Analytic and numerical considerations for velocity grid smoothing in ray based modelling and migration

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ABSTRACT

Ray tracing is a fast and effective general purpose method for propagating seismic waves through a geological model. According to Snell's law, rays bend relative to the gradient vector that is produced by the local velocity contrast. Velocity model smoothing allows for a more accurate computation of the gradient directions close to discontinuities in the velocity model. However, smoothing also changes the velocity gradient at points farther away from these boundaries. Care must be taken when smoothing a velocity model to ensure the rays refract in the right direction without changing the position of the reflector interface. We describe possible strategies to compute accurate gradient directions so as to minimize the need for velocity model smoothing. A simple, special purpose method for computing more accurate gradient vectors at a dipping interface while removing the artifacts produced by smoothing is presented.

INTRODUCTION

Ray tracing is an important component of many modelling and inversion processes. The accurate computation of ray paths, and travel times along these paths is useful for Kirchhoff migration (Gray and May, 1994) as well as turning wave tomography (Zhu and McMechan, 1989; Bijwaard and Spakman, 2000) and many seismic modelling algorithms (Gjoystdal et al., 2002). These methods generalize Snell's law, which quantifies the refraction of a ray as it passes between two homogeneous materials, to a continuously variable velocity field (Cerveny, 2001; Krebes, 2009).

The normal vector to the boundary between two surfaces generalizes to the gradient of the velocity model, which is necessary to compute the change in the direction of the ray. When this continuous velocity function is sampled onto a grid, it can become difficult to accurately compute these gradient vectors across dipping interfaces. Velocity model smoothing is one simple solution to this problem. Smoothing enforces the assumption that the velocity function varies smoothly in space (Gray, 2000). A smoothed velocity model gives more accurate gradient directions near a material boundary, but introduces some variation away from the boundary which can affect the quality of the final result. The resulting rays are smoother and less likely to cross, and refraction directions are more accurate, but the rays will be incorrectly positioned as they travel toward the surface (Versteeg, 1993).

THEORY AND METHODS

An effective general method of computing the gradient of a velocity model at an arbitrary point, either at a given grid point or at any arbitrary position along a ray path, would allow us to compute more accurate ray paths when the medium is not well behaved. Here we introduce the basic process of ray tracing. We then present a number of numerical approaches to computing gradients and discuss their limitations.

Ray tracing for seismic modelling

Ray tracing is performed by solving the coupled system of first degree differential equations given by,

$$\frac{d\vec{x}}{dt} = v^2\vec{p} \tag{1}$$

$$\frac{d\vec{p}}{dt} = -\frac{1}{v}\vec{\nabla}v. \tag{2}$$

Here $\vec{x}$ and $\vec{p}$ are the position and slowness of the ray, and $v = v(\vec{x})$ is the P-wave velocity of the elastic medium (Krebes, 2009). These equations can be solved numerically using any number of methods for solving initial value problems, the most popular of which is the 4th order Runge-Kutta method (Burden and Faires, 2001). Equation 1 describes the speed and direction of the ray. Equation 2 describes how the direction of the ray varies in relation to the gradient of the velocity field, similar to how Snell's law describes how the direction of the ray changes with respect to the gradient of the velocity at an interface.

Computation of gradient vectors

In order for the ray to bend in the correct direction, we need to be able to accurately compute the direction of the gradient of the velocity model. This requires us to estimate partial derivatives of the velocity model from the information given at the grid points. Many elementary methods for estimating the gradient of a velocity field give unsatisfying results in many seismic settings.

Finite Difference

The simplest method of computing gradient vectors is to use a finite difference derivative. The 2D 1st-order centred difference approximation of a sampled velocity field is given in Equations 3-5.

$$\vec{\nabla}v = \left(\frac{\partial v}{\partial x}, \frac{\partial v}{\partial z}\right) \tag{3}$$

$$\frac{\partial v}{\partial x} \cong \frac{1}{2\Delta x}\left[v_{x+\Delta x} - v_{x-\Delta x}\right] \tag{4}$$

$$\frac{\partial v}{\partial x} \cong \frac{1}{2\Delta z}\left[v_{z+\Delta z} - v_{z-\Delta z}\right]. \tag{5}$$

Consider a simple two layer model with dip angle $\tan^{-1}\left(\frac{1}{5}\right)$, as shown in Figure 1. The true gradient vectors are indicated in green, and the centred difference approximation is given in yellow. When computing gradient vectors at the boundary, this method sees either a change in only the z direction, resulting in a vertical gradient vector (Figure 2(a)), or an equal change in both directions, resulting in a gradient vector pointing 45 degrees in the direction of the higher velocity segment (Figure 2(b)). Since the computation does not

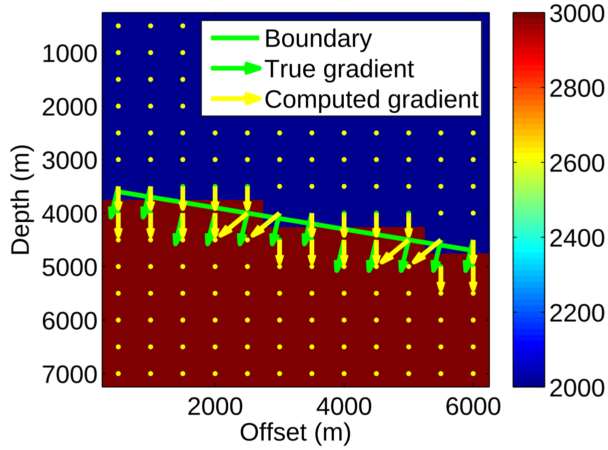

FIG. 1. A simple dipping interface. The correct gradient of the model is denoted by the green arrows, and the computed gradient by the yellow arrows.

look farther than one grid point away from the target in any direction, it is unable to see the correct dip of the interface.

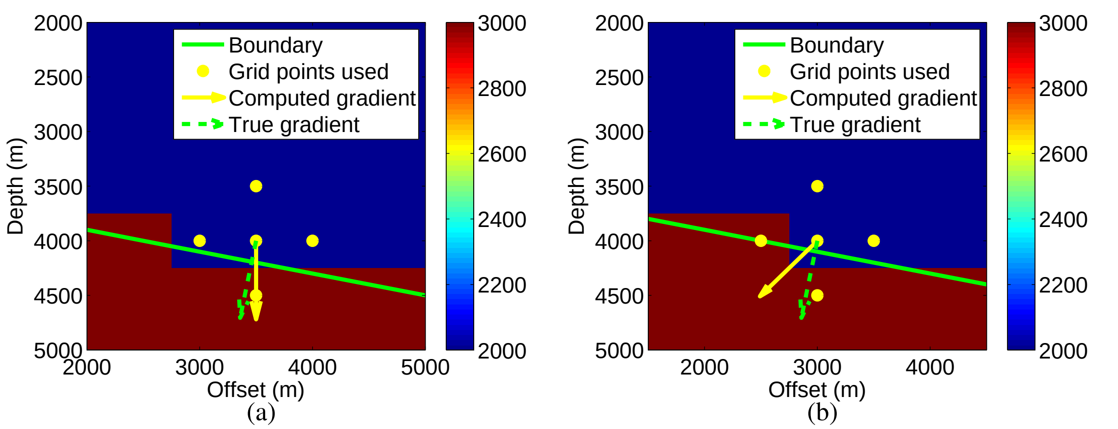

FIG. 2. This finite difference method gives two different gradient vectors along the same dipping interface, depending on whether the point is along the horizontal (a), or near a step (b).

Smoothing

The velocity model is generally smoothed to account for the above phenomenon (Pacheco and Larner, 2005; Gray, 2000). Smoothing replaces the value at each node with a weighted average of the values at a number of nearby nodes. As a result, each node in the smoothed model contains information about many surrounding nodes in the original model, so the finite difference method sees more information about the dip of the interface.

Figure 3 shows a ray incident on a dipping surface, and several transmitted rays computed with respect to velocity models that have undergone different degrees of smoothing. The dashed yellow line shows the ideal transmission direction as predicted by Snell's law. When no smoothing is applied, this ray refracts as if the interface is horizontal, so it travels in the wrong direction. Smoothing the boundary slightly corrects the direction of the

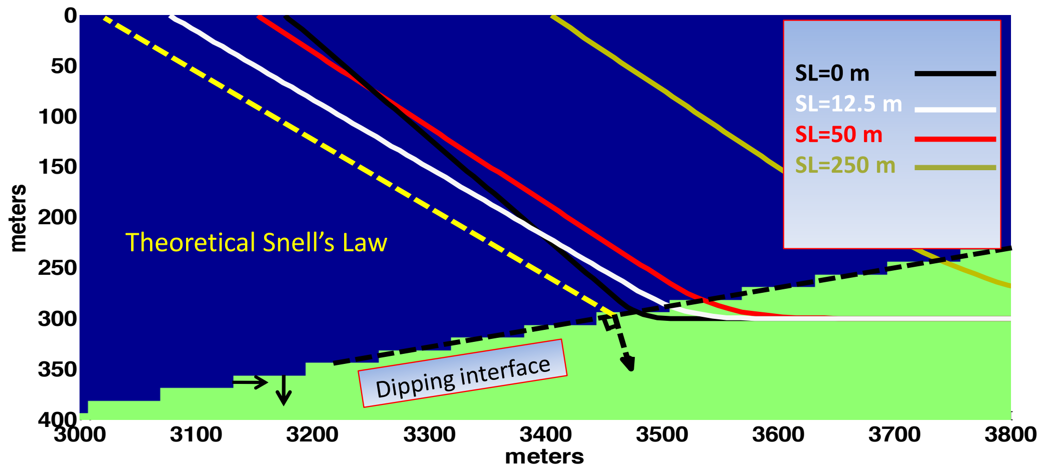

FIG. 3. Several different computed transmitted raypaths from the same incident ray. The dotted yellow line indicated where the ray should go by Snell's law. SL refers to the length of the operator used to smooth the model. The black line is calculated on the unsmoothed model. Gently smoothing the model results in rays that travel more in the right direction, although not quite along the right path. Over smoothing results in the ray turning much too early, which could negatively affect the quality of our modelling or migration.

gradient at the interface, so the ray is deflected in the right direction, but the ray begins to bend earlier, so its path is perturbed laterally from the ideal path. Using a larger smoother worsens this effect, as the ray begins to bend way too early. These rays will not reach the surface at the appropriate location, which may result in artifacts in ray based modelling and migration algorithms (Versteeg, 1993).

Figure 4(b) shows the gradient computed on a smoothed velocity model. Compared to the unsmoothed model in Figure 4(a), the gradient vectors near the interface are closer to the true gradient vectors. However, farther from the interface, we can see that the calculation has introduced a nonzero gradient where there was none before. More smoothing is still needed here to make the gradient correctly reflect the dip of the interface, but we have to strike a balance between correct gradient vectors at the boundary and the artifacts produced nearby.

Moser and Pajchel (2011) suggests that after computing ray paths on a smoothed model, travel times can be computed with respect to the unsmoothed model. The resulting Kirchhoff migration gives better continuity of reflectors below salt bodies. If we can find an alternative to smoothing that computes better gradient vectors, and therefore better ray paths, we should be able to further improve upon these results.

Higher order finite difference

Since smoothing works by allowing us to see farther away from the given grid point, it is natural to consider using a higher-order finite difference method. Using a higher order method carries a larger computational cost, but since we only need to compute the gradient once for a given velocity model, this cost is generally negligible compared to the cost of solving the ray tracing equations. The 2nd-order centred finite difference approximation is

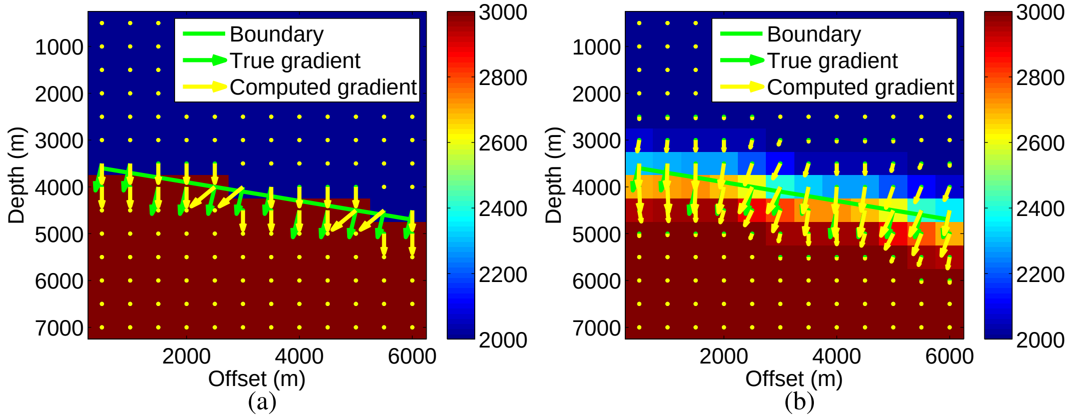

FIG. 4. Gradient vectors computed on (a) an unsmoothed model and (b) a smoothed model. Vectors near the interface are closer to the true gradient in the smoothed model, but extra variation is introduced farther from the boundary.

given in Equations 6 and 7.

$$\frac{\partial v}{\partial x} \cong \frac{1}{48\Delta x}\left[v_{x-3\Delta x} - 27v_{x-\Delta x} + 27v_{x+\Delta x} - v_{x+3\Delta x}\right] \tag{6}$$

$$\frac{\partial v}{\partial z} \cong \frac{1}{48\Delta z}\left[v_{z-3\Delta z} - 27v_{z-\Delta z} + 27v_{z+\Delta z} - v_{z+3\Delta z}\right] \tag{7}$$

For our simple two layer model, the result of computing the gradient by this approximation are shown in Figure 5(b). The desired effect is not achieved. The directions of the new computed gradient vectors are no better than those from the 1st-order difference method (Figure 5(a)). Furthermore, we have again introduced nonzero gradient vectors farther away from the interface, although these are much less severe than those introduced by smoothing.

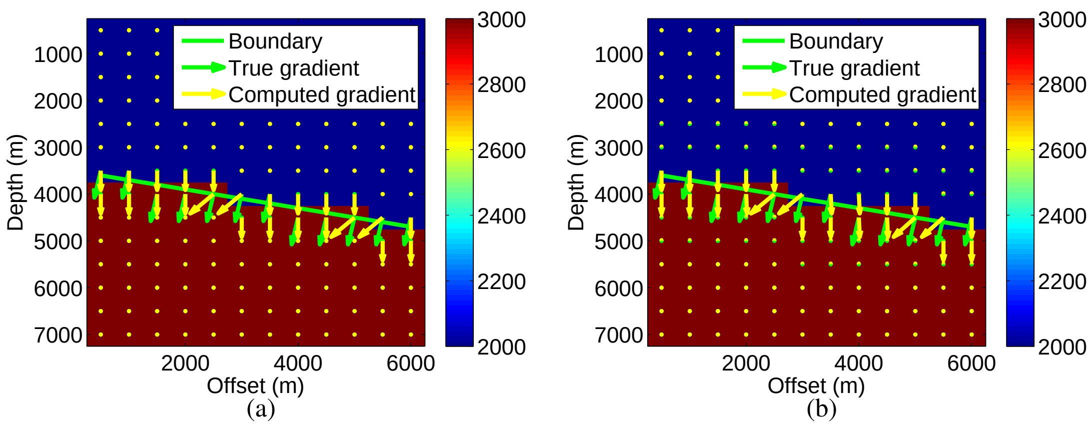

FIG. 5. Gradient vectors computed using (a) 1st order centred difference and (b) 2nd order centred difference. Gradient vectors near the boundary are no more accurate with the higher order method.

The issue here is that the finite difference approximation can not see the difference between the change in velocity in the x-direction and the change in the z-direction, similar to the situation in Figure 2. While smoothing allowed both finite difference calculations to see more of what was happening in both directions, this finite difference scheme considers each principle grid direction separately.

This example helps illustrate that finite difference derivatives and smoothing operators both take the form of a weighted average of function values on the grid near the target node. With this information we can generalize all the gradient computation methods we have seen so far using Equations 8 and 9.

$$\frac{\partial v}{\partial x} \cong \sum_{j=-m}^{m} \sum_{k=-n}^{n} a_{jk} v(x + j\Delta x, z + k\Delta z) \tag{8}$$

$$\frac{\partial v}{\partial z} \cong \sum_{j=-m}^{m} \sum_{k=-n}^{n} b_{jk} v(x + j\Delta x, z + k\Delta z) \tag{9}$$

The coefficients a_{jk} and b_{jk} should be chosen so that, by expanding each $v(x + j\Delta x, z + k\Delta z)$ in a two-dimensional Taylor series, the right side reduces to the left side, plus some function that is negligible when Δx and Δz are small enough (Krebes (2009)). Furthermore, since we found that smoothing gave the better results than directional derivatives alone, we should require that there are nonzero coefficients a_{jk} and b_{jk} with $j \cdot k \neq 0$.

Interpolation

The characterization of the gradient in Equations 8 and 9 suggest that these methods we have shown should converge to the correct result when the distance between nodes is very small. That is, it may be effective to upsample the velocity model, interpolating between the grid points, until the rugged boundary between the two segments becomes smooth. It should then be simple to compute accurate gradient vectors, as shown in Figure 6(a). If we could replace the discontinuity in the model with a smooth transition, our methods should be able to see that the change is faster in the z-direction and more gradual in the x-direction. Figure 6(b) shows a simple linear interpolation, which fails to remove the rugged edge from the model. Many other interpolation methods exist, but we have yet to determine an effective general purpose interpolation scheme that will produce accurate derivatives.

Proximal smoothing

When smoothing this simple velocity model, it is not difficult to tell which gradient vectors are affected positively and which are affected negatively. Smoothing improves the vectors near the interface, where the gradient is large, and degrades the quality in the area immediately surrounding the interface, where the gradient is small. With this in mind, we can perform a proximal smoothing of the model at the interface. We smooth the model as normal, and compute the gradient vectors. Then, anywhere our unsmoothed model is not varying quickly, we simply replace the gradient vectors in the smooth model with the corresponding vectors in the unsmoothed model. The resulting gradient vectors are more

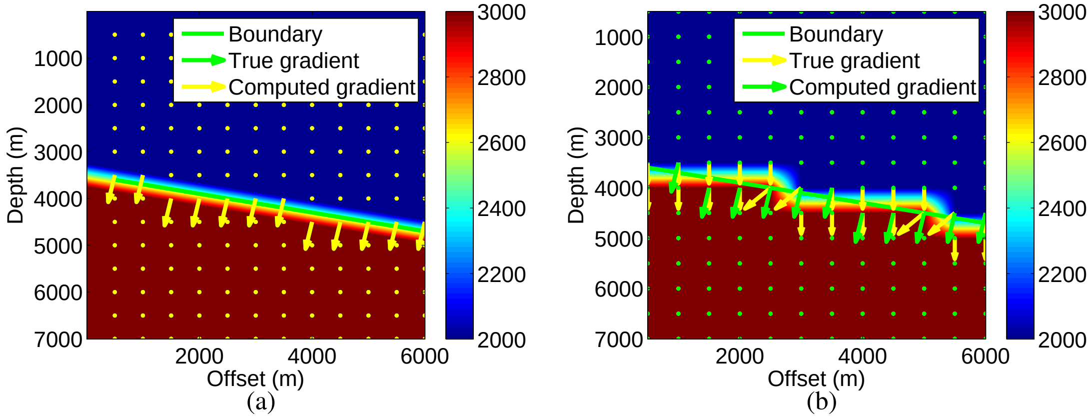

FIG. 6. (a) A conceptual representation of a smoothed boundary, where finite difference can accurately predict the dip angle of the interface. (b) A simple linear interpolation, which fails to restore the dip of the reflector to its proper shape.

accurate both near the interface and farther away.

Since we had to pick and choose which gradient vectors we liked from both the smoothed and unsmoothed model, it is not clear how we would generalize this technique to a general velocity model containing both smoothly varying areas and abrupt boundaries. It will allow us to experiment with how computing better gradient vectors affects our ability to draw more accurate rays. The result of the proximal smoothing technique is given in Figure 7(b). We have a tighter transition between the two segments, and the computed gradient vectors are closer to the true dip of the interface.

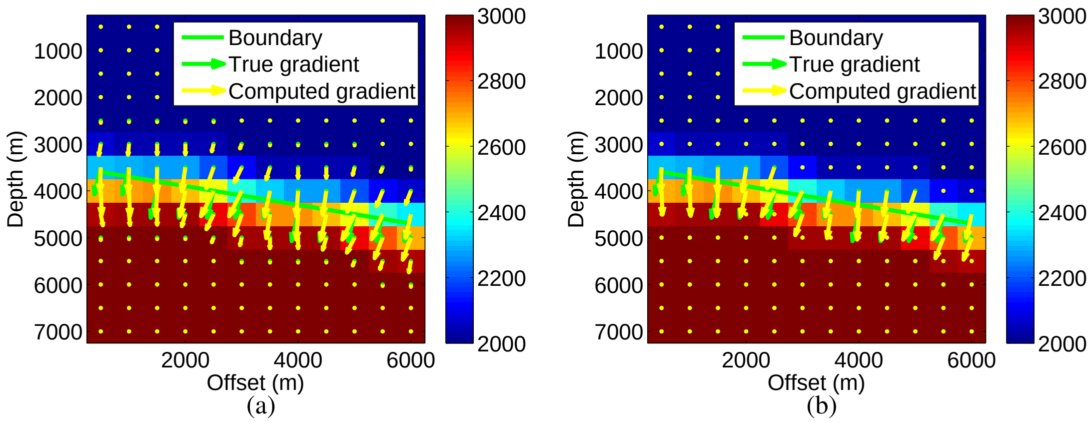

FIG. 7. (a) Gradient vectors computed on a globally smoothed velocity model (b) Gradient vectors computed on a proximally smoothed velocity model.

EXAMPLE

Figure 8 shows a simple velocity model with flat and dipping interfaces, sampled onto a 12.5m grid. We shot three groups of rays from a single scatter point, using three different gradient computations. The black rays were shot through the model after it was smoothed using a 50m smoother. The yellow rays correspond to a 300m smoother, and the blue rays were calculated using a 300m proximal smoother. Similar to Figure 3, a small amount of smoothing makes the ray paths more consistent with each other. However, it is often still necessary to oversmooth a model. The oversmoothed yellow rays are turning very early before they strike the interface, and will likely be incorrectly positioned at the surface. The proximally smoothed blue rays use the same smoother as the yellow rays but are less affected by the artifacts related to smoothing.

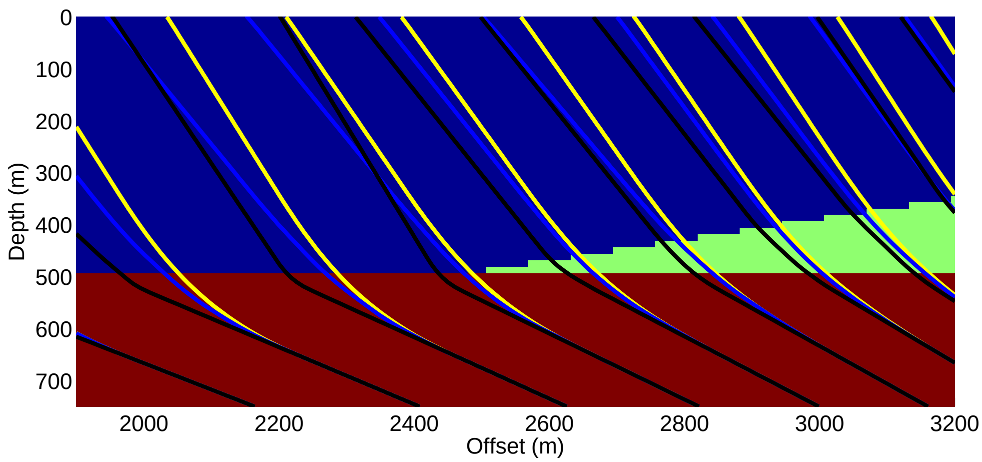

FIG. 8. A simple velocity model with a dipping interface, sampled onto a 12.5m grid. Several rays are shot from a single scatter point, using different smoothing lengths. The black rays are generated on a 50m smoothed model, the yellow rays on a 300m smoothed model, and the blue rays on our modified proximal 300m smoothed model. The blue rays refract more consistently than the black rays, and are less affected by the artifacts of smoothing than the yellow rays.

FUTURE WORK

By casting the problem of velocity model smoothing in terms of the accurate computation of gradient vectors, we hope to improve the utility of ray tracing methods to more challenging geological settings. Ideally, we would like to be able to characterize a velocity grid as a sampling from a collection of differentiable basis functions that correctly model the shapes of real subsurface structures. This would allow us to accurately compute the gradient of any geological model. By developing a general purpose method for accurately computing the gradient of a velocity model, we can minimize the amount of smoothing necessary, which we hope will give us better results when modelling and migrating. Armed with such a technique, we will reproduce a number of benchmark imaging tests to see if better gradient vectors give us more accurate images in complex media.

ACKNOWLEDGEMENTS

The authors wish to thank the sponsors, faculty, staff and students of the Consortium for Research in Elastic Wave Exploration Seismology (CREWES), and the Natural Sciences and Engineering Research Council of Canada (NSERC, CRDPJ 379744-08) for their support of this work.

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