More numerical experiments in high frequency edge diffraction theory

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ABSTRACT

Formulae related to the diffraction of seismic waves by linear edges in elastodynamic media, based on an extension of the high frequency, zero order Asymptotic Ray Theory (ART) formulation are presented. Theoretical aspects of the problem have been minimized, as these have been developed in numerous notable works. The intention here is to present the basic methodology for numerical implementation into synthetic seismogram software.

Schematics indicating relevant details such as the shadow boundary and the boundary ray are have been included. The identification of the boundary ray is required as the argument of the diffraction coefficient is dependent on the angle between the shadow boundary ray and the diffracted ray or equivalently the difference between the diffracted and direct arrival times. The direct geometrical arrival does not exist in the shadow region and its travel time is required to determine the argument of the diffraction coefficient which is done within the context of analytic continuation and the aforementioned limits of minimal theoretical discussion.

A basic problem is considered to give a brief overview of the theory of edge diffractions. The geometry of this problem involves a wedge embedded in a halfspace in a manner such that the plane of incidence is the (r,z) plane and the wedge is such that its leading edge is perpendicular to this plane, i.e., parallel to the y axis. Both the source and receivers are located in the (r,z) plane.

INTRODUCTION

The work of Klem-Musatov (1980) dealing with elastic waves diffracted by the linear edges of seismic interfaces, using an extension of zero order Asymptotic Ray Theory (ART) (Červený and Ravindra, 1970 and Červený, 2001, among others) was employed to obtain a high frequency approximation for elastodynamic waves diffracted by linear edges. The method presented uses modifications of ART incorporating the boundary layer method. This theory is applicable to a three dimensional case of rays, emanating from a point source, propagating in a geological model with homogeneous layers separated by curved interfaces. The Russian text by Klem-Musatov (1980), once relatively inaccessible may be found now in the English translation, Klem-Musatov (1995) as well as the translation of a more recent work, Klem-Musatov et al. (2007). A dissertation based on the original text (1980) is the topic of the Ph.D. thesis of Chan (1986), where many of the subjects contained in that book are discussed. The papers by Bakker (1990), Hron and Chan (1995) and Gallop and Hron (1997) pursue some aspects of the theory presented in the works of Klem-Musatov. The work of Bakker (1990) approaches the edge diffraction problem using a paraxial or Dynamic Ray Tracing (DRT) (Červený and Hron, 1980) approximation. Hron and Chan (1995) arrive at essentially the same results using a more conventional ART modified by the inclusion of boundary layer theory. As stated above, it is the intent of this paper to engage in a minimal amount of theoretical development and employ previously derived formulae to introduce concepts in a form intended for numerical implementation. An extensive review of the literature would suggest for the purposes of this work the paper by Klem-Musatov and Aizenberg (1984) would be the most useful as the theory is presented in a compact and understandable manner.

Formulae for use in the computation of numerical results would ideally be such that (a) they would allow for simple physical interpretation, (b) provide a practical means for the efficient calculation of the diffracted field and (c) in light of (b), also maintain a reasonable degree of accuracy when describing the diffracted wavefield. The ART approximation to the edge diffracted arrival consists of the zero order asymptotic expression for the incident wavefield at the diffractor, due to a point source, multiplied by what was termed a diffraction coefficient. The resulting diffracted wavefield was obtained by considering the diffractor as a secondary source of seismic energy and tracing its path to receivers in accordance with a modified form of Snell's Law and zero order ART. The theory may be referred to as 2.5*D* due to the fact that out of plane geometrical spreading is included.

The accuracy of a method similar in development to the one described above was determined in the work of Chan (1986) where an approximate method for SH waves was compared with the highly numerically accurate results obtained by the Alekseev Mikhailenko method (AMM), a pseudo-spectral method (see for example, Mikhailenko, 1985 and 1988). This same procedure was used to test the program of the ART type results presented here and in use for several decades.

The problem considered here will be defined in the next section along with some comments and a restatement of the final formulae required for computation of the diffracted wavefield due to an edge diffractor will be given.

The model that will be considered in the next section is shown in Figure 1 consisting of a wedge that is infinite in the y direction with ray propagation constrained to the (r,z) plane. The receivers in both geometries are along a vertical receiver profile. Another model of a similar type will be introduced when computing numerical results.

THEORY

Consider a three dimensional Cartesian coordinate system (r,y,z), in an isotropic homogeneous elastic halfspace $z>0$ with an explosive point source of compressional (P) waves located at the origin, (Figure 1). An infinite wedge is assumed to be located at a depth $z_{\mathcal{D}}$ below the surface, occupying the three dimensional space $(z_{\mathrm{D}} \le z < \infty\ ,\ 0 \le r \le r_{\mathrm{D}}, -\infty < y < \infty)$. Receivers are placed in a vertical array in the (r,z) plane containing the source. The direct ray from the source to the receiver, the ray transmitted through the wedge and the diffracted P wave from the edge at $\mathcal{D} = (r_{\mathcal{D}}, z_{\mathcal{D}})$

are the only arrivals considered in the resulting synthetic seismograms. The compressional (P) wave velocities in the half plane, α_1, and in the wedge strip, α_2, are chosen such that $\alpha_2 > \alpha_1$.

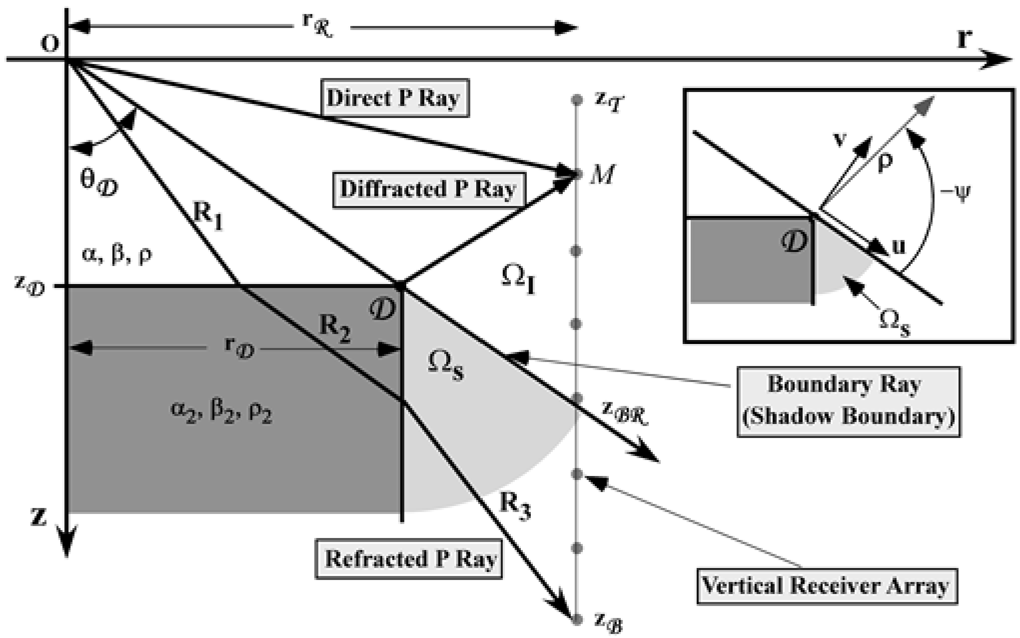

Fig 1. Geometry of Model 1 for a point source and a vertical array of receivers. The wedge (dark gray) occupies the $3D$ space $(z_{\mathcal{D}} \le z < \infty\ ,\ 0 \le r \le r_{\mathcal{D}}, -\infty < y < \infty)$. The point of edge diffraction is in the (r, z) plane at $(r_{\mathcal{D}}, z_{\mathcal{D}})$ Ray propagation is assumed to lie in this plane. The shadow region Ω_S (light gray) for the direct ray is defined by the boundary ray. The direct arrivals at a vertical receiver, $M(z)$, such that $(z_T \le z \le z_B)$ are seen on the synthetic traces. The diffracted arrival appears at all depths.

When considering diffraction from the wedge there are two regions to be considered in the (r, z) plane, the illuminated (Ω_I) and the shadow (Ω_S) regions (Figure 1). The illuminated and shadow regions are separated from one another by what is termed the "boundary ray". In the uncomplicated situation being considered here the direct arrival only appears on the synthetic traces in the illuminated regions, the transmitted ray is present only in the shadow region, while the diffracted arrival exists at the receivers in both regions.

The Fourier time transformed vector particle displacement of due to a compressional wave generated by a point source at the origin and recorded at the vertical receiver array may be written in terms of the zero order ART approximation as

$$\mathbf{U}_G(\mathbf{r},\omega)=\frac{F(\omega)\Pi}{L_G(\mathbf{r})}\exp\left[i\omega\tau_G(\mathbf{r})\right]\mathbf{e} \tag{1}$$

where the subscript "G" (geometrical) indicates either the direct or transmitted P arrival, $\mathbf{r}$ is the generally three dimensional position vector of some point in the halfspace that may, when convenient, be denoted as M, ω is the circular frequency, $\tau_G(\mathbf{r})$ $\left[\tau_G(M)\right]$ is the travel time along the ray from the source to a point $\mathbf{r}$ in the vertical receiver array. The quantity $L_G(\mathbf{r})$ $\left[L_G(M)\right]$ is the three dimensional geometric spreading of the ray between the source and the point $\mathbf{r}$ which for the direct arrival is given by

$$L_G(M)=\left[r_{\mathcal{M}}^2+z_{\mathcal{M}}^2\right]^{1/2}. \tag{2}$$

and the direct ray arrival time may be written as

$$\tau_G(M)=\frac{\left[r_{\mathcal{M}}^2+z_{\mathcal{M}}^2\right]^{1/2}}{\alpha_1}. \tag{3}$$

The term Π in equation (1) is the product of all reflection and transmission coefficients encountered along the geometrical ray from source to receiver. In the direct ray case, $\Pi=1$, as it does not interact with any interface. The unit vector $\mathbf{e}$ is a vector which partitions the incident particle displacement at a receiver into its constituent vertical and horizontal components. $F(\omega)$ is the Fourier time transform of the band limited source wavelet, $f(t)$, t being time. A spherically symmetric radiation pattern of the source function is assumed.

The particle displacement of the ray transmitted through the wedge is formally given as

$$\mathbf{U}_G^{(T)}(\mathbf{r},\omega)=\frac{F(\omega)\Pi}{L_G^{(T)}(\mathbf{r})}\exp\left[i\omega\tau_G^{(T)}(\mathbf{r})\right]\mathbf{e}_G^{(T)} \tag{4}$$

where Π is the product of the two PP transmission coefficients, at the top of the wedge, into the wedge and from the wedge flank back into the halfspace. The geometrical spreading for this ray is of the form (Červený and Ravindra, 1970)

$$\frac{1}{L_G^{(T)}(\mathbf{r})}=\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]. \tag{5}$$

The related travel time is

$$\tau_G^{(T)}(\mathbf{r}) = \frac{R_1}{\alpha_1} + \frac{R_2}{\alpha_2} + \frac{R_3}{\alpha_1} \tag{6}$$

where the lengths $R_i\ (i = 1,2,3)$ are defined in Figure 1. It is of note that in the limit as $R_2 \to 0$, the transmitted arrival tends to the boundary ray.

The travel time of the diffracted arrival from source to receiver consists of two parts; the time it takes the ray to travel form the source to the diffraction edge in the (r,z) plane at $(r_{\mathcal{D}}, z_{\mathcal{D}})$ plus the time taken for the ray to progress from the diffraction point $(\mathcal{D})$ to the receiver at *M*. This infers that a diffracted arrival generally has a different ray parameter (horizontal component of the slowness vector) on either side of the edge diffraction point.

From observations of field data and numerical results obtained from numerical modeling with finite difference methods a number of attributes of edge diffracted arrivals may be realized; (a). the diffracted wavefield is frequency dependent and (b). angular dependent measured at some unit distance about the diffraction point on the diffracting edge and (c). the point on the diffracting edge appears to act as a secondary source. A pseudo solution may be written in terms of the reflected arrival at some distance about the point $\mathcal{D}$, which amounts to multiplying the reflected arrival at the point $\mathcal{D}$ by some function, $I(\omega,\psi)$ yet to be determined. This function is assumed to be a frequency (ω) and angular (ψ) dependent in the (r,z) plane. Based on empirical observations, the particle displacement of the diffracted arrival may be written as

$$\mathbf{U}_{\mathcal{D}}(\mathbf{r},\omega) = \frac{F(\omega)I(\omega,\psi)\Pi}{L_{\mathcal{D}}(\mathbf{r})}\ \exp\left[i\omega\tau_{\mathcal{D}}(\mathbf{r})\right]\mathbf{e}. \tag{7}$$

The three dimensional geometrical spreading $L_{\mathcal{D}}(\mathbf{r}) = L_{\mathcal{D}}(M)$ is the sum of the spreading from the source to the point of diffraction plus the addition of the spreading from this point to M .

Another Cartesian coordinate system, (u,v), whose origin is at $\mathcal{D}$, together with a related polar coordinate system, (ρ,ψ) are convenient to be introduced. Here, ψ is positive/negative in the shadow/illuminated regions which are defined by the boundary ray. The quantity ρ_M is the distance from the point of diffraction at $\mathcal{D}$ to the observation point M fully defined by the additional coordinate, ψ_M within the context of the Cartesian system (u,v), (Figure 1).

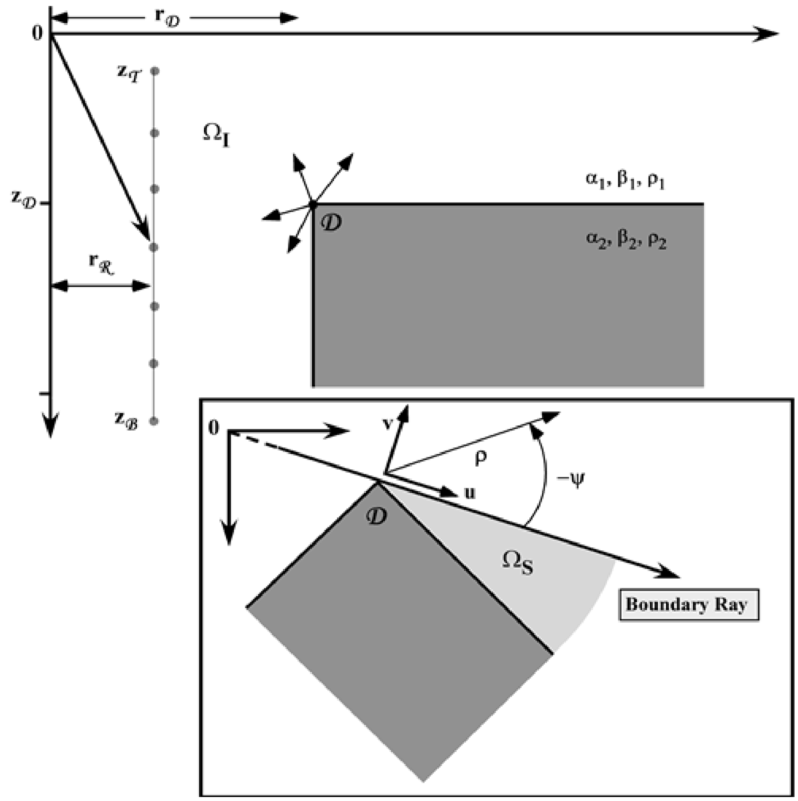

Fig. 2. Geometry of Model 2 for a point source and a vertical array of receivers. The insert shows the introduction of a boundary ray where one apparently does not exist.

With the above Cartesian and polar coordinate systems definitions, the function $I(\omega,\psi)$ will be replaced in Equation (4) by a more general related function, $\mathrm{W}(\check{z})$ with $\check{z} = \check{z}\left(\omega, w\left(\omega, \rho_M, \psi_M\right)\right)$, termed the diffraction coefficient.

The diffraction coefficient requires the solution of a Cauchy type integral which leads to what in diffraction theory has come to be known as the Sochotskiy-Plemel[1] problem (Rektorys, 1969).. The text by (Rektorys, 1969) deals with this in possibly the most rigorous manner of all the works of complex variable theory that could be mentioned. This coefficient is derived in several of the previously cited works, with varying degrees of complexity to which interested readers are referred, as its inclusion here would introduce unnecessary length and be redundant.

The diffracted $P-$wave vector particle displacement at the point M has the form

$$\mathbf{U}_{\mathcal{D}}\left(M,\omega\right)=\frac{F\left(\omega\right)\mathrm{W}\left(\check{z}\right)\Pi}{L_{\mathcal{D}}\left(M\right)}\exp\left[i\omega\tau_{\mathcal{D}}\left(M\right)\right]\mathbf{e}\,. \tag{8}$$

The geometrical spreading $L_{\mathcal{D}}\left(M\right)$ and travel time $\tau_{\mathcal{D}}\left(M\right)$ of the diffracted $P-$wave arrival may be written as

$$L_{\mathcal{D}}\left(M\right)=\left[z_{\mathcal{D}}^{2}+r_{\mathcal{D}}^{2}\right]^{1/2}+\rho_{M} \tag{9}$$

and

$$\tau_{\mathcal{D}}\left(M\right)=\frac{\left[z_{\mathcal{D}}^{2}+r_{\mathcal{D}}^{2}\right]^{1/2}}{\alpha_{1}}+\frac{\rho_{M}}{\alpha_{1}}\,, \tag{10}$$

respectively. In addition, $\check{z}=\check{z}\left(\omega,w_{M}\right)$ and $w_{M}=w\left(\omega,\rho_{M},\psi_{M}\right)$, with ω being the circular frequency and

$$\mathrm{W}\left(\check{z}\right)=\pm e^{-\check{z}^{2}}erfc\left(-i\check{z}\right),\quad \text{with}\quad \check{z}=e^{i\pi/4}\sqrt{\omega/2}\,w \tag{11}$$

with the "$-$" and "$+$" signs associated respectively with the illuminated and shadow regions, and $\mathrm{W}\left(\check{z}\right)$ is the scaled complementary error function. For small values of $\check{z}$

$$\mathrm{W}\left(\check{z}\right)\approx\mathrm{W}\left(0\right)+\frac{i\check{z}}{\sqrt{\omega}},\quad \mathrm{W}\left(0\right)=\frac{1}{2} \tag{12}$$

and the asymptotic expansion of $\mathrm{W}\left(\check{z}\right)$-for large values of $\left|\check{z}\right|$, $\left|\arg\left(\check{z}\right)\right|<3\pi/4$ is

(1) Gabor wavelet: $f\left(t\right)=\sin\left(2\pi f_{0}t\right)\exp\left[-\left(2\pi f_{0}t/\gamma\right)^{2}\right]$ where γ is a dimensionless damping that controls the amplitude spectrum width in the frequency domain and the side lobes of the wavelet in the time domain.

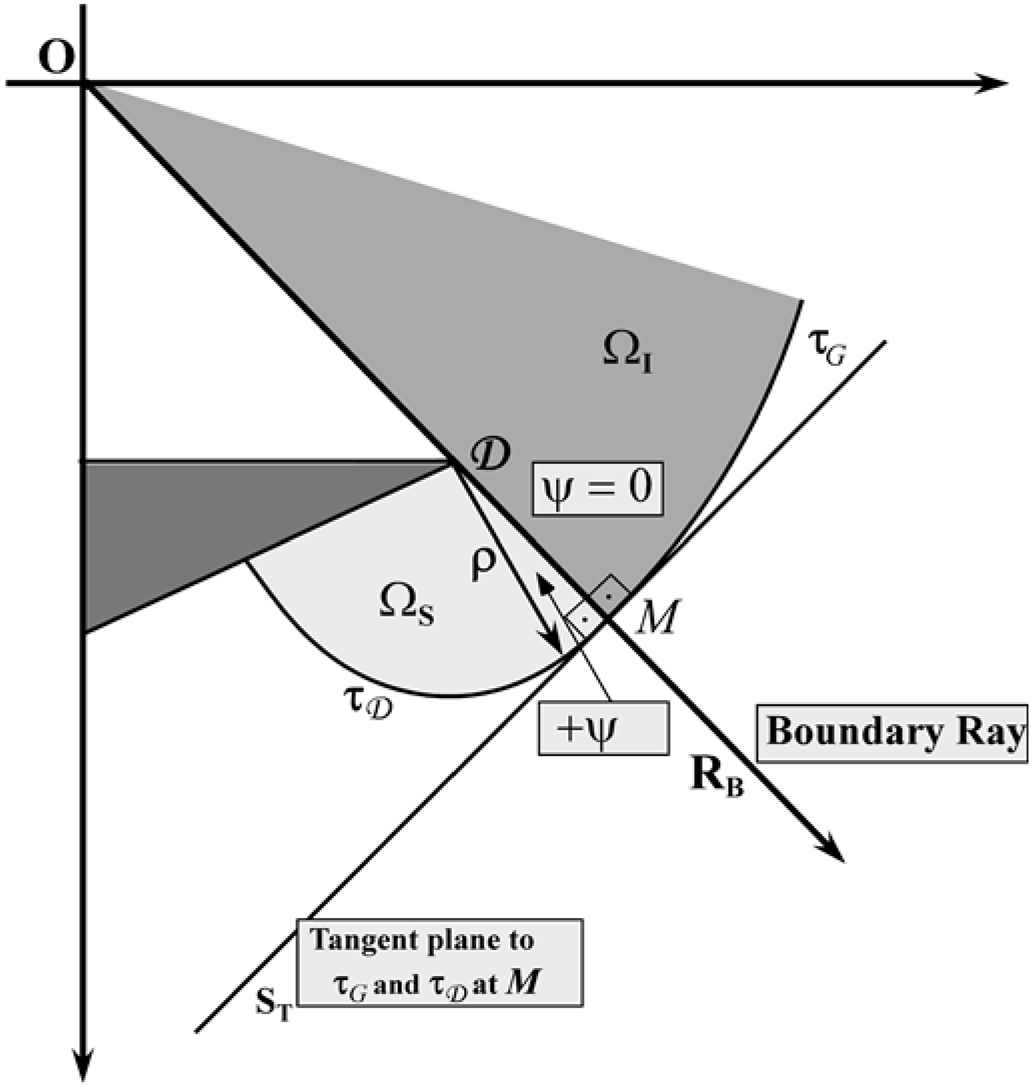

Fig. 3. Analytic continuation of the geometrical wavefield into the shadow region to obtain some simulated measure of the geometrical arrival time in this region. A more complete discussion may be found in Appendix A.

$$\mathrm{W}(\check{z}) \approx \mathrm{i}\big/\sqrt{\pi}\left(e^{i\pi/4}\sqrt{\pi/2}\,w\right) = e^{i\pi/4}\Big/\sqrt{\pi\omega\left(\tau_{\mathcal{D}} - \tau_R\right)}\,, \tag{13}$$

indicating that the amplitude of the diffracted arrival is of the order $O\left(1\big/\sqrt{\omega}\right)$ for large values of the argument of $\check{z}$. The meaning of the quantity w is found by studying the works dealing with solution method for edge diffracted waves and is defined as

$$w^2 = \begin{cases} (a). \quad \dfrac{2\omega}{\pi}\left(\tau_{\mathcal{D}}\left(M\right)-\tau_G\left(M\right)\right) & - \text{ illuminated zone} \\ (b). \quad \dfrac{2\omega}{\pi}\left(\dfrac{\rho_M}{\alpha_1}\left(1-\cos\psi_M\right)\right) & - \text{ shadow zone} \end{cases} \tag{14}$$

Equation (14.b) for w^2 results from the fact that at points along the vertical receiver array no direct arrival exists in the shadow zone. Equation (14.b) was obtained using analytic continuation. Appendix A contains a discussion of its derivation in a simplified manner avoiding excessive mathematical rigor. For anyone deciding to pursue work in this area a Cauchy type integral problem known as the Sochotskiy-Plemel integral (Rectorys, 1969) should be given more than cursory consideration. Figure 3 depicts schematically the discussion presented in Appendix A. As before, the compressional wave velocity in the halfspace, except for the wedge, is α_1.

NUMERICAL RESULTS

Parameters defining the the geological model used in this report are given in Table 1. Additional quantities required for model definition are the distance from the surface to the tops of the wedges, $h = 400m$, the horizontal distance from the source location to the points of diffraction, $r_{\mathcal{D}} = 1000m$ and the horizontal distance from the source to the points the boundary ray in. These together with a time scale and a brief description of what is shown in a given figure appear on all of the synthetic seismograms. A Gabor wavelet[(1)] is used when producing the synthetic traces. The function $\mathrm{W}\left(\check{z}\right)$ is a standard function available in most mathematical libraries such as IMSL, where it is denoted as *ZERFE* (Gautschi, 1979a and 1979b).

Schematic of the two model used are given in Figures 1 and 2 where additional information may be found in the caption. The synthetic seismograms presented include both the vertical and horizontal components of displacement along the vertical line of receivers composed of an elastic halfspace with an embedded wedge. To keep matters simple only the direct and diffracted arrival are shown in the synthetics. The transmitted arrival through the wedge or reflected arrival from the flank of the wedge is not included.

Figures 4 through 7 are associated with Models 1 and 2 described in Table 1 and display the vertical and horizontal components of particle displacement. There are 3 panels in each of the Figures 4 – 7 showing: (a). the direct arrival, (b). the diffracted arrival and (c). the combination of the two.

CONCLUSIONS

Simple geological models have been used for the investigation of some of the basic concepts of diffraction theory in elastic media were presented. This was done within the framework of asymptotic ray theory (ART) and certain extensions thereof. ART produces results equivalent to those derived using the high frequency geometrical optics solution method and it has been shown in previous numerical experiments to be reasonably accurate when compared to “exact” methods. The models were designed to investigate properties of edge diffraction from a 3D wedge. The introduction of the boundary ray for different geometries and ray types were considered. This ray is such that it defines the illuminated and shadow zones for the reflected arrival. The diffracted ray exists in both regions while the reflected ray exists only in the illuminated region. The formulae for the edge diffracted arrival were obtained from other works cited in the Introduction. The smooth transition of the diffracted arrival across the boundary ray from one region to another was taken as an indication that the formulae being used satisfied at least that constraint. Comparison of the modified ART solution for this problem has been checked for more complex media using finite difference and related methods and as such has not been included here. The diffraction coefficient is a function of the difference of the diffracted and direct travel times. As the direct arrival does not exist in the shadow zone it was necessary to introduce the concept of analytic continuation to provide an appropriate value for this quantity. What has been presented here is not a definitive source of all theory that is required for the introduction of edge diffracted arrivals into synthetic traces for complex structures, but rather a simple introduction to the topic to provide a basis for the numerical implementation of diffraction theory. The presentation here, based on more theoretically complete works, was only for the purpose of providing a foundation for the numerical implementation of diffraction theory into ART type software.

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	P Velocity (km/s)	**S Velocity (km/s)**	**Density (gm/cm^3)**
Wedges I & II	**2.50/1.60**	**1.44/0.92**	**2.20/1.50**
Halfspace	**2.00**	**1.15**	**1.80**

Table 1. Elastic parameters for Models I and II.

APPENDIX A

Assume a point source of compressional (P) waves located at the origin of an isotropic 3 dimensional Cartesian half space in which there is embedded a wedge of infinite dimensions in the y direction (Figure 3). At a time after an impulsive excitation of the point source the direct wavefront will have progressed to the define the wavefront surface at τ_G and the diffracted wavefront, defined by $\tau_{\mathcal{D}}$, begins at the time the direct arrival reaches $\mathcal{D}$, the point of diffraction (Figure X). The ray associated with the point source at the origin which is such that it passes some small distance $\varepsilon : \varepsilon \to 0$ front the point $\mathcal{D}$ and denoted R_B will be called the boundary ray. This ray separates the illuminated zone (Ω_I) from the shadow (Ω_S) zone for the direct wavefront which originates from a point source at some arbitrary origin, O.

In the region of the boundary ray, where $\psi = 0$, $\left[\tau_D(M)\right]_{\psi=0} = \left[\tau_G(M)\right]_{\psi=0}$ and from simple geometrical considerations $\left[\frac{\partial \tau_D(M)}{\partial \psi}\right]_{\psi=0} = \left[\frac{\partial \tau_G(M)}{\partial \psi}\right]_{\psi=0} = 0$ so that a Taylor series expansion of $\tau_G(M)$ in this region in the variable ψ is of the form

$$\tau_G(M) = \tau_D(M)\big|_{\psi=0} + \frac{1}{2}\left[\frac{\partial^2 \tau_D(M)}{\partial \psi^2}\right]_{\psi=0} \psi^2 \quad \text{or}$$
$$\frac{1}{2}\left[\frac{\partial^2 \tau_D(M)}{\partial \psi^2}\right]_{\psi\approx 0} \psi^2 \approx \left[\tau_G(M) - \tau_D(M)\right]_{\psi\approx 0} \quad . \quad \text{(A.2)}$$

The above approximation for the second derivative of τ_D with respect to ψ is inherent in the solution of the Sochotskiy-Plemel problem (Rectorys, 1969) for the determination of the diffraction coefficient.

In the shadow zone the direct geometrical arrival does not exist even though its travel time in this region is required to determine the argument of the diffraction coefficient, $\mathrm{W}(\check{z})$, where

$$\check{z} = e^{i\pi/4}\sqrt{\frac{\omega}{2}}\, w \quad \text{(A.1)}$$

and w is defined (tentatively) by the relation

$$w^2 = \left[\frac{2\omega}{\pi}\left(\tau_{\mathcal{D}}(M) - \tau_G(M)\right)\right]. \quad \text{(A.2)}$$

In the shadow region the travel time of the geometrical arrival to the point M must be determined by analytic continuation of $\tau_G(M_0)$ from the shadow/illuminated boundary into the shadow region. As S_T is the tangent plane to $\tau_G(M_0)$ on the boundary ray R_B it may be interpreted as the local representation of $\tau_G(M_0)$ there, which is to say that from the view point of seismic energy partitioning due to encounters of the ray with interfaces it is identical to a seismic plane wave at that point. From a mathematical point of view the analytic continuation of the travel time along the plane representation of the wave front S_T requires that the travel time is the same along the plane wave front as it is at M_0. The same argument may be used to infer that the amplitude along the plane wave front S_T must also be the same as its value at M_0.

Using the coordinate system (ρ,ψ) defined in the text which is centered at the diffraction point $\mathcal{D}$, assuming that all rays are constrained to lie in the (r,z) plane of the (r,y,z) Cartesian system initially assumed here, the travel time of the direct geometrical arrival on the seismic boundary ray R_B, is $\tau_G(M_0)$ and as a result of the preceding argument

$$\tau_G(M) = \tau_G(M_0) \quad \text{where} \quad M \in S_T . \qquad \text{(A.3)}$$

If τ_1 is the time taken for the direct geometrical arrival to travel from the point source to the edge diffraction point $\mathcal{D}$, then it may be seen from Figure 3 that

$$\tau_{\mathcal{D}}(M) = \tau_1 + \rho_M / \alpha \qquad \text{(A.4)}$$

and

$$\tau_G(M) = \tau_1 + \rho_M \cos\psi_M / \alpha \qquad \text{(A.5)}$$

where α was defined in the text as the P wave velocity in the half space. Substituting equations (A.4) and (A.5) into (A.2) the quantity ž expressed in terms of w may be obtained from

$$w^2 = \left[\frac{2\omega}{\pi}\left(\tau_{\mathrm{D}}(M) - \tau_G(M)\right)\right] = \left[\left(\frac{2\omega}{\pi}\right)\left(\frac{\rho_M}{\alpha}\right)\left(1 - \cos\psi_M\right)\right]. \qquad \text{(A.6)}$$

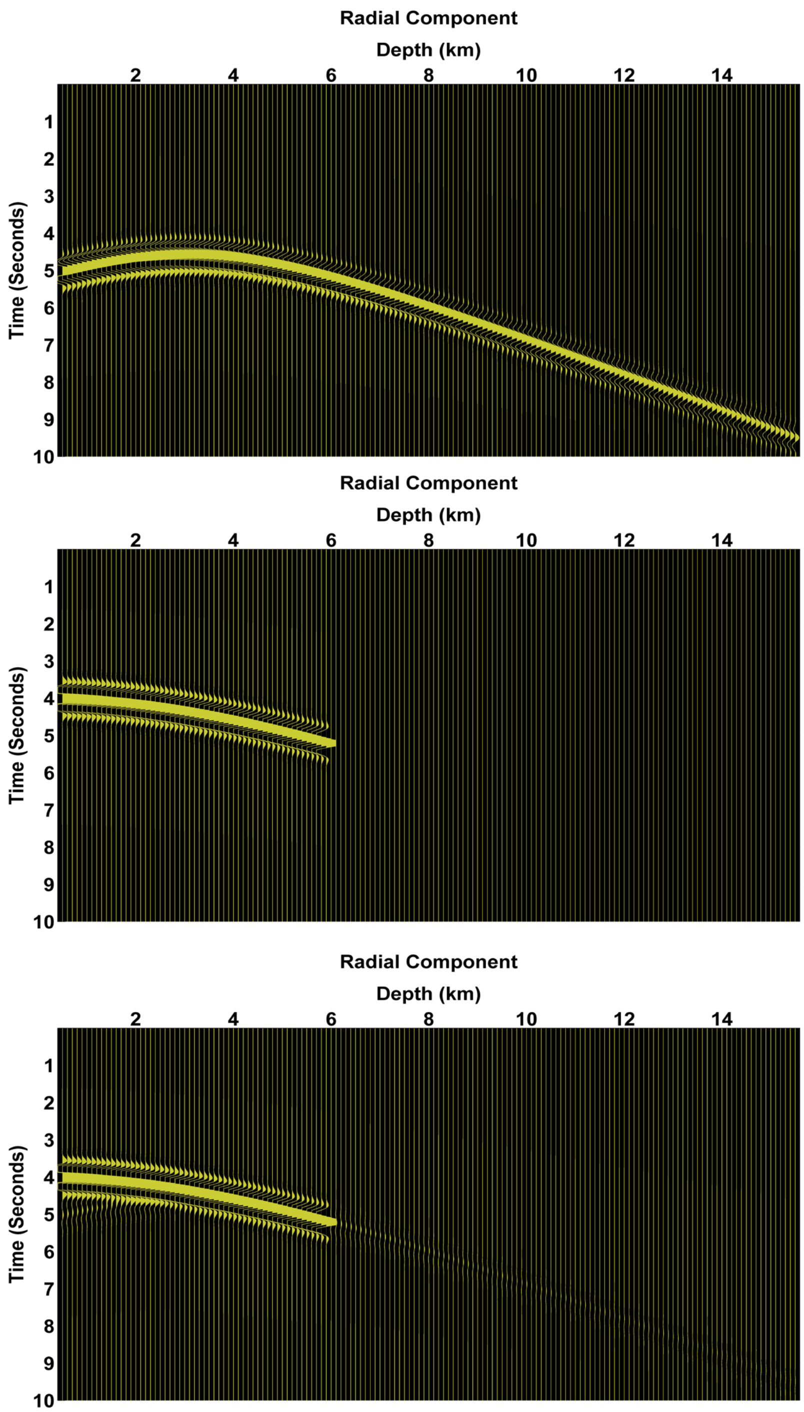

Fig. 4. Radial component of displacements of the (a). diffracted, (b). direct and (c). combined arrivals, for Model I with receivers located at the model surface.

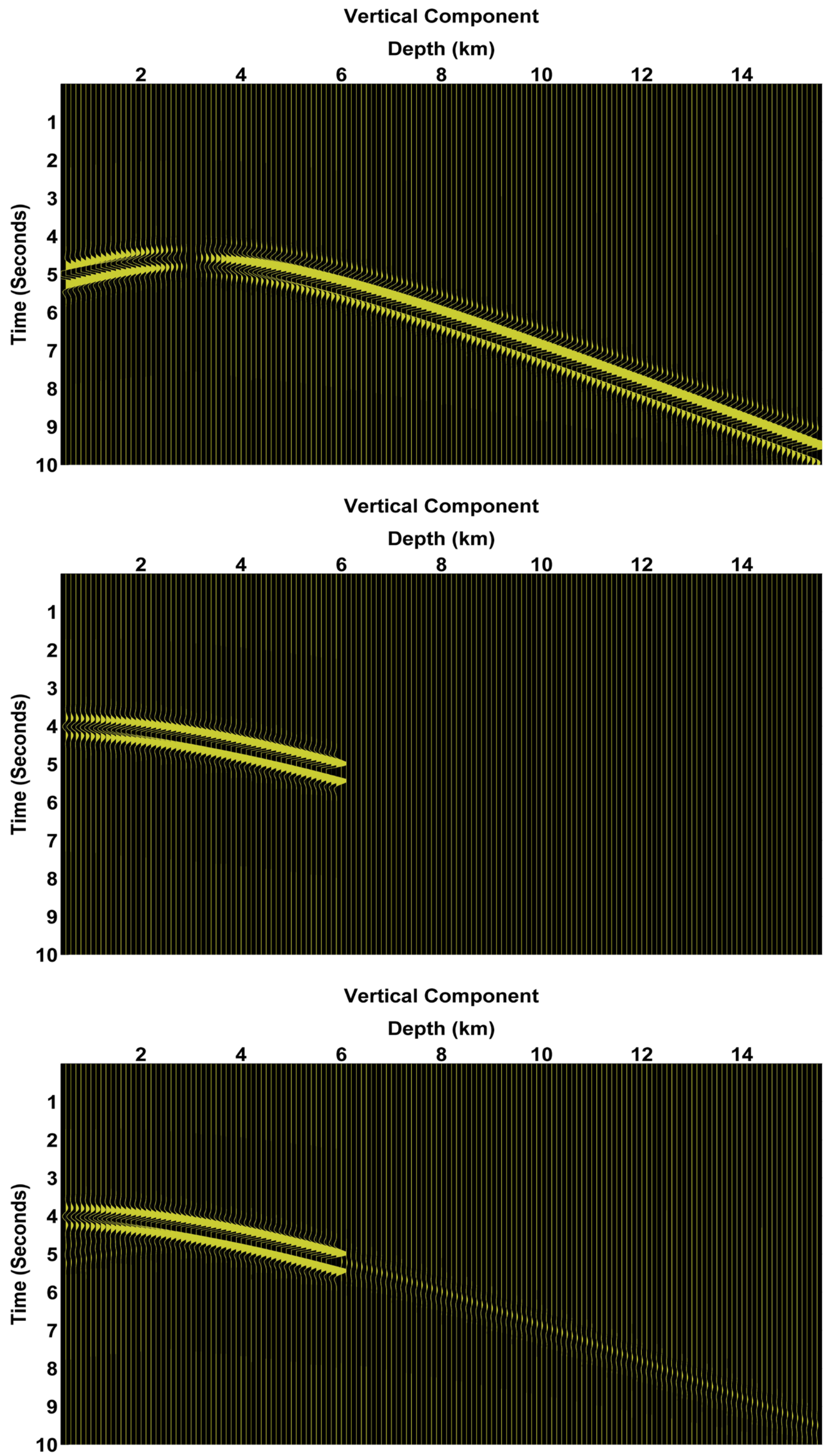

Fig. 4. Vertical component of displacements of the (a). diffracted, (b). direct and (c). combined arrivals, for Model I with receivers located at the model surface.

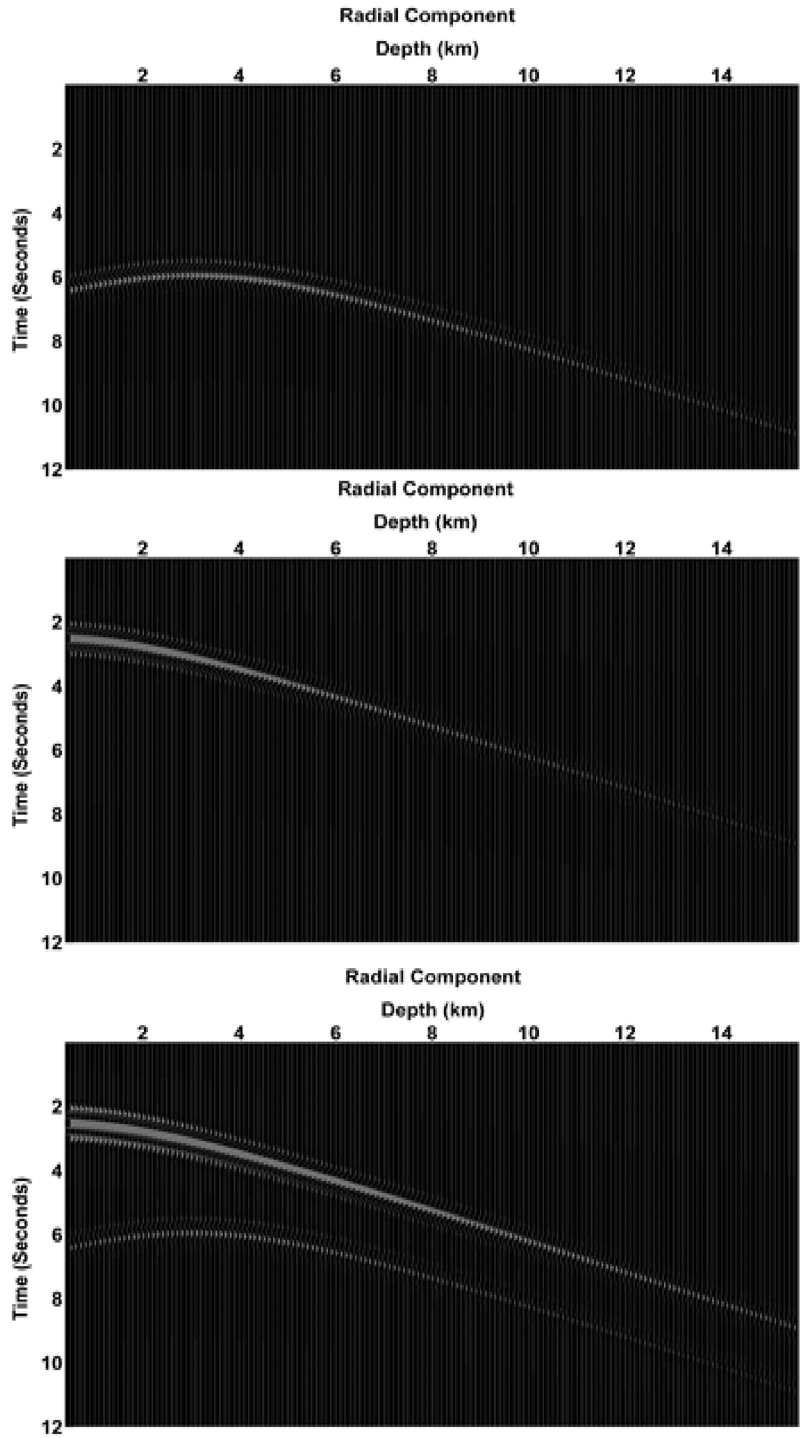

Fig. 6. Radial component of displacements of the (a). diffracted, (b). direct and (c). combined arrivals, for Model II with receivers located at the model surface.

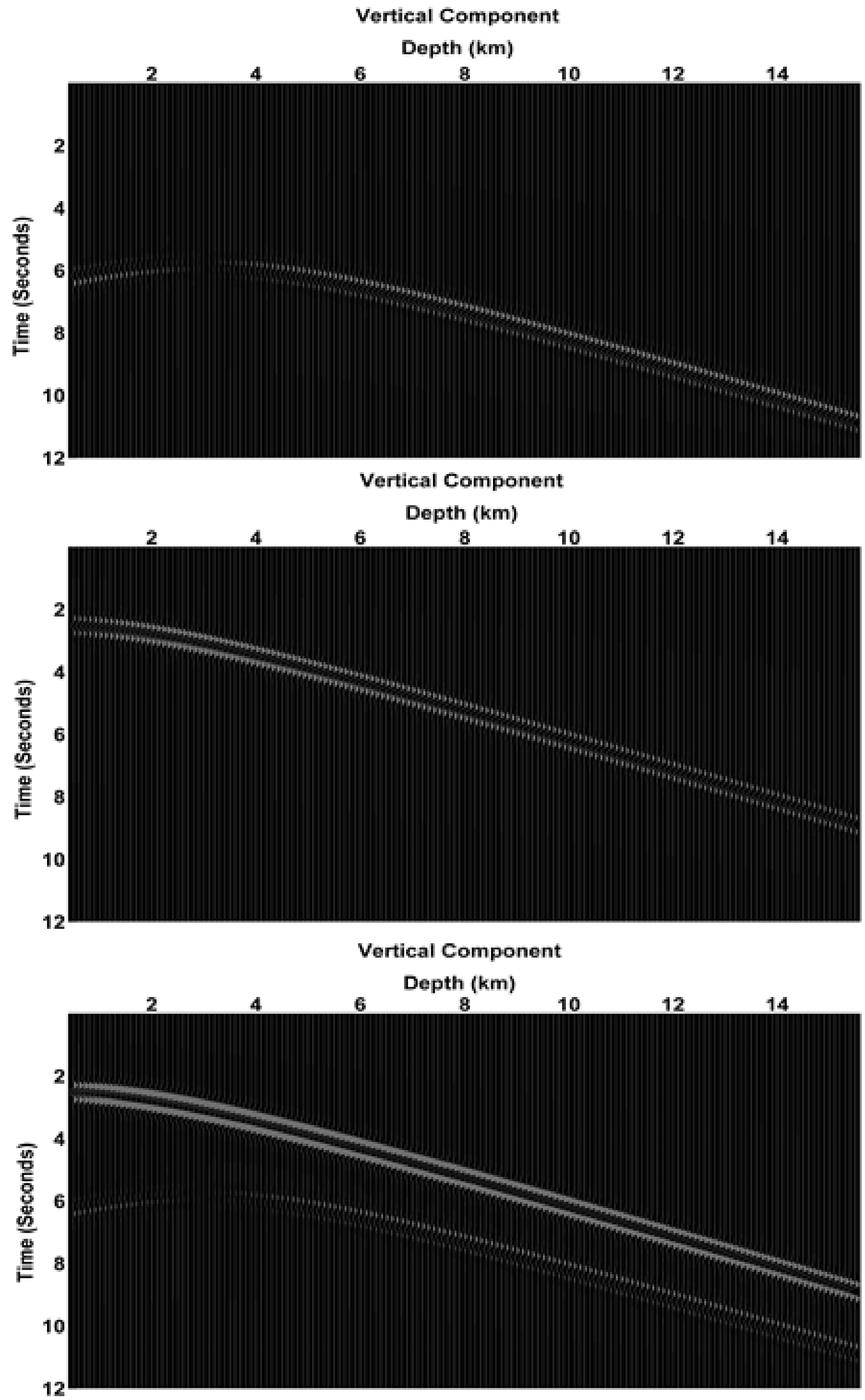

Fig. 7. Vertical component of displacements of the (a). diffracted, (b). direct and (c). combined arrivals, for Model II with receivers located at the model surface.