Grid scaling 2-D acoustic full waveform inversion with a high frequency impulsive source

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ABSTRACT

Spatial grid multi-scaling with domain decomposition is developed in this study in order to obtain better convergence rate for the acoustic inverse problem for velocity field with self-adjoint dispersive operator. The model used in the study is based on the 2D acoustic wave equation boundary problem approximated with fourth-order spatial finite-difference approximation and factorization in orthogonal spatial directions. Combinations of different spatial scaling in the case of one impulse point source and limited number of observation points (virtual geophones) is implemented in the paper to verify the ability to estimate the velocity field with suitable synthetic data.

INTRODUCTION

The velocity coefficients inverse problem for acoustic equation theory is well studied in last 40 years by many geophysicists (Tarantola, 1984; Shin 2007). There are many approaches solving this problem in time-frequency domain (Virieux 2009) with signal deconvolution (Margrave et al., 2011) or in time-spatial domain solution of the wave equation with finite-differential or finite-element approximation.

In present study we use a finite-differential approximation in the acoustic forward and adjoint boundary problems for numerical solution of the forward problem. Then we apply a gradient method inversion approach (Tarantola, 1984) to velocity coefficient estimation problem with one impulse point source. With this approach we try to obtain a good observation angle from point source position. The main purpose of the study is to verify numerically with the synthetic data the inversion approach which is based on multi-scaling also developed by many other researchers (Bunks et al., 1995) and domain decomposition with overlap. We also simulate the consequences of limitations in the observation equipment (virtual geophones) as well as in sources (considering just one shot for each subdomain).

Grid scaling considered in the present paper is known as multi-scaling and means the use of finer grids for velocity finite-differential approximation in forward acoustic propagation and coarser grid for velocity inverse problem solution with gradient method. It also requires interpolation operators to transfer data from finer grid to the coarse one and back to the finer grid.

Velocity coefficients inverse problem solution is implemented with the first-order gradient method. The choice of minimization direction on each step is made in order to avoid non-linearity of the inversion operator while performing a linear gradient search.

FORWARD WAVE EQUATION BOUNDARY PROBLEM

Let us consider forward boundary problem for acoustic wave equation in 2-D spatial domain Ω (1)-(3):

$$\frac{\partial^2}{\partial t^2}u = div(c^2 \nabla u) + f, \quad (1)$$

$$u(t=0) = 0, \ \ \frac{\partial u}{\partial t}(t=0) = 0, \quad (2)$$

$$u\big((x,y) \in \partial\Omega\big) = 0, \quad (3)$$

where

$u(x,y,t)$ – wave equation forward propagation solution in the area $\Omega\times[0,T]$;

$c^2(x,y)$ – velocity field defined in Ω;

$f(x,y,t)$ – source function defined both in time and space.

In the current study homogeneous initial and boundary conditions (2)-(3) are considered for simplicity of the problem and can be replaced with certain inhomogeneous conditions on $u(x,y,t)$.

The source function $f(x,y,t)$ is given with the formula (4):

$$f(x,y,t) = \delta(x-x_0, y-y_0)e^{-\lambda^2(t-t_0)^2}sin\big(\omega\lambda(t-t_0)\big) \quad (4)$$

with appropriate ω and λ so that $|f(x_0,y_0,0)| < \varepsilon$ for the small ε which is considered close enough to 0. The time interval [0,T] for both forward and adjoint wave propagation is limited in the model with the distance from the source point (x_0,y_0) to $\partial\Omega$ over the maximum of the wave propagation speed $c^2(x,y)$ allowed in the model which makes us put the source point (x_0,y_0) far away from the border $\partial\Omega$.

In the inverse problem solution, where $c^2(x,y)$ is an unknown function, we assume that there is projection operator $P_{ground\ level}$ defined on the set of virtual geophones on the boundary layer between air and soil which gives us some information about exact velocity propagation field $c^2_{EXACT}(x,y)$ as a result of observation the information recorder in virtual geophones point:

$$d_{obs} = P_{ground\ level}u, \quad (5)$$

FINITE-DIFFERENTIAL APPROXIMATION SCHEME

On the regular time-spatial grid we are using standard second-order factorizing scheme for second derivative in time and fourth-order scheme in space approximating differential equation (1) with the following scheme

$$(I - \Delta t^2 \cdot \sigma\Lambda_x)(I - \Delta t^2 \cdot \sigma\Lambda_y)u(t^{n+1}) = \Big(2I + \Delta t^2(1-2\sigma)\big(\Lambda_x + \Lambda_y\big)\Big)u(t^n)$$
$$- \Big(I - \Delta t^2 \cdot \sigma\big(\Lambda_x + \Lambda_y\big)\Big)u(t^{n-1}) + \Delta t^2 \cdot f, \quad (6)$$

where Δt is a step of the time grid; Λ_x and Λ_y are forth order differential analogs of second spatial derivatives $\frac{\partial^2}{\partial x^2}$ and $\frac{\partial^2}{\partial y^2}$ respectively; I is an identity operator; σ is a weighting factor which allows as allows a tradeoff between a pure explicit scheme without factorizing ($\sigma = 0$) and an absolutely stable semi-implicit scheme ($\sigma = 0.25$). Both schemes can be implemented but the semi-implicit absolutely stable scheme has some advantages and it is used in numerical experiments below.

ADJOINT BOUNDARY PROBLEM

Let us consider the standard adjoint problem corresponding to (1)-(3) defined in the same time-spatial area $\Omega\times[0,T]$:

$$\frac{\partial^2}{\partial t^2}\varphi = div(c^2\nabla\varphi) + g, \quad (7)$$

$$\varphi(t = T) = 0, \ \ \frac{\partial\varphi}{\partial t}(t = T) = 0, \quad (8)$$

$$u\big((x, y) \in \partial\Omega\big) = T, \quad (9)$$

where $g(x, y, t)$ is a source function for adjoint differential operator usually considered as a misfit data $\Delta d = d_{obs} - d_{cal}$ or a part of it.

The adjoint operator is traditionally used in solving linear problems in finite spaces with gradient method whenever the gradient is proportional to adjoint operator as in this case.

INVERSION ALGORITHM

We are using the standard inversion approach proposed by Tarantola (Tarantola, 1984) which is well developed in his papers and used by many other researchers.

For this we are considering the following integral $I_{t_0}(x, y)$ as a gradient of the velocity field $c^2(x, y)$ with the misfit function φ:

$$I_{t_0}(x, y) = \int_0^{t_0}(\nabla u, \nabla\varphi)dt, \quad (10)$$

where $[0,t_0]$ is a window in time in which φ is defined as

$$\varphi = \begin{Bmatrix}\Delta d, & t < t_0 \\ 0, & t \geq t_0\end{Bmatrix}. \quad (11)$$

Minimizing this gradient on the special subarea of Ω (defined below) we are trying to obtain the local minimum of the misfit Δd which is close to the global one as soon as inverse problem is ill-posed and very sensitive to all kind of noise and global minimum is difficult to get.

The iterative minimization problem can be formulated as the following:

$$min_{(x,y)\in\partial\Lambda}\left|I_{t_0}(x, y)\right|^2, \quad (12)$$

where Λ is a subarea of Ω far from the source point (x_0,y_0) in which $I_{t_0}\big((x, y) \in \Lambda\big) < \varepsilon$ for and on the border of Λ $\left|I_{t_0}\big((x, y) \in \partial\Lambda\big)\right| \geq \varepsilon > 0$. In other words we are considering the set of independent grid points on which the projection of Hessian function is close to diagonal (there are no guarantee that the diagonal is zero-vector) enough and the problem locally behaves like a set of independent problems for each grid point of $\partial\Lambda$, standard first order gradient method automatically gives us a good approximation accuracy. In numerical experiments Λ is just a set of spatial coarse grid points and can be easily determined.

The process is iterative, the minimization problem is very non-linear and the interval $[0, t_0]$ is extended monotonically as iterations proceed to involve more information about misfit data Δd.

In this minimization problem grid scaling plays a key role because we consider the velocity field function $c^2(x, y)$ defined on the coarse grid and both u and φ are defined on a much finer grid (Figure 1). This gives us an opportunity to approximate the source function f and generate forward and adjoint solutions which are able to detect all gradients of the velocity field defined on the coarse grid. In this case a higher frequency source f is more preferable over lower frequency sources.

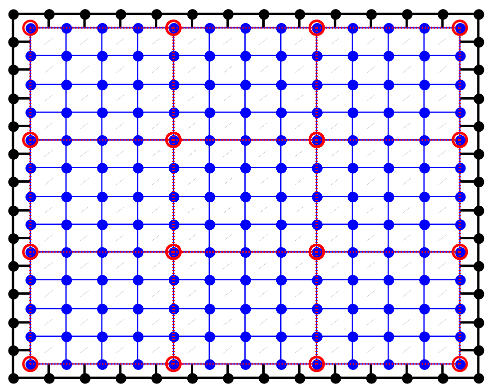

FIG 1. Regular grids used in grid scaling: Black –grid for wave propagation (u and φ); Blue – fine grid for velocity field used c^2 in finite-difference approximation of (1); Red – coarse grid for c^2 used in gradient minimization (12)

As we see from the figure, velocity $c^2(x, y)$ is originally defined on the coarse grid and then interpolated to the fine grid to be applicable in approximating (1) and (6). This also ensures the velocity field is smooth.

Minimization is obtained by gradient method with fixed step implemented on a coarse sub-grid ($\partial\Lambda$). The fixed-step gradient method has advantages over other search methods – it is very fast and effective due to high nonlinearity of the posed inverse problem and due to high sensitivity of the misfit data to the changes in the velocity field. Excessively large steps can result in unnecessary fluctuations in velocity and affect the convergence as well as accuracy.

NUMERICAL EXPERIMENT

In this practical part we combine sources of different frequencies together imaging different sub-domains of given spatial computational domain with synthetic data. For this, we will make a set of basic assumptions which determine the problem and define the way to solve it numerically:

- The computational area consists of an air layer and a soil layer which defines the topography of the ground.
- The acoustic model in air layer is known and determined with velocity field;
- The very first layer in soil is also known as soon as it is considered physically observable and set of virtual geophones (observation points) is put on the upper border of this layer (on the surface of the ground);
- High frequency source is located in the middle of known first layer in soil and source signature is known;
- The velocity field in soil is determined on a locally refined rectangular grid which consists of two regular grids of different resolution put together such that upper part of the soil is determined on a finer grid and lower part on a coarser grid (Figure 2);
- The velocity-coefficient inverse problem is solved separately in each sub-domain to decrease a total computational expense;
- Low frequency source is located in the area observable with high frequency source inversion and its signature is also known;
- a minimal density of virtual geophones for both inversion problems on fine and coarse grids is taken the same as corresponding velocity grid size but for a quicker convergence (with synthetic data approximation error is equal to zero and convergence rate is the only critical to density) it is considered higher than minimal;

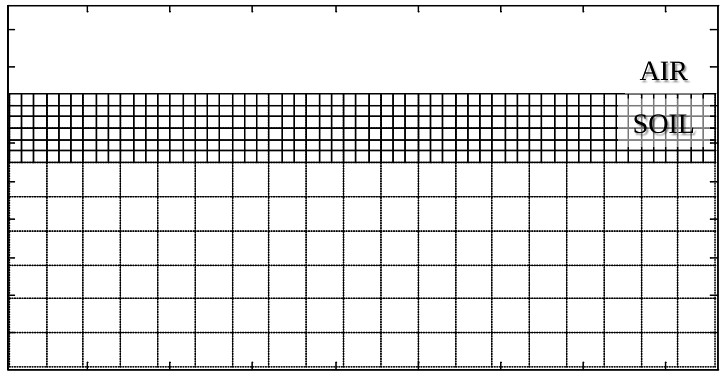

FIG 2. Spatial grid for velocity coefficient problem defined in soil layer.

Based on key assumptions listed above, numerical experiment is run in the following way:

1. For a given exact velocity propagation field (Figure 3) and some initial approximation of the velocity field (Figure 4), given source function (Figure 5) in upper sub domain of the soil (which plays the role of weathered layer) with fine grid of velocity field as well as corresponding waves propagation, the imaging problem is solved with 7 high frequency point sources regularly distributed along the subsurface layer (Figure 6). It is important to put the source in the known velocity field and not to put it directly on the surface (in this case most of energy goes into the air, the accuracy of imaging as well as the convergence rate decrease significantly) on the other side. All 7 imaging problems for high frequency sources are solved together via domain decomposition with overlapping.

2. After the successful convergence of the velocity field in upper layer to some approximation of the exact velocity (Figure 7) we put low frequency source somewhere in already known velocity field and solve the problem for lower soil layer with coarse spatial grid of velocity (Figure 8).

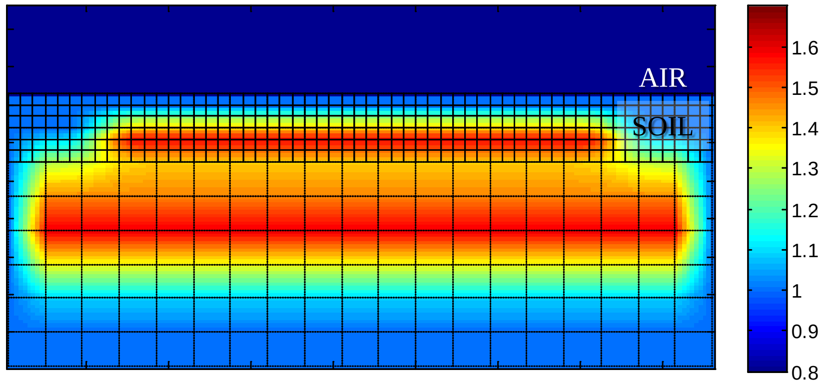

FIG 3. Exact velocity field $c^2_{EXACT}(x,y)$.

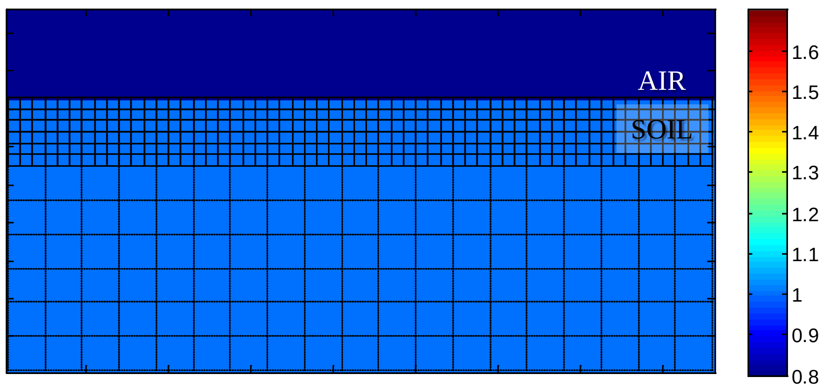

FIG 4. Initial approximation of the velocity field $c^2_{APPRX}(x,y)$.

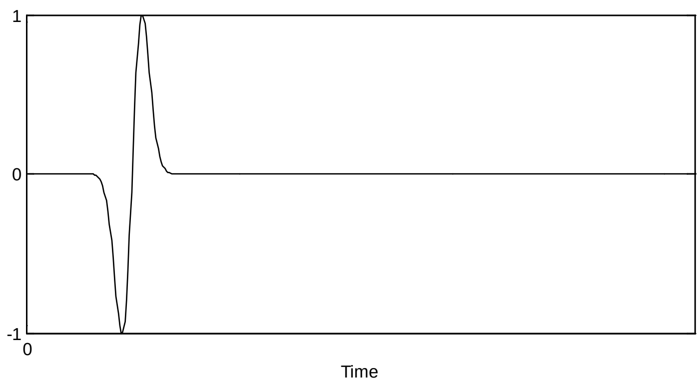

FIG 5. Time component of the impulse point source $f(x,y,t)$.

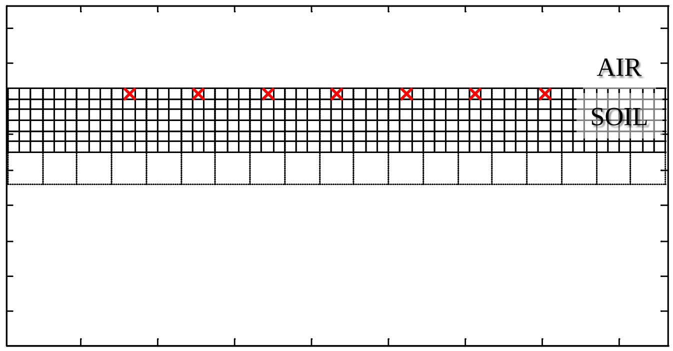

FIG 6. High frequency sources distribution and corresponding velocity field sub grid with overlapping.

From the Figure 6 we see that each inversion problem of 7 considered on the fine grid due to domain decomposition should proper overlap not only with its neighbours from sides but also with the coarse grid sub domain. As a result, one row of the coarse grid is attached to each fine grid sub domain.

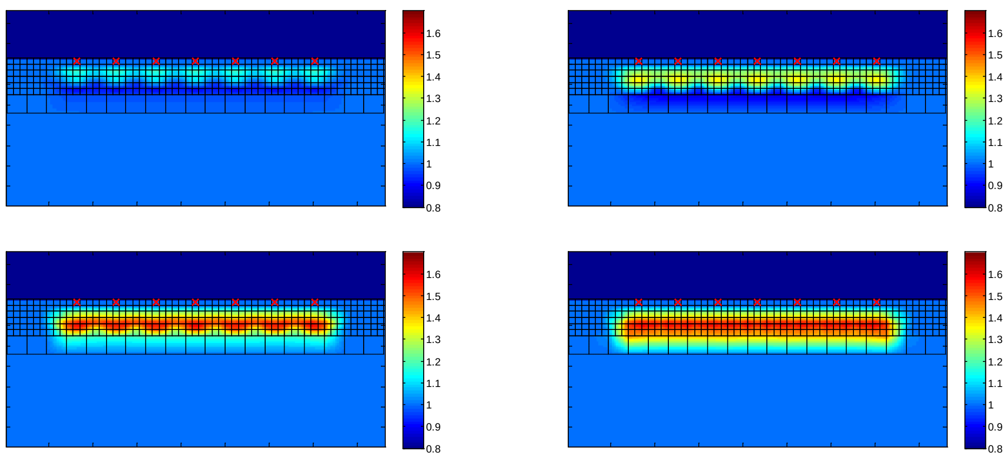

FIG 7. The convergence of an approximate solution $c^2_{APPRX}(x,y)$ on a fine grid.

Based on Figures 7 and we can numerically confirm that the approximate solution $c^2_{APPRX}(x,y)$ converges to the projection of $c^2_{EXACT}(x,y)$ on the fine grid.

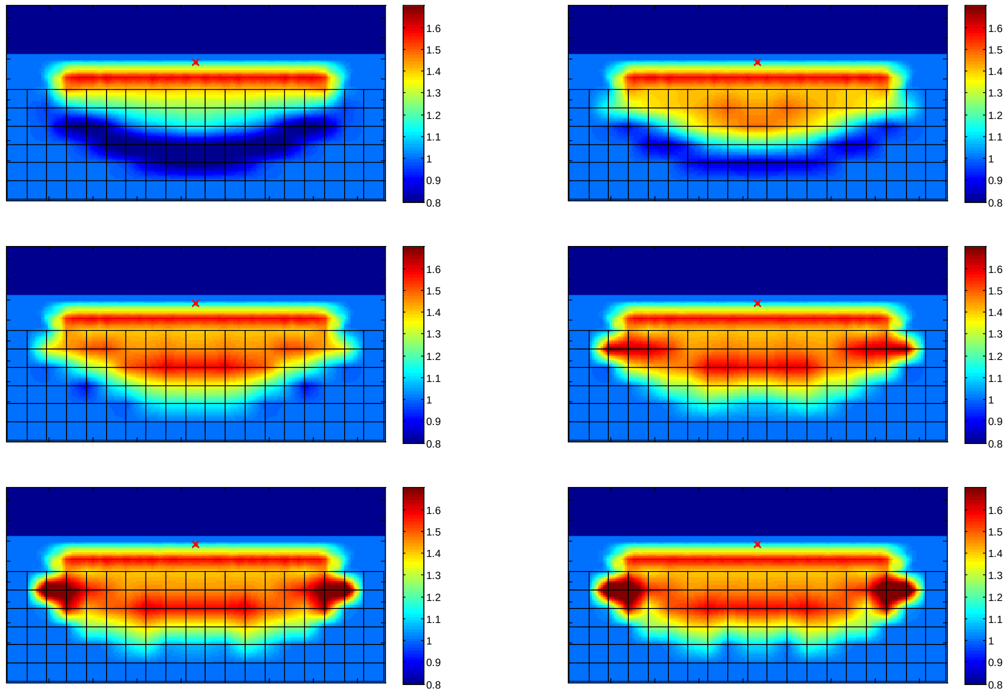

FIG 8. The convergence of an approximate solution $c^2_{APPRX}(x,y)$ to $c^2_{EXACT}(x,y)$ on a coarse grid.

Comparing the convergence results presented in Figure 8 with exact velocity field presented in Figure 3, we can conclude that approximate velocity successfully converges to the exact velocity almost without oscillations. All techniques implemented in this study work perfect mostly due to simplicity of the considered inversion problem and indicate the potential of their applicability for the elastic equation.

The total number of iterations depends on grid resolution, gradient method stepping limitations and fine grid scaling. In this particular numerical experiment there where performed about 5000 iterations of gradient method.

CONCLUSION

The multi-scaling approach performed together with the domain decomposition method shows its effectiveness and numerical stability while applied to acoustic equation inverse coefficient problem. The ability to combine different scaling grids together via overlapping allows solution of inversion problems for high and low-frequency sources simultaneously on multi-core computer and increases the convergence rate. A higher resolution grid defined on surface layer increases accuracy of deeper layers velocity coefficient inversion.

High frequency source inversion is considered as a precondition of the low-frequency source problem. On the other hand, low-frequency source inversion can be used as high-frequency source post-processing or in some inversion verification procedure. A number of sources of different frequencies processed together can significantly decrease the total computation cost of inversion on the one hand, and provide the reasonable accuracy of the approximate velocity field on the other hand.

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