews the earlier work of Innanen and Naghizadeh (2010) to establish a basic work to investigate the role of higher order terms in removing the spurious terms in the reference data.

INTRODUCTION

This paper reproduces earlier work of Innanen and Naghizadeh (2010), which is being reviewed as a preliminary step to expanding and implementing it further for investigating the role of higher order terms.

Up-to-date information of a reservoir provides programs to optimize the management of a reservoir and extends the useful life of an oilfield. A time lapse survey introduces an important contribution to the production of hydrocarbons around the world. A time lapse, or 4D seismic monitoring enables us to monitor the changes in the behavior of a reservoir over time. Comparison of repeated seismic surveys over months, years, or decades adds the forth dimension, time, to the seismic data (Greaves and Fulp, 1987; Lumley, 2001). Prior to utilize a reservoir, a first seismic experiment called the baseline survey is acquired and after a particular interval of time following several geological/geophysical changes, another seismic survey, called monitor survey, is acquired. In a time lapse seismic survey the baseline survey is compared with the monitoring survey. The difference data is the difference between the baseline survey and the following monitor surveys. Seismic trace can differ in amplitude, frequency, polarity, or the location of the interfaces from the baseline survey to the monitor survey. The difference data between a baseline and a monitor survey is categorized as either amplitude or the location of the boundary (Innanen and Naghizadeh, 2010).

The scattering theory can be used as a framework to model these difference data in a time lapse survey. The main idea in scattering theory is computing a wavefield in an inhomogeneous medium using a wavefield in a reference medium perturbed with a function which is related non-linearly to the earth properties. This innovation is used to describe the difference data in time-lapse through resembling the baseline survey as the reference medium and the monitoring survey as the perturbed medium. The difference data are presented as the scattered wavefield data (Zhang, 2006).

Setting the baseline survey as a reference wavefield encounters us with two particular difficulties. Reference medium is as complicated as perturbed medium, and therefore assigning an smooth reference medium to simplify the problem is not an option any more. Another concern is due to the reflected data in the baseline survey which are absent in the reference medium for a standard scattering method (Innanen and Naghizadeh, 2010).

The study described here focuses on describing the difference data for structural change in a reservoir with scattering theory. A forward operator which is mapping the earth model to the seismic data is derived. Then an adjoint operator is defined using the same process. The forward and adjoint operators are derived from Kaplan's thesis (Kaplan, 2010). A linear Born approximation is used to predict the difference data.

WAVE EQUATION AND BORN APPROXIMATION

The Helmholtz wave equation describes a wavefield propagating through an acoustic medium

$$\left[\frac{\partial^2}{\partial x^2} - \left(\frac{1}{c^2(x,z)}\right)\left(\frac{\partial^2}{\partial t^2}\right)\right] P(x,z|x_s,z_s;\omega) = f(\omega)\delta(z-z_s)\delta(x-x_s) \tag{1}$$

Where c(x,z) is wave speed with $x = (\mathrm{x}, \mathrm{y})$, and $f(\omega)$ is the frequency distribution of a point source at (x_s, z_s) with $x_s = (\mathrm{x}_s, \mathrm{y}_s)$. Scattering theory, which is described in companion paper (Jabbari and Innanen, 2011), has been used to solve this wave equation. The result is

$$P(x_g,z_g|x_s,z_s;\omega) = P_d(x_g,z_g|x_s,z_s;\omega) + P_s(x_g,z_g|x_s,z_s;\omega)$$

Where P_d describes the first term in Born series and is a direct wavefield propagating from the source at (x_s, z_s) to the receiver at (x_g, z_g) and is calculated as

$$P_d(x_g,z_g|x_s,z_s;\omega) = f(\omega)G_0(x_g,z_g|x_s,z_s;\omega) \tag{2}$$

G_0 is the Green's function satisfying

$$\left[\frac{\partial^2}{\partial x^2} - \left(\frac{1}{c_0^2(x,z)}\right)\left(\frac{\partial^2}{\partial t^2}\right)\right] G_0(x,z|x_s,z_s;\omega) = \delta(z-z_s)\delta(x-x_s) \tag{3}$$

and P_s is the scattered wavefield

$$P_s(x_g,z_g|x_s,z_s;\omega)=f(\omega)(\int\int_{-\infty}^{\infty}G_0(x_g,z_g|x',z';\omega)(\frac{\omega}{c_0(x',z')})^2\alpha(x',z')G_0(x',z'|x_s,z_s;\omega)dx'dz'$$
$$+\int\int_{-\infty}^{\infty}G_0(x_g,z_g|x',z';\omega)(\frac{\omega}{c_0(x',z')})^2\alpha(x',z')$$
$$\left[\int\int_{-\infty}^{\infty}G_0(x',z'|x'',z'';\omega)(\frac{\omega}{c_0(x'',z'')})^2\alpha(x'',z'')G_0(x'',z''|x_s,z_s;\omega)dx''dz''\right]dx'dz'+...)$$
$$=P_1+P_2+... \tag{4}$$

In these equation α is a dimensionless quantity called perturbation

$$\alpha(x,z)=1-\frac{c_0^2(x,z)}{c^2(x,z)} \tag{5}$$

Where c_0 and c are velocities in the reference and inhomogeneous mediums.

When the reference velocity, c_0 is constant, Green's functions are derived applying residue theory (Saff and Snyder, 1993). More detail can be found in (Kaplan, 2010).

$$G_0(k_{gx},z_g|x',z',\omega)=-\frac{1}{i4k_{gz}}e^{-ik_{gx}.x'}e^{ik_{gz}|z'-z_g|} \tag{6}$$

$$G_0(x',z'|k_{sx},z_s,\omega)=-\frac{1}{i4k_{sz}}e^{ik_{sx}.x'}e^{ik_{sz}|z'-z_s|} \tag{7}$$

where

$$\begin{aligned} k_{gz}&=sgn(\omega)\sqrt{(\frac{\omega}{c_0})^2-k_{gx}.k_{gx}}\\ k_{sz}&=sgn(\omega)\sqrt{(\frac{\omega}{c_0})^2-k_{sx}.k_{sx}} \end{aligned} \tag{8}$$

These equations relate horizontal wave-numbers k_{sx} at source and k_{gx} at receiver to their respective vertical wave numbers k_{sz} and k_{gz} and are called dispersion relations.

The Born series is a consequence of the solving the wave equation and is fully covered in the companion paper (Jabbari and Innanen, 2011). Truncating the Born series after the second term forms the Born approximation in which the measured data are linear with model, meaning that the wavefield are linearly related to the perturbation α, The Born approximation is valid when the perturbation is small (Matson, 1996).

$$P_s(x_g,z_g|x_s,z_s;\omega)\approx f(\omega)\int\int_{-\infty}^{\infty}G_0(x_g,z_g|x',z';\omega)(\frac{\omega}{c_0})^2\alpha(x',z')G_0(x',z'|x_s,z_s;\omega)dx'dz' \tag{9}$$

We will use the Born approximation all over in this paper.

Forward Operator

A forward operator is an operator which maps the earth model to the seismic data. To derive a forward operator, we take into account the variation of the reference medium velocity c_0 in space for the vertical(depth) and lateral directions using the Born approximation. The problem is treated in two separate steps, first the velocity variation in depth is taken into consideration using the Gazdag migration method with the assumption of no variation in the lateral direction (Gazdag, 1978). In Gazdag modeling, the reference velocity is only a function of depth. The entire depth is divided into n_z layers

$$D_l = [z \in R | 0 \leq z_{l-1} \leq z < z_l], \quad l = 1...n_z, \tag{10}$$

$c_{0(l)}$ and $G_{0(l)}$ are approximating a constant reference velocity and a Green's function respectively Within each layer. The total scattered wavefield is the sum of wavefields within each layer, $P_{s(l)}$

$$P_s = P_{s(1)} + P_{s(2)} + ... + P_{s(n_z)} \tag{11}$$

where $P_{s(1)}$, $P_{s(2)}$, and $P_{s(n_z)}$ are the contributions to the scattered wavefield from the corresponding layer in the depth.

To compute $P_{s(1)}$, we assume $z_s = z_g = z_0$. Substituting Green's function for the first layer from equations 6 and 7 into the Born approximation in equation 9 gives the scatter wavefield in the first layer

$$P_{s(1)} \left(k_{gx}, z_0 | k_{sx}, z_0; \omega \right) = f(\omega) \int_{z_0}^{z_1} u_{p(1)} \left(k_{gx}, k_{sx}, z', \omega \right) \frac{\omega^2}{c_{0(1)}^2} \alpha \left(k_{gx} - k_{sx}, z' \right) dz' \tag{12}$$

where k_{sx} and k_{gx} are the Fourier conjugate variables of x_s and x_g, respectively, and the function $u_{p(1)}$ is

$$u_{p(l)} \left(k_{gx}, k_{sx}, z', \omega \right) = -\frac{e^{i(k_{gz(l)} + k_{sz(l)})(z' - z_{l-1})}}{16 k_{gz(l)} k_{sz(l)}} \tag{13}$$

for $l = 1$ and where $k_{gz(1)}$ and $k_{sz(1)}$ are given by the dispersion relations

$$\begin{aligned} k_{gz(l)} &= sgn(\omega) \sqrt{(\frac{\omega}{c_{0(l)}})^2 - k_{gx}.k_{gx}} \\ k_{sz(l)} &= sgn(\omega) \sqrt{(\frac{\omega}{c_{0(l)}})^2 - k_{sx}.k_{sx}} \end{aligned} \tag{14}$$

To construct the scattered wavefield in the second layer, we compute the direct propagation of the wavefield from the source to the boundary of the first layer using $G_{0(1)}$. This wavefield, then goes under perturbation and is scattered in the second layer. We use $G_{0(2)}$ for Green's function for propagation of the scattered wavefield here. This scattered wavefield, propagates back to the receiver through the first layer, therefore the energy is propagated according to $G_{0(1)}$. These steps are presented in the following equations

$$\left[\frac{\partial^2}{\partial x^2} - \left(\frac{1}{c^2_{0(2)}(x,z)}\right)\left(\frac{\partial^2}{\partial t^2}\right)\right] P_{(2,1)}(x,z|x_s,z_0;\omega) = f(\omega)G_{0(1)}(x,z|x_s,z_0;\omega)\delta(z-z_1) + \frac{\omega^2}{c^2_{0(2)}}\alpha(x,z)P_{(2,1)}(x,z|x_s,z_0;\omega) \tag{15}$$

$$\begin{aligned} P_{(2,1)}(x,z|x_s,z_0;\omega) = f(\omega)\int_{-\infty}^{\infty} G_{0(1)}(x',z_1|x_s,z_0;\omega)G_{0(2)}(x,z|x',z_1;\omega)dx' \\ + \int_{-\infty}^{\infty}\int_{z_1}^{z_2} G_{0(2)}(x,z|x',z';\omega)(\frac{\omega}{c_{0(2)}})^2\alpha(x',z')P_{(2,1)}(x',z'|x_s,z_0;\omega)dx'dz' \end{aligned} \tag{16}$$

Writing $P_{(2,1)}$ to first order in α leads to

$$\begin{aligned} P_{s(2,1)}(x,z|x_s,z_0;\omega) \approx f(\omega)\int\int_{-\infty}^{\infty} G_{0(1)}(x'',z_1|x_s,z_0;\omega) \\ \times \int_{z_1}^{z_2} G_{0(2)}(x,z|x',z';\omega)(\frac{\omega}{c_{0(2)}})^2\alpha(x',z')G_{0(2)}(x',z'|x'',z_1;\omega)dz'dx''dx' \end{aligned} \tag{17}$$

This is the scattered wavefield measured in D_2. To determine the scattered wavefield measured at the surface, we apply $P_{s(2,1)}$ as a boundary condition at the bottom of D_1

$$\left[\frac{\partial^2}{\partial x^2} - \left(\frac{1}{c^2_{0(1)}(x,z)}\right)\left(\frac{\partial^2}{\partial t^2}\right)\right] P_{s(2)}(x,z|x_s,z_0;\omega) = P_{s(2,1)}(x,z|x_s,z_0;\omega)\delta(z-z_1) \tag{18}$$

$$\begin{aligned} P_{s(2)}(x,z|x_s,z_0;\omega) = f(\omega)\int\int\int_{-\infty}^{\infty} G_{0(1)}(x,z|x',z_1;\omega)\int_{z_1}^{z_2} G_{0(2)}(x',z_1|x'',z'';\omega) \\ \times (\frac{\omega}{c_{0(2)}})^2\alpha(x'',z'')G_{0(2)}(x'',z''|x''',z_1;\omega)dz''G_{0(1)}(x''',z_1|x_s,z_0;\omega)dx'''dx''dx' \end{aligned} \tag{19}$$

Evaluating this at the measurement surface (x_g, z_0) and taking the Fourier transforms over x_g and x_s we have

$$\begin{aligned} P_{s(2)}(k_{gz},z_0|k_{sx},z_0;\omega) = f(\omega)\int\int\int_{-\infty}^{\infty} G_{0(1)}(k_{gx},z_0|x',z_1;\omega)\int_{z_1}^{z_2} G_{0(2)}(x',z_1|x'',z'';\omega) \\ \times (\frac{\omega}{c_{0(2)}})^2\alpha(x'',z'')G_{0(2)}(x'',z''|x''',z_1;\omega)dz''G_{0(1)}(x''',z_1|k_{sx},z_0;\omega)dx'''dx''dx' \end{aligned} \tag{20}$$

Substituting the Green's function in equations 6 and 7 and recognizing the Fourier transform over x' and x"' gives

$$P_{s(2)}(k_{gz}, z_0|k_{sx}, z_0; \omega) = f(\omega) \int_{-\infty}^{\infty} \left(-\frac{e^{ik_{gz(1)}(z_1-z_0)}}{i4k_{gz_{(1)}}}\right) \int_{z_1}^{z_2} G_{0(2)}(k_{gx}, z_1|x'', z''; \omega)$$
$$\times (\frac{\omega}{c_{0(2)}})^2 \alpha(x'', z'') G_{0(2)}(x'', z''|k_{sx}, z_1; \omega) dz'' \left(-\frac{e^{ik_{sz(1)}(z_1-z_0)}}{i4k_{sz_{(1)}}}\right) dx'' \tag{21}$$

By rearranging this equation we can have a summary form as follow

$$P_{s(2)}\left(k_{gx}, z_0|k_{sx}, z_0; \omega\right) =$$
$$f(\omega) u_{p(1)}\left(k_{gx}, k_{sx}, z_1, \omega\right) \int_{z_1}^{z_2} u_{p(2)}\left(k_{gx}, k_{sx}, z', \omega\right) \frac{\omega^2}{c_{0(2)}^2} \alpha\left(k_{gx} - k_{sx}, z'\right) dz' \tag{22}$$

where $u_{p(2)}$ is giving by equation 13 for l=2.

The scattered wavefield for the l^{th} layer is generalization of $P_{s(2)}$ to the l^{th} layer

$$P_{s(l)}\left(k_{gx}, z_0|k_{sx}, z_0; \omega\right) =$$
$$f(\omega) u_{p(1)} u_{p(2)} ... u_{p(l-1)} \int_{z_{l-1}}^{z_l} u_{p(l)}\left(k_{gx}, k_{sx}, z', \omega\right) \frac{\omega^2}{c_{0(l)}^2} \alpha\left(k_{gx} - k_{sx}, z'\right) dz' \tag{23}$$

Split-step modeling

Split-step modeling is used to account for lateral variation in the reference velocity in our calculation. We let $k_{gz(l)}$ and $k_{sz(l)}$ be the functions of slowness $c_{0(l)}^{-1}$ which is allowed to vary in the lateral dimensions. Taking the Taylor expansions of $k_{gz(l)}$ and $k_{sz(l)}$ about a constant $c_{1(l)}^{-1}$ which is the average of $c_{0(l)}^{-1}$, and truncating it to the first term gives

$$k_{gz(l)}(c_{0(l)}^{-1}) \approx k_{gz(l)}(c_{1(l)}^{-1}) + \omega \left[1 - |c_{0(l)}(x_g) k_{gx}/\omega|^2\right]^{-1/2} \left(c_{0(l)}^{-1}(x_g) - c_{1(l)}^{-1}\right)$$
$$\approx k_{gz(l)}(c_{1(l)}^{-1}) + \omega \left(c_{0(l)}^{-1}(x_g) - c_{1(l)}^{-1}\right) \tag{24}$$

Here we have set $c_{0(l)}(x_g) k_{gx}/\omega$ to zero. This term is zero if either ω is large or k_{gx} is small. This is valid for near vertical traveling plane wave components and when the lateral variation in the reference velocity is small. Applying this equation allows to split the dispersion relation into two parts which depend on the wavenumber and lateral space independently. With this split, the l^{th} term in equation 23 in Gazdag modeling can be rewrite as

$$P_{s(l)}\left(x_g, z_g|x_s, z_s; \omega\right) = \left(\frac{1}{2\pi}\right)^{4l} f(\omega) \left(u_{s(1)} F_{gs}^* u_{p(1)} F_{gs}\right) ... \left(u_{s(l-1)} F_{gs}^* u_{p(l-1)} F_{gs}\right)$$
$$\times \int_{z_{l-1}}^{z_l} \left(u_{s(l)}(x_g, x_s, z', \omega) F_{gs}^* u_{p(l)}(k_{gx}, k_{sx}, z', \omega) F_{gs}\right) \frac{\omega^2}{c_{1(l)}^2} \alpha\left(x_g, x_s, z'\right) dz' \tag{25}$$

where F_{gs} is the four dimensional Fourier transform over x_s and x_g and F^*_{gs} is its adjoint, such that the corresponding inverse Fourier transform is $(1/2\pi)^4 F^*_{gs}$. The phase shift, $u_{s(l)}$, and split-step correction, $u_{p(l)}$, are

$$u_{s(l)}\left(x_g, x_s, z'; \omega\right) = e^{i\omega\left(c^{-1}_{0(l)}(x_g)+c^{-1}_{0(l)}(x_s)-2c^{-1}_{1(l)}\right)(z'-z_l)} \tag{26}$$

$$u_{p(l)}\left(x_g, x_s, z'; \omega\right) = -\frac{i\left(k_{gz(l)}+k_{sz(l)}\right)(z'-z_{l-1})}{16k_{gz(l)}k_{sz(l)}} \tag{27}$$

Equation 11, 25 and 26 are split-step wavefield modeling in which the reference velocity varies in all dimensions and constitute a linear forward operator which maps perturbation α to the measured wavefield P_s.

Adjoint Operator

In preceding section we constituted a forward operator mapping the model earth, or perturbation, to the measured data, also called de-migration. In seismology we are rather looking for techniques which can do the inverse, mapping the measured data to the model or perturbation. In inversion, this task is done using adjoint operator. The adjoint of equation 9 is

$$\alpha^\dagger(x', z') = \int\int\int_{-\infty}^{\infty} f^*(\omega) G^*_0(x_g, z_g|x', z'; \omega)\frac{\omega^2}{c^2_0(x', z')} P_s(x_g, z_g|x_s, z_s; \omega) G^*_0(x', z'|x_s, z_s; \omega)\, dx_g dx_s d\omega \tag{28}$$

Fredholm integral equation has the form of

$$g(x) = \int u(x, z)h(z)dz \tag{29}$$

and its adjoint is

$$h^\dagger(z) = \int u^*(x, z)g(z)dx \tag{30}$$

where u, g, and h are arbitrary functions, and u^* is the adjoint of u. Correspondingly in equation 11 and 25 we can recognize the form of

$$P_s\left(x_g, z_0|x_s, z_0; \omega_j\right) = \sum_l u(x_g, x_s; \omega_j, z_l)\alpha(x_g, x_s; z_l) \tag{31}$$

Similarly its adjoint will be

$$\alpha^\dagger(x_g, x_s; z_l) = \sum_l u^*(x_g, x_s; \omega_j, z_l) P_s\left(x_g, z_0|x_s, z_0; \omega_j\right) \tag{32}$$

where

$$u^*(x_g, x_s; \omega, z_l) = (\frac{1}{2\pi})^{4l} f^*(\omega)\left(u^*_{s(l)}F^*_{gs}(\frac{\omega}{c_{1(l)}})^2 u^*_{p(l)}F_{gs}\right) ... \left(u^*_{s(2)}F^*_{gs}u^*_{p(2)}F_{gs}\right)\left(u^*_{s(1)}F^*_{gs}u^*_{p(1)}F_{gs}\right) \tag{33}$$

$u^*_{s(l)}$ and $u^*_{p(l)}$ are the conjugate of $u_{s(l)}$ and $u_{p(l)}$.

A STRUCTURAL PERTURBED TIME-LAPSE PROBLEM

We will consider two seismic experiments involved in a time-lapse survey, the baseline survey, followed by a monitoring survey. The acoustic medium In this project,is one-dimensional problem varying in depth with a normal incident plane source. The only parameter changes from the time of baseline survey to monitoring survey is the depth of the reflectors. The depth of the interface location at the time of the baseline survey is z_I. In a time lapse problem, the reference wavefield, or the Green's function, is the wavefield of the baseline survey and can be expressed from equation 1 as

$$\left[\frac{d^2}{dz^2} + \frac{\omega^2}{c_I^2(z)}\right] G_0(z, z_s; \omega) = \delta(z - z_s) \tag{34}$$

For simplicity, the source is a pulse which is presented by a delta function at $z = z_s$, and the $1/c_I^2(z)$ is defined as

$$\frac{1}{c_I^2(z)} = \begin{cases} c_I^{-2}, & z > z_I \\ c_0^{-2}, & z < z_I \end{cases} \tag{35}$$

c_0 and c_I are the incidence and target wavefield velocities. The perturbed medium has the same target medium parameter in which interference location is changed to z_F

$$\left[\frac{d^2}{dz^2} + \frac{\omega^2}{c_F^2(z)}\right] P(z, z_s; \omega) = \delta(z - z_s) \tag{36}$$

where

$$\frac{1}{c_F^2(z)} = \begin{cases} c_I^{-2}, & z > z_F \\ c_0^{-2}, & z < z_F \end{cases} \tag{37}$$

The baseline wavefield which is the reference wavefield for the time lapse problem is calculated as (Innanen and Naghizadeh, 2010)

$$P_0 = \frac{1}{i2k_0} + R_I \frac{e^{i2k_0 z_I}}{i2k_0} \tag{38}$$

Where $k_0 = \frac{\omega}{c_0}$, $R_I = (c_I - c_0)/(c_I + c_0)$, and $z_s = z_g = 0$. The first term in this equation is the direct wavefield propagating from the source to the receiver, and the second term is the reflection from the interface location z_I.

The solution for the first order scattered wavefield or the Born linear approximation, which is the difference data in time-lapse interpretation is obtained as (Innanen and Naghizadeh, 2010)

$$P_s \approx \frac{\alpha_{TL}}{4}\left[\frac{e^{i2k_0 z_F}}{i2k_0} - \frac{e^{i2k_0 z_I}}{i2k_0}\right] \tag{39}$$

The α is

$$\begin{aligned} \alpha_{TL}(z) &= 1 - \frac{c_I^2(z)}{c_F^2(z)} \\ &= \begin{cases} 0, & z < z_F \\ 1 - c_0^2/c_I^2, & z_F < z < z_I \\ 0, & z_I < z \end{cases} \\ &= \alpha_{TL}\left[H(z - z_f) - H(z - z_I)\right] \end{aligned} \tag{40}$$

where H is Heaviside function and $\alpha_{TL} = 1 - c_0^2/c_I^2$ is the time lapse perturbation. The full Born series can be written as

$$P = \frac{1}{i2k_0} + R_I \frac{e^{i2k_0 z_I}}{i2k_0} + \frac{\alpha_{Tl}}{4}\left[\frac{e^{i2k_0 z_F}}{i2k_0} - \frac{e^{i2k_0 z_I}}{i2k_0}\right] + ... \quad (41)$$

The second term in the right describes a reflection from the interface z_I in the reference medium. But in reality, there is not such an event in P. This fact produces a difficulty in interpretation of the time lapse structural problem using the scattering theory. There is a possibility that this spurious event is canceled out with higher order terms in the Born series. The future work for this project will be computing higher order terms and investigating this possibility.

So far in all parts of this project, we consider the first or linear term in the Born series. We have derived equations for the forward and adjoint operator based on the linear Born approximation for a standard seismic reflection survey. A forward and adjoint operator for the difference data based on linear Born approximation can be derived and tested on a model data for a time lapse problem. Also the Born series will be investigated and derived full series solution to map the earth model to the measured data and the measured data to the model. The possibility of the removing the spurious events on the Born terms will be investigated using higher terms in the Born series and tested on the model.

CONCLUSION

Employing the scattering theory in many geophysical area such as time-lapse is worthwhile. Time-lapse measurements provide a tool to monitor the dynamic changes in subsurface properties during the time of the exploitation of a reservoir. For the future work we will derive a forward and adjoint operator for the difference data in a structural change time lapse problem. One of the main obstacle on using linear scattering theory to predict the model for the difference data in the structural change time-lapse problem is producing spurious events due to the complexity of the reference medium which is the baseline survey. A possible solution to this is calculating the higher order terms in the Born series and investigating if these events can be removed through the involving higher order terms.

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